SIMULATION STUDY OF ELECTRON CLOUD MULTIPACTING IN STRAIGHT SECTIONS OF PEP-II*

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Abstract
Simulation studies of electron cloud multipacting were performed for the SLAC B-factory vacuum chamber for different bunch trains and solenoidal magnetic field. Results show that increasing the number of positron bunches by a factor of two and keeping the same current per bunch means that the solenoidal field needs to be doubled in order to keep the same electron cloud density. Fortunately there are regions that are “free of multipacting” at smaller values of field where the electron cloud density can not get very high. To build the electron cloud, positron bunches have to lose some amount of their kinetic energy through the longitudinal electric field. When the cloud is already established, this longitudinal field acts as an oscillating force. This action is similar to the action of RF fields in a cavity. As a result the positron bunches will have different lengths throughout the train.

INTRODUCTION
The electron cloud at PEP-II in the low-energy positron ring is built up from multipacting electrons in the straight section vacuum chamber and secondary emission of electrons from the vacuum antechamber in the arcs. Placing solenoidal magnetic fields around the ring successfully reduced multipacting and damped the electron cloud instability [1]. PEP-II has an upgrade plan that is leading toward higher luminosity by doubling the number of bunches and halving the bunch spacing [2]. Here we present an attempt to understand the effect from this new bunch pattern on electron cloud multipacting.

THE MODEL OF ELECTRON CLOUD
The physical nature of the multipacting process leads us to use the phase distribution function for the best description of the electron cloud and for a precise modelling of secondary electron emission. It is worth noting that the usual approach of particle tracking cannot accurately describe the probability process of secondary electron emission. The energy distribution of the secondary electrons, which are emitted from a surface bombarded with primary electrons, has a narrow peak of order 5-6 eV. However, in order to have a secondary emission yield of more than one electron, the primary electrons must have energies of several tens of electron volts. This means the emitted electrons have to be accelerated by the field of positron bunches up to these energies in order to build up the multipacting process. The force vector of a positron bunch determines the vector of secondary electron momentum, so the initial angular distribution of emitted electrons does not play a significant role. Finally, without multipacting the number of primary electrons coming from photoemission is not enough to create a noticeable effect on the positrons.

In the straight sections of PEP-II the vacuum chamber is round and made from stainless steel. The electric field of a positron bunch moving along the axis of a round tube gives a radial kick to the cloud electrons. If the surface of the tube wall is azimuthally homogeneous (secondary emission yield is the same everywhere), then we can imagine that the electron cloud will also be azimuthally homogeneous. Therefore we only need a two-dimension phase distribution function of radius and radial momentum to get a complete description of the electron cloud in a straight section.

Vlasov equation and electromagnetic forces
The phase distribution function \( \Psi(t,r,V) \) describes the electron cloud density on the phase plane of radius and radial momentum as illustrated in Fig. 1.

![Figure 1: Phase distribution function with radial momentum on the left stand and radius on the right.](image)

The phase distribution function obeys the Vlasov equation
\[
\frac{\partial}{\partial t} \Psi + \frac{F}{m_e} \frac{\partial}{\partial V} \Psi + V \frac{\partial}{\partial r} \Psi = 0
\]
where \( m_e \) is the electron mass, \( F \) is the total force acting on the electrons from a positron bunch field, from space charge field and from the solenoid magnetic field. The electric field of a positron bunch with a Gaussian shape is
\[
E_{\text{bunch}}(r,t) = \frac{c}{2\pi r} \left( \frac{Z I_{\text{bunch}}}{f_{\text{rev}}} \right) \frac{1}{\sigma \sqrt{2\pi}} \exp\left( -\frac{(ct)^2}{2\sigma^2} \right)
\]
The solenoidal radial force is
\[
F_{\text{sol}} / m_e = r \Omega^2 \left( \frac{a}{r} \right) \left( 1 - \frac{\phi_0}{\Omega} \right)^2
\]
where \( I_{\text{bunch}} \) is the positron bunch current, \( \sigma \) is the bunch length, \( f_{\text{rev}} \) is the revolution ring frequency and 
\[
Z_0 = 120\pi, \quad \Omega = \frac{e}{2mc}H
\]
is the Larmor frequency, \( a \) is the radius of the vacuum pipe and \( a\phi_0 \) is the initial azimuth velocity. Azimuthal motion has this invariant 
\[
r^2(\phi - \Omega\tau) = a^2(\phi_0 - \Omega\tau).
\]
The space charge field has radial and longitudinal components. In the case of a periodical series of positron bunches i.e. 
\[
E(t,z,r) = E(t-z/c,0,r),
\]
these components are:
\[
E_r^n(r,\tau) = Z_0 \int_a^r j_r(r',\tau)dr',
\]
\[
E_z^n(r,\tau) = Z_0 \int_a^r j_z(r',\tau)dr'.
\]

\( j_r \) is the radial electron cloud current. These formulas give an estimation of the average electron cloud density 
\[
\overline{n}_e = \frac{2}{a^2} \int_0^a n_e r dr = \frac{2}{\pi a^2 f_{\text{rev}} c T} I_{\text{bunch}}^+.
\]
and the energy gain for cloud electrons at the chamber wall from the kick of a positron bunch 
\[
W = \frac{mc^2}{2} \left( \frac{Z_0 I_{\text{bunch}}^-}{mc f_{\text{rev}} c T} \right)^2.
\]
The spacing between positron bunches \( cT \) is equal to the RF wavelength multiplied by the spacing number \( N \). For 
\[
I_{\text{bunch}} = 2mA, \quad f_{\text{rev}} = 136 \, \text{kHz}, \quad a = 47mm \quad \text{and} \quad cT = 1260mm \quad (N=2)
\]
we get \( W = 31eV \) and \( \overline{n} = 2 \times 10^{13} \, \text{m}^{-3} \). This means that accelerated electrons that hit the wall can produce more than one secondary particle. The cloud density is of the order of the density of the residual gas in vacuum chamber \( (\rho = 1n\text{Torr}) \).

**Computer algorithm**

A double-step semi-implicit finite-difference scheme with an artificial diffusion parameter is used to numerically solve the Vlasov equation. The scheme has a very good dispersion relation up to the mesh-size wavelength and a small enough diffusion is needed to compensate any oscillation effects at the mesh-size frequency. The wall conditions are described by the probability function 
\[
\Psi_{in}(r=a) \cdot V_{in} = \Psi_{out}(r=a) \cdot V_{out} \times P(\varepsilon_{in},\varepsilon_{out})
\]
The probability function \( P(\varepsilon_{in},\varepsilon_{out}) \) is a combination of secondary emission yield and a spectrum of secondary electrons. We use the secondary emission yield as a function of the primary electron energy, which was measured by R.E. Kirby for stainless steel [3]. We choose zero yields for the zero energy of primary electrons. The energy spectrum of the secondary electrons includes inelastically backscattered and elastically reflected electrons. Detailed information about algorithm can be found in [4].

**MULTIPACTING AT SMALL FIELDS**

First simulations were carried out for small solenoidal fields to study the growth rates of cloud density due to multipacting. We start with the same initial distribution of photoelectrons then let positron bunches appear periodically in time and watch how the electron cloud density changes in time. Fig.2 shows the dynamics of the cloud density with a positron train that has bunch spacing of every other RF bucket (by 2) and bunch current of 2mA for different values of solenoidal field. At the beginning, the cloud density increases somewhat like an exponential, but then saturates due to the action of space charge forces.

![Figure 2: Dynamics of electron cloud density](image)

**MAIN RESONANCE**

While studying the behaviour of the electron cloud for different solenoidal fields we found a strong resonance. This resonance happens when the time interval between the positron bunches is equal to the flight time of the secondary particles back to wall. The flight time is primarily determined by the solenoidal field \( H \) but there is a contribution from the cloud size and intensity. Naturally the resonance depends strongly upon the secondary emission function. The resonance is the boundary between completely different behaviours of the electron cloud. Multipacting happens when the flight time is a little bit smaller than the positron time interval, when the solenoidal field is a little bit higher than the resonance field \( H > H_{res} \). There is no multipacting when \( H < H_{res} \). Fig.3 illustrates this effect.

![Figure 3: Cloud density behaviour in resonance region](image)

**A difference of only 1 Gauss in solenoidal field completely changes the behaviour of the cloud.** Corresponding phase photos (high plots) of the clouds are shown at Fig.4. Clouds are “shot” just before the positron
bunch arrives. Secondary particles, produced by high-energy particles previous positron bunches are ready to be accelerated by the next positron bunch. After acceleration they will come back to the wall and produce more new particles.

Figure 4: Electron clouds on the phase plane just before a positron bunch arrives. The left picture is for a solenoidal filed H= 37G and the right picture for H=38G. It is possible to see that the “right” cloud mainly consists of secondary particles. However the “left” cloud has an additional high-energy peak, which will be decelerated by the positron bunch and will arrive at the wall with very little energy and hence will not produce new secondary particles. Therefore the density will go down and eventually the “left” cloud will decay away.

OTHER RESONANCES

We can assume that there can be some other resonances. A resonance can also happen if the flight time of the secondary particle is equal to an integer number of time intervals between positron bunches.

Figure 5: Electron cloud saturated density (N= 2, 4).

Figure 6: Growth/damping rates for (N=2, 4).

These resonances happen at smaller values of the solenoidal field. In our case we have a second resonance at solenoidal field strength of 23G. Other resonances are in the region below 10 G. Fig.5 and Fig. 6 show the saturated values of the electron cloud as a function of the solenoidal field and growth/damping rates. Negative values mean that the cloud decay away after some time. There is no multipacting when the solenoidal field is more than 60 G. Clear regions are also in the gap of 26-36 G and 14-22 G. For comparison we present analogue curves for the positron bunch spacing by 4. The main resonance is moved to 16 G, other resonances are in the region below 8 G. No multipacting after 30 G and in the gap 8-15 G. It is interesting to note that in the region of 39 G there is a jump in the damping rate. It suggests that there is a half integer resonance: the forced frequency from the field of positron bunches is two times smaller than the repetition rate of the secondary electron emission.

LONGITUDINAL ELECTRIC FIELD OF THE ELECTRON CLOUD

Positron bunches have to lose some of their kinetic energy in order to build the electron cloud. The field that is responsible for the energy transformation is the longitudinal electric field. When the cloud is already built, this longitudinal field acts as an oscillating force on the cloud electrons and gives, at the same time, an additional energy variation inside the positron bunches. The head of the positron bunch is accelerated and the tail is decelerated. This action of the longitudinal field is similar to the action of RF fields in a cavity and it has the same sign. As a result the positron bunches will have different lengths throughout the train. Figure 11 shows a longitudinal field together with the positron bunch shape. The energy gradient along a positron bunch is more than 2.5kV/m². The total length of all the straight sections in the positron storage ring is around 740 m, so the total effect of the electron cloud can be of the order of 1.85MV/m, which is equivalent to 185 kV of RF voltage at 476 MHz. It is easy to make an analytical estimation for this effect. The additional energy variation in a positron bunch can be easily estimated:

\[
\Delta W / l = \frac{2}{\lambda_{RF} \cdot N} m_e c^2 \left( \frac{Z_0 l_{bunch}}{m_e c \cdot 2\pi a} \right)^2
\]

This formula gives the same result we obtain from computer simulations.

Figure 7: Longitudinal electric field in the electron cloud. Solenoidal field is 38 G and the bunch spacing N=2.

REFERENCES