AN EXPERIMENTAL STUDY OF MICROWAVE STABILITY NEAR TRANSITION IN THE PSR*

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Abstract

We have undertaken a study of microwave stability in the PSR storage ring under a variety of beam conditions near transition, including variable intensity and machine impedance, which can be independently controlled in the PSR. Results indicate that the general features of a linear stability model are valid, namely that the instability threshold becomes very small sufficiently close to transition. In addition, many nonlinear features are apparent and the results suggest an extended operating regime is possible with saturated, but otherwise benign, longitudinal fluctuations. Details of the linear model experimental results and corresponding simulations will be presented.

INTRODUCTION

Beams in circular accelerators that must cross transition have long been observed to suffer significant, though often manageable, emittance growth. While the cause for the emittance growth is not completely understood, it is a widely-held belief that it arises from a longitudinal microwave instability at or near transition energy.

Recent theoretical work [1] has suggested that beams are always unstable longitudinally near transition. Since the first-order frequency dispersion goes to zero at transition, it is necessary to include second-order effects to determine the stability properties. For a Gaussian beam, the following dispersion relation is found:

\[
Z_m / m = \frac{2\pi^{1/2}\epsilon_o^2 i \sqrt{k_o^2 - m\omega_o - \Omega}}{N^{1/2} \epsilon_0 \Xi} \left( \epsilon_i - \Xi t \epsilon_0 \right)
\]

where the first and second-order frequency dispersion is given by

\[
\epsilon_{1,2} = -\frac{k_o}{k_i} \pm \sqrt{k_o^2 - m\omega_o - \Omega} \cdot \frac{4k_i^2}{m\omega_o - \Omega}
\]

\[
k_o = -\omega_o \frac{\eta_o}{\beta_o^2 E_o}
\]

\[
k_i = -\omega_o \frac{\eta_o}{2} \left( \alpha_o - \alpha_1 - \frac{3}{2} \eta_o \frac{\beta_o^2}{\gamma^2} \right) \frac{1}{\beta_o^2 E_o^2}
\]

with \( \Delta\omega = \omega_o \Delta E + k_i \Delta E^2 \) and where \( \Xi \) is the complex error function and \( \epsilon_o \) is the RMS beam energy spread. Other parameters are given in ref. [1]. We note that the higher-order correction to \( \eta \) effectively adds another branch to the stability criterion, such that whenever it exists there is a region of instability. The stability criterion formally is found by imposing \( \text{Im}(\Omega) = 0 \) and solving for the associated complex impedance. This is parameterized by the quantity \( \alpha = k_i \epsilon_o / k_o \) which denotes the proximity to transition. \( \alpha = \infty \) occurs at transition. The stability boundary is plotted on the impedance plane in Fig. 1.

Fig. 1 Stability boundaries in the complex impedance plane near transition. As transition is approached, a new unstable region appears within the familiar Keil-Schnell boundary. Plot is for \( \alpha \sim 0.15 \).

It is seen that for \( \eta \) sufficiently small, there is always a region of instability near transition, as evidenced by the branch internal (smaller impedance) to the usual Keil-Schnell limit. At issue, however, is the growth rate and the relevant emittance growth. Solving for the growth rate associated with the Keil-Schnell impedance limit, we find the growth rate approaches a constant value as transition is approached (Fig. 2). We note, in particular, that the entire impedance plane is expected to be unstable, but the growth rates are bounded by the isochronicity of the dynamics.
Fig. 2 Maximum growth rate for longitudinal modes normalized to revolution harmonic as a function of the parameter $\alpha$, i.e. proximity to transition. Transition occurs at $\alpha = \infty$. Instability occurs for $\alpha > 0.05$.

**EXPERIMENTAL SETUP**

We have designed an experiment to quantify longitudinal stability near transition on the Proton Storage Ring (PSR) at LANL. The experimental setup is shown schematically in Fig. 3. Longitudinal oscillations are picked up by a pair of beam position monitor strip lines and summed. In addition, the capability for beam transfer function measurements exists using an amplified network analyzer and pick-up as shown.

![Fig. 3 Experimental setup on the PSR. Approximately 90 db of gain was used in the circuit.](#)

Care was taken to ensure that the lattice was tuned so that $\gamma_t$ was close to the injection energy ($\gamma = 1.85$), which required careful tuning of the injection orbit and lattice quadrupoles. Subsequent measurements showed that the operating point was within 0.05 units of transition for the data shown in this paper. To facilitate a sufficiently long storage time, the buncher cavity (h=1) was detuned and unpowered. Moreover, the ring was filled with multiple-turn injection, allowed to debunch and was extracted after 1 msec.

**EXPERIMENTAL RESULTS**

A primary result of this study is that we found the beam to be longitudinally unstable for very small beam currents when the beam was within 0.2 units of transition. In Fig. 4, we show the envelope of longitudinal oscillations following injection and debunching. It is characterized by a rapid growth followed by saturation and envelope oscillation at much lower frequencies. The corresponding frequency spectrum in Fig. 5 shows a broad range over which oscillations occur.

![Fig. 4 Envelope of longitudinal oscillations near transition. I = 7 mA. Initial growth is followed by low-frequency oscillations. Vertical scale is signal amