THE ANALYSIS OF THE CROSS-TALK IN A RF GUN SUPERCONDUCTING CAVITY

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Abstract
A project to develop an all Niobium Superconducting RF Gun is underway at Brookhaven National Laboratory in collaboration with Advanced Energy Systems. The geometry of the gun requires that the power input and the pickup probes are on the same side of the cavity, which causes direct coupling between them, or crosstalk. At room temperature, the crosstalk causes serious distortion of the RF response. This paper addresses the phenomenon, the analysis and the simulation results as well as the measurements. A method is provided on how to extract the desired information from the confusing signal and allow accurate measurements of the coupling between the probes and the cavity.

INTRODUCTION
Superconducting cavities (SCC) have been used in many accelerator facilities. The great success of the superconducting technology has encouraged many other applications like microwave guns. For example, DESY has applied it for a microwave gun[1,2] In Brookhaven National Lab, an “Electron Cooling” project is underway[3]. As a first experiment, a microwave gun with superconducting cavity is employed [4,5].

A SCC in an accelerator usually has its input antenna (or launcher) on one side and the pickup probe on the other side. The coupling between them is only through the cavity cells, and has a perceivable coupling only near the resonant frequency. For a cavity of a microwave gun, one end must be the cathode; thus, both launcher and pickup may be on the same side, as is the case here. We found there was strong cross-talk between the launcher and the pick-up in room temperature measurement [6]. Fig. 1 shows a typical response of S21 by virtue of a network analyzer. Obviously, it is too much distortion from a typical resonant curve. How to extract the useful information from the undesirable signal is a challenge.

To this end, we had to make a model for simulation. Analyses are also necessary in order to understand the relationship between parameters and the responses in the measurements. Fortunately, we found the crosstalk is not important when the cavity is cooled down to the superconducting status.

This article summarizes the results of the analysis and the measurements. It also provides a method to deal with crosstalk in case it is not negligible.

MODELING AND SIMULATION
We chose the equivalent circuit model as shown in Fig. 2, where C0-L0-R0 represents the cavity. C1 and C2 represent the input and pickup couplers, respectively, with each connected to a 50 ohm cable. A capacitance C12 is added to represents a direct coupling between input and pickup and introduces crosstalk.

![Fig. 1. A typical phenomenon of crosstalk](image1)

![Fig. 2. The equivalent circuit](image2)

![Fig. 3. The Pspice simulation result](image3)

Here all the couplings are attributed to capacitance, because the probes are of rod shape and located in the electric field area.

Fig. 3 shows the results from PSpice. Evidently, without direct coupling, the curves display normal resonance (symmetric green curve). The phase changes 180 degrees when the frequency crosses the resonance (not shown). When C12 is added (C12 = 0.001pF is...
chosen in this curve), both the magnitude and phase display are distorted, exactly as we measured.

The dip frequency is due to the interference of two signals. One is the normal coupling through the cavity, of which the phase is very frequency sensitive. The other is due to the direct coupling, which is not sensitive to frequency. When the two signals have opposite phases, the signals cancel each other and thus form a dip.

The simulation also demonstrates that if one increases the Q of the cavity by cooling it down from room temperature to its superconducting state, in the vicinity of the resonant frequency the coupled signal through the cavity becomes much stronger than that through C12, and the crosstalk becomes less important.

### ANALYSIS

The simulation gives a clear response, but doesn't give an explicit relationship between its parameters. Therefore, it is necessary to analyze the equivalent circuit. Fig. 4 gives a generalized form, which resembles a bridge circuit.

![Fig. 2. The generalized circuit](image)

Applying Kirchhoff law, after algebraic manipulation one obtains the following matrix equation.

\[
\begin{pmatrix}
Y_1 & -B_{12} & -B_1 & V_1 \\
-B_{12} & Y_2 & -B_2 & V_2 \\
-B_1 & -B_2 & Y_0 & V_c
\end{pmatrix}
= \begin{pmatrix}
G_S \\
0 \\
0
\end{pmatrix},
\]

where \( Y_1 = G_S + B_1 + B_{12} \)
\( Y_2 = G_L + B_2 + B_{12} \)
\( Y_0 = Y_C + B_1 + B_2 \).

\( Y_0 \) is the total admittance of the cavity including the coupling capacitance. In our special case, \( G_S = G_L = G_0 = 1/50(\text{ohm}) \), and the couplings are very weak such that \( B_1, B_2 \) and \( B_{12} \) are negligible in comparison with \( G_0 \). Then approximately, \( Y_1 = Y_2 = G_0 = 0.02 \text{ mho} \).

Solving the equation (1), one obtains:

\[
V_2 = \frac{Y_1 B_{12} + B_1 B_{12}}{Y_0 G_0 - B_1^2 - B_2^2} V_g . \tag{2}
\]

The coupling coefficient related to the coupling capacitance is:

\[
\beta_2 = Q_2^2 \frac{R_s C_2^2}{R_c C_c^2}, \quad \beta_1 = Q_2^2 \frac{R_s C_2^2}{R_c C_c^2}. \tag{3}
\]

Note that \( R_s = R_2 = 1/G_0 \), and \( R_c \omega_0 C_c = Q_0 \).

Note that only \( Y_0 \) is frequency sensitive. The other parameters can be regarded as approximately constant.

The fraction (2) includes a pole and a zero. Without crosstalk (i.e. \( B_{12} = 0 \)), its numerator becomes constant and thus has no zero, corresponding to a simple resonance.

Substituting the parameters \( B_1 = j \omega C_1, B_2 = j \omega C_2, B_{12} = j \omega C_{12} \), formula (2) can be rewritten in the form:

\[
\frac{V_2}{V_g} = \frac{j \omega C_{12}}{G_0(1 + \beta)} \cdot \frac{1 + j Q_1 \cdot 2(f - f_0)/f_0}{1 + j Q_L \cdot 2(f - f_0)/f_0}, \tag{4}
\]

where \( \beta = \sum \beta_i \), \( Q_1 = Q_0/(1 + \beta) \), \( Q_L = Q_0(1 + \delta_2) \),

\( f_z = f_0(1 - \frac{1}{2} \delta_2) \), \( \delta_2 = C_1 C_2 / C_0 C_{12} \).

Evidently a pole is at \( f = f_0 \), and a zero at \( f = f_z \). This verifies what we have observed in the measurement. If the crosstalk is serious, that is \( \delta_2 \) is small, then \( f_z \) is close to \( f_0 \), and the resonant curve is so distorted that the peak frequency \( f_{\text{max}} \) is not exactly at \( f_{0_{\text{max}}} \) but not exactly at \( f_z \), the measured \( Q \) from 3 dB bandwidth is not exact \( Q_L \).

The introduced “crosstalk parameter” \( \delta_2 \), is also a measure of relative frequency separation of \( f_z \) and \( f_0 \).

\[
\delta_2 = \frac{2(f_0 - f_z)}{f_0} \tag{5}
\]

The closer the two frequencies, the more the distortion of the resonant curve, implying more serious crosstalk.

The agreement with observation once again verifies the circuit model. In order to make a further normalization, Let’s define

\[
\delta = \frac{2(f_0 - f_z)}{f_0}, \quad F = Q_0 \delta, \quad F_z = Q_0 \delta_2. \tag{6}
\]

\( \delta \) is the relative frequency deviation. \( F \) is the normalized frequency deviation, such that \( F = 1 \) corresponds to a frequency at the edge of the 3-dB bandwidth. \( F_z \) is the normalized crosstalk parameter that measures the deviation of the dip frequency from the resonant frequency.

Substituting (3) and (6), then (4) can be rewritten as

\[
\frac{V_2}{V_g} = \frac{j \sqrt{\beta_1 \beta_2}}{1 + \beta} \cdot \frac{1}{1 + j k_1 F} \cdot \frac{1 + j k_2(F + F_z)}{F_z}, \tag{7}
\]

where \( k_2 = 1 + \delta_2 \approx 1, k_1 = 1/(1 + \beta) \approx 1 \).

Equation (7) has a clear physical meaning. On the RHS, the first two fractions represent the response in the absence of crosstalk. The first one is the magnitude of the coupling at resonant frequency. The second manifests the resonant performance. The third fraction involving \( C_2 \) manifests the effect of the direct coupling or the crosstalk. It approaches unity for \( F_z \rightarrow \infty \), i.e. whenever either \( Q_0 \) or \( \delta_2 \) is very large. Just like parameter \( \delta_2 \), \( F_z \) is also a measure of crosstalk.

At room temperature, \( F_z \) is in a range that the crosstalk must be taken into account. When cooled down to the superconducting state, \( Q_0 \) increases a few orders of
magnitude, such that the condition $F_Z >> 1$ is realized, and the crosstalk vanishes.

THE RELATIONSHIP TO THE TESTED PARAMETERS

Using the analyses above, we now can calculate the parameters we want from the parameters that we can measure.

The parameters we want to know are the resonant frequency $f_0$, $Q_0$, coupling coefficient $\beta$, and external $Q$.

Refer to Fig.1, the measurable parameters are $f_{\text{max}}$, $f_{\text{min}}$ and corresponding $S_{21\text{max}}$, $S_{21\text{min}}$. $S_{21\text{min}}$ usually is very weak that may not be read out precisely. Instead, we can measure a middle point $f_M$ and its $S_{21M}$. From network analyzer, one may also read out $Q_{\text{read}}$, which is not the real loaded $Q$ due to distortion.

From (7) we obtain:

$$
\begin{equation}
S_{21} = 10 \log \left[ \frac{4\beta_1\beta_2}{(1+\beta)^2} \cdot \frac{1}{1+k_F^2(F + F_c)^2} \right] \tag{8}
\end{equation}
$$

Note that both $k_F$ and $k_Z$ are very close to unity and $\beta = \beta_1 + \beta_2 << 1$. Obviously, if crosstalk is negligible (i.e., $F_Z >> 1$), the maximum $S_{21}$ occurs at resonance or $F = 0$, one obtains $\beta_1\beta_2$ by testing $S_{21\text{max}}$.

$$
\begin{equation}
S_{21\text{max}} = 10 \log (4\beta_1\beta_2). \tag{9}
\end{equation}
$$

If the crosstalk is not negligible, then

$$
\begin{equation}
10 \log (4\beta_1\beta_2) = S_{21\text{max}} - \Delta S_0, \tag{10}
\end{equation}
$$

where

$$
\Delta S_0 = -20 \cdot \log \left[ 1 - \frac{1}{C^2} \right] \tag{11}
$$

$$
C = 10^{\Delta S_{0\text{th}}/20} \tag{12}
$$

$$
\Delta S_{0\text{th}} = S_{21\text{max}} - S_{21\text{M}} \tag{13}
$$

In order to determine each coupling coefficient, one has to use an extra probe $C$. Combing with the input probe $A$ and pickup $B$, one can measures three products $\beta_A\beta_B$, $\beta_A\beta_C$, and $\beta_C\beta_B$. Then it is ready to find each individual $\beta$ value by

$$
\begin{equation}
\beta_i = \frac{(\beta_A\beta_B) \cdot (\beta_A\beta_C) \cdot (\beta_C\beta_B)}{(\beta_i\beta_{i'})} \tag{14}
\end{equation}
$$

The loaded is:

$$
\begin{equation}
Q_L = G_q Q_{\text{read}} \tag{15}
\end{equation}
$$

Where

$$
\begin{equation}
G_q = \sqrt{2 \left( \frac{C^2 - 1}{C^2 - 2} \right)^2 - 1} \tag{16}
\end{equation}
$$

The deviation of the resonant frequency is:

$$
\begin{equation}
f_{\text{max}} - f_0 = \frac{f_0}{2Q_0} \sqrt{F_Z^2 + 4 - F_z} \tag{17}
\end{equation}
$$

where

$$
\begin{equation}
F_Z = C - \frac{1}{C} \tag{18}
\end{equation}
$$

SUMMARY

The frequency response with an asymmetric was found due to crosstalk. A model with direct coupling along with PSpice simulation successfully demonstrated this phenomenon. A detail analysis gives the formulas, from which one can calculate the coupling parameters, although the frequency response is serious distorted.

This work was aimed to solve the crosstalk problem in a superconducting gun. But of course, this method is also applicable for any crosstalk phenomena.

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REFERENCES


