RECTANGULAR DIELECTRIC-LINED TWO-BEAM WAKEFIELD ACCELERATOR STRUCTURE*

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Abstract
A novel dielectric structure is described for a two-beam wake field accelerator which consists of three or four rectangular dielectric slabs positioned within a rectangular conducting pipe. This structure is equivalent to two symmetric, dielectric-lined, three-zone, rectangular waveguides, joined side-by-side. The design mode in the two-beam structure is the LSM$_{31}$ mode, a combination of two symmetric LSM$_{31}$ modes in each of the three-zone waveguides. This two-channel mode can be employed to decelerate drive particles in one channel and accelerate test particles in the other, without need for additional power extraction/transfer structures. It is possible to find structure parameters that in principle give a ratio $T$, of acceleration gradient for the test beam to deceleration gradient for the drive beam, even as high as $T = 100$.

INTRODUCTION
In this paper, a novel dielectric structure is described for a two-beam wake field accelerator (WFA) that goes beyond past WFA ideas because of its obtainable high ratio $T$ of acceleration gradient for the test beam, to deceleration gradient for the drive beam. The ratio $T$, which is termed here the “transformer ratio” is close to two in the classic wake field model [1], and is usually not greater than about five, even using sophisticated step-up couplers [2]. For accelerated particles to reach a final energy $W_f$ in a two-beam accelerator, the number of accelerator sections $N$ is given by the simple expression $N \geq W_f / (TW_d)$, where $W_d$ is the energy loss per section by each particle in the drive beam. Thus a higher value of $T$ can lower the number of sections, thereby leading to reduced complexity and cost. Another distinctive feature of the structure described here is that rf power produced by the drive beam is directly coupled into the acceleration channel, without need for an array of coupling structures, as for example in CLIC [3]. Moreover, rectangular geometry allows slots to be cut in the top and bottom walls, centered in each channel, both for suppression of undesired modes and for continuous vacuum pumpout.

BASIC PRINCIPLE
The dielectric two-beam WFA structure described here is shown in cross-section in Fig. 1. It consists of four rectangular dielectric slabs positioned within a rectangular conducting pipe. This structure can be thought of as equivalent to two symmetric dielectric-lined three-zone rectangular waveguides, joined side-by-side.

A single three-zone waveguide supports symmetric and anti-symmetric LSM and LSE modes, where the symmetry is that of axial electric field $E_z$ with respect to horizontal coordinate $x$ (perpendicular to the dielectric interfaces). The preferred accelerating mode for the single three-zone structure is LSM$_{31}$ (the subscript $s$ signifies “symmetric”), in which the axial wake field $E_z$ reaches an extremum at the center of the structure. In the two-beam WFA structure, the design mode is LSM$_{31}$, which is made up of two such LSM$_{41}$ modes that have vanishing electric field components tangential to the interface (dashed line in Fig. 1) and the same axial wave-number.

![Figure 1: Dielectric two-beam WFA structure, with one vacuum channel for a decelerated (drive) beam and the other for an accelerated (test) beam.](image-url)

Figure 2 shows the calculated dependence of drive beam channel half-width $a_2$ and transformer ratio $T$ on drive channel structure half-width $b_2$ for a two-beam WFA structure, which operates in the LSM$_{31}$ design mode at a frequency of 102.816 GHz for a phase velocity equal to the vacuum light speed $c$. The frequency chosen is the third-harmonic of 34.272 GHz, at which the Yale/Omega-P magnicon could serve as the rf source to power the drive beam. The plots in Fig. 2 are for the case $a_1 = 0.5$ mm and $b_1 = 0.75$ mm for the acceleration channel; the common waveguide half-height $d$ is taken to be 20 mm and the dielectric constants are both taken to be 5.7.

It is seen from Fig. 2 that $a_2 = 11.53$ mm and $b_2 = 11.56$ mm when $T = 10$, while $a_2 \approx 38.58$ mm and $b_2 \approx 38.60$ mm when $T = 100$. Of course, the drive and acceleration channels have the same dimension when $T = 1$.

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NUMERICAL EXAMPLE

A code has been developed for calculation of wake fields excited by rectangular rigid bunches in the structure described. Examples from analysis are presented here for a two-beam WFA structure with \( T = 10 \).

Figure 3 shows the mode spectrum, excited by a train of ten identical 500-MeV, 100-nC electron bunches, with the first bunch located at \( z = 100 \) cm, and with an inter-bunch period of \( L_{b} = 8.75 \) mm (corresponding to a 34.272 GHz injector). The bunch size is taken as \( 20 \times 10 \times 0.5 \) mm\(^3\) (1 nC/mm\(^3\)), centered in the drive channel. A total of 50 waveguide modes are included in the analysis (1 \( \leq n_{x}, n_{y} \leq 5 \)), 25 LSM and 25 LSE modes, noting that LSE\(_{m0}\) modes make no contribution. It is seen that the design LSM\(_{31}\) mode has the largest radiation power, namely 3.422 GW. Competing modes, in descending order of power, are the LSM\(_{13}\) (35.36 GHz, 926 MW), LSM\(_{11}\) (19.62 GHz, 40.1 MW), LSM\(_{21}\) (88.51 GHz, 30.5 MW), LSM\(_{32}\) (107.53 GHz, 17.1 MW), and LSM\(_{35}\) (193.61 GHz, 3.1 MW); others are completely negligible. These powers can be compared with the 1.714 TW beam power.

The radiation power into a given mode can be shown to be proportional to the bunch interference factor
\[
C_{b} = \sin^{2}(N_{b}\xi)/\sin^{2}\xi, \text{ where } N_{b} \text{ is the number of bunches and } \xi = \pi\lambda_{mm}/L_{b} \text{ with } \lambda_{mm} \text{ the wavelength of the } m\text{-}n^{th} \text{ mode. Radiation into competing (non-synchronous) modes from a periodic train of bunches will be suppressed if the ratio } \overline{C}_{b} = C_{b}/N_{b}^{2} \text{ is small enough.}
\]

For example, the LSE\(_{11}\) mode (37.67 GHz) is completely suppressed by the 10-bunch train (\( \overline{C}_{b} = 0.01 \)), while for a 5-bunch train (\( \overline{C}_{b} = 0.5 \)) its power is still 11\% that of the LSM\(_{31}\) design mode. The competing LSM\(_{13}\) mode at 35.36 GHz, has \( \overline{C}_{b} = 0.7 \) for a 10-bunch train, \( \overline{C}_{b} = 0.23 \) for a 20-bunch train, and 0.007 for a 30-bunch train.

Figure 4 shows the dependence of axial wake force \( F_{z} \) on \( x \) for the design LSM\(_{31}\) mode for a total of 50 modes superimposed, for a test particle located at \( y = 0 \) (the structure mid-plane) and \( z = 91.25 \) cm (namely a distance of 30 wavelengths of the LSM\(_{31}\) mode from the center of the first drive bunch). The acceleration gradient in the acceleration channel is 223.5 MeV/m, while the deceleration gradient is 22.2 MeV/m in the drive channel. These values imply that the 500 MeV drive beam could travel about 22 m, during which an accelerated bunch.
could gain about 5 GeV. It is seen from Fig. 5 that the total $F_z$ deviates somewhat from that of the pure LSM$_{31}$ mode because of multimode interference, where the deviation results mainly from the LSM$_{13}$ mode.

It should be indicated that the design LSM$_{31}$ mode is different from all the other interfering modes in that it has no wall current at the center of the top and bottom walls of the drive channel ($J_z = 0$). Therefore, slots may be cut along these positions to suppress undesired modes.

Figure 6 shows the dependence of axial wake field forces in the centers of the drive and acceleration channels for the design LSM$_{31}$ mode. The first bunch is positioned at $z = 100$ cm and the $10^{th}$ bunch is located at $z = 92.12$ cm. The force amplitudes jump after every three periods where the ten drive bunches are injected, but after the $10^{th}$ bunch they remain constant. If a bunch with a size of 0.6 mm×0.6 mm×0.2 mm is located at the $z = 91.25$ cm peak in the acceleration channel, the maximum transverse wake field force in the bunch will be 0.04 MeV/m while the maximum gradient is about 224 MeV/m.

**GRADIENT SCALING**

The gradient in the acceleration channel falls with an increase in the transformer ratio $T$ for a given bunch charge, since the width of the drive channel must increase with $T$. Figure 7 shows the dependence of gradient $G_q$ in the acceleration channel, produced by a unit charge in the drive channel, on transformer ratio $T$ for different half waveguide heights $d$. The plots are drawn by keeping $\Delta x_b/(2b_2) = 0.64$, $\Delta y_b/(2d) = 0.25$, and $\Delta z_b = 0.2$ mm, where $\Delta x_b$, $\Delta y_b$, and $\Delta z_b$ are, respectively, the bunch width, height, and length. It is seen that $G_q$ increases as $d$ decreases for a given $T$, since the two-beam structure becomes more compact. However, a smaller $d$ leads to less uniformity of axial wake field forces in beam channels and stronger transverse wake field forces, which could be harmful to the stability of the bunch.

Since the ratio $d/a$ in the drive channel falls with an increase in $T$, the drive bunch has considerable effect on the gradient in the acceleration channel. Figure 8 shows the dependence of the gradient $G_d$ on the normalized bunch width $\Delta x_b/(2b_2)$ for $T = 10$ and $T = 100$, both with $d = 20$ mm.

**REFERENCES**