Abstract
Electron cooling of 8.9 GeV/c antiprotons in the Fermilab's Recycler ring requires precise matching of electron and antiproton velocities. While the final match can be done by optimisation of the cooling process, for the very first cooling one should rely on the knowledge of absolute values of electron and antiproton energies. The upper limit for the energy uncertainty of both beams is determined by the Recycler's momentum aperture and is equal to 0.3 %. The paper discusses a method of the electron energy calibration that is based on the measurement of the electron's Larmor wavelength in the field of the cooling section solenoid. The method was tested in an 18 m long cooling section prototype with 3.5 MeV electrons. An accuracy of 0.4 % was demonstrated.

INTRODUCTION
The Recycler Electron Cooling (REC) [1] at Fermilab is suggested to be applied to 8 GeV antiprotons and, therefore, requires a DC beam of 4.3 MeV electrons. A general layout of the REC system is shown in Fig. 1. The Pelletron [2] accelerates an electron beam. Then the beam is bent in two planes to bring it into the cooling section (CS). The cooling section is immersed into the 100 G solenoidal field. After the CS, the electrons make a U-bend down the cooler, and finally come back to the Pelletron where they are decelerated and dumped into a collector. The Recycler Electron Cooler has been installed in the Recycler tunnel and is under commissioning now.

![Figure 1: The schematic layout of the Fermilab electron cooler.](image)

A prototype of the actual REC was assembled and tested in the Wideband laboratory at Fermilab for R&D purpose [3]. Some parameters of the REC cooler and the prototype are listed in Table.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>REC</th>
<th>Prototype</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terminal voltage</td>
<td>4.3 MV</td>
<td>3.5 MV</td>
</tr>
<tr>
<td>Terminal voltage ripple</td>
<td>&lt; 500 V</td>
<td>&lt; 500 V</td>
</tr>
<tr>
<td>Electron beam current</td>
<td>0.5 A</td>
<td>0.5 A</td>
</tr>
<tr>
<td>Cooling section length</td>
<td>20 m</td>
<td>18 m</td>
</tr>
<tr>
<td>Cooling section field</td>
<td>70, 75, 100 G</td>
<td>100,190 G</td>
</tr>
<tr>
<td>Beam radius in the CS</td>
<td>6 mm</td>
<td>6-8 mm</td>
</tr>
</tbody>
</table>

A scenario of the first cooling [4] proposes to fill the entire momentum aperture of the Recycler, which is 0.3%, with antiprotons. To observe cooling, energies of electron and antiproton beams must be matched within this value. The electron energy is determined by the Pelletron high voltage that is measured by a generating voltmeter (GVM). While GVM readings are highly linear, its absolute precision at 4.3 MeV level is estimated to be 2 %. In this paper we propose to measure the electron energy by an independent method, use the obtained data for a precise calibration of GVM and rely on its readings thereafter.

THE ALGORITHM OF THE MEASUREMENTS

General Idea
The proposal for energy measurements consists in the measurement of the wavelength of electron’s Larmor precession in the field of the cooling section.

The trajectory of electron beam can be excited by a dipole kicker located upstream of the cooling section. The difference of the initial and excited beam’s trajectories (a “differential trajectory”) is the Larmor helix in the CS. The wavelength of the helix ($\lambda$) is determined by the momentum of the beam ($p$) and the average value of the CS solenoidal field ($B$) [5].

$$\lambda = \frac{2\pi pc}{eB}$$

Here $c$ is the speed of light and $e$ is the electron charge.

The precision of the suggested energy measurement depends on the measurement precisions of $B$ and $\lambda$. The dynamics of electron cooling process requires a highly uniform solenoidal field in the cooling section. For this purpose, the value of longitudinal magnetic field in the CS was measured with absolute precision of 0.1 % [6].

The CS is equipped with 10 beam position monitors (BPM). The BPMs are longitudinally positioned with a precision better than 1 mm. For $\lambda$ approximately equal to 10 m, it gives the possibility to find $\lambda$ with 0.01 % precision. Therefore, in case of perfectly precise BPMs, the energy can be found with 0.1 % precision.
Theoretical Consideration

Let us consider the motion of an electron in a uniform longitudinal magnetic field taking into account the effect of image charges.

The motion of an electron in electro-magnetic field [5] is described by (1):

\[ \frac{dp}{dt} = -e \left( \frac{v}{c} \times B + E \right) \]  

(1)

Here \( v \) is electron’s velocity, and \( E \) is an electric field.

In the Cartesian coordinate system \((x,y,z)\) with \( z \) coinciding with the CS axis, we introduce the following notations: \( \theta = \theta_n + i \theta_b \) and \( \zeta = x + iy \), where \( i \) is the imaginary unit, \( \theta_n \) and \( \theta_b \) are the \( x \) and \( y \) components of the transverse angle of an electron. One can show that from equation (1) follows:

\[ \begin{align*}
\zeta &= \theta \\
\theta' &= i k \theta + \Lambda \zeta
\end{align*} \]  

(2)

Here \( \zeta = d\zeta/dz \), \( \theta' = d\theta/dz \), \( k = 2\pi/\lambda \), and \( \Lambda = (2Ie)/(b^2 \gamma \beta c e) \) present the effects of image charges in case of a DC beam. Here \( I \) is the electron current, \( b \) is the radius of the vacuum chamber, \( \gamma \) and \( \beta \) are the standard relativistic parameters. In some measurements the pulsed electron beam (instead of DC) was used. The length of the pulse (2 \( \mu \)s) is significantly smaller than the time of magnetic diffusion in the wall of the vacuum chamber (about 300 \( \mu \)s), thus \( \Lambda \) for the pulsed beam is suppressed by an additional factor of \( \gamma^2 \).

The solution of equation (2) is:

\[ \zeta(z) = e^{ikz} \left( \xi_0 (K-k) - 2i \theta_0 \right) + e^{i(kz)} \left( \xi_0 (K+k) + 2i \theta_0 \right) \]  

(3)

Here \( K = \sqrt{k^2 - 4\Lambda} \), \( \theta_0 \) and \( \xi_0 \) are the beam’s angle and displacement after the entrance into the cooling solenoid respectively.

So far we have not been taking the nonuniformity of the real solenoidal field into account. The effect of nonuniform magnetic field on the electrons trajectory can be calculated from the comparison of the simulated trajectories in the measured magnetic field and in the uniform field (equal to the integral average of the measured field). With this amendment the readings of BPM \( \# n \) in the cooling section are given by:

\[ \tilde{z}_n(\xi_0, \theta_0, k) = \xi(\xi_n, \tilde{\xi}_0, \theta_0, k) + d\tilde{z}_{n,sol}(\xi_0, \theta_0, k) \]  

(4)

Where \( z_n \) is the position of the particular BPM, \( d\tilde{z}_{n,sol} \) is an additional trajectory’s displacement in the BPM, caused by the nonuniformity of the CS magnetic field, and \( d\tilde{z}_{n,sol} \) is the linear function of \( \theta_0 \) and \( \tilde{\xi}_0 \).

Measurement Algorithm

We find the fit of a differential trajectory in the least-squares sense by minimisation of the goal function:

\[ \chi^2 = \frac{1}{\eta} \sum_{n} \left( \frac{\Xi_n - \tilde{\Xi}_n}{\sigma_{\Xi_n}} \right)^2 \]  

(5)

Here \( \Xi_n = X_n + iY_n \), where \( X_n \) and \( Y_n \) are the readings of BPM \( \# n \), \( \sigma_{\Xi_n} \) is the respective error, and \( \ast \) means the complex conjugation; the sum is taken over all BPMs. In all the calculus we used standard deviations instead of the statistical errors. The fact is that the systematic non-linear effects (not taken into account in (3)) are of the value of standard deviations. The number of degrees of freedom \( \eta \) is equal to the number of measured points (\( N \)) minus the number of parameters of the fit (\( M \)); in the prototype setup the number of BPMs was 9, thus \( N=18 \), there are 5 parameters of the fit \((x_0, y_0, \theta_0, \theta_b, k)\), so \( \eta = 13 \). The errors of the found fit parameters are given by:

\[ \sigma_{a_m} = \left| H^{-1}_{m,m} \right| \]  

(6)

where \( a_m \) is the parameter number \( m \) [7].

Ergo, substituting (4) into (5) and minimizing (5) one can find the Larmor wave number \( k \), and from it the energy of the electron beam. The error of the found energy is given by (6).

### THE RESULTS OF THE MEASUREMENTS

<table>
<thead>
<tr>
<th>#</th>
<th>B [G]</th>
<th>Beam current [mA]</th>
<th>Dipole kicker</th>
<th>Nominal &amp; measured energy (E) [MeV]</th>
<th>Error [MeV]</th>
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<tbody>
<tr>
<td>100</td>
<td>160</td>
<td></td>
<td></td>
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<tr>
<td>1</td>
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<td>100 pulsed.</td>
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</table>

The typical results of the application of the derived algorithm are shown in Fig. 2. The two upper plots show the differential trajectories of BPMs (error bars) and the fitting functions (solid lines) for \( x \) and \( y \) coordinates. Two lower plots show the corresponding residuals (dots) and the standard deviations of BPM readings (error bars).

The residuals are too high, and as a result \( \chi^2 \) is about 20 instead of 1. An additional analysis shown that the residuals are linear functions of \( \theta_0 \) and \( \tilde{\xi}_0 \). Therefore, the most probable reason for the high value of the residuals is unsatisfactory precision of BPM calibration and BPM tilts that were not taken into account in the fit (4).
Figure 2: The typical result of the applied fitting algorithm. $B = 75.7$ G, nominal energy = 3.530 MeV, $I = 22$ mA. The standard deviations (error bars in the lower plot) were calculated from 20 consecutive BPM readings. The scatter of the readings is caused by the beam’s oscillations.

CONCLUSION

We have devised and tested the algorithm of beam-based energy measurements. The obtained precision of the measurements of electrons’ energy is better than 0.4%. The measurements were done in the prototype of the Recycler Electron Cooler.

By the present time the electron cooler has been moved into the Recycler tunnel. We will repeat the same measurements and hope to improve the precision of the energy measurements to 0.1 % (the precision of magnetic field measurements). Improvements of BPM calibration are planned to be done by measuring the trajectory of the antiproton beam, which is a straight line inside the cooling section.

Authors acknowledge the critical contribution of V. Tupikov to measurements of magnetic field in the CS and are thankful to the entire Electron Cooling group for help and fruitful discussions.

REFERENCES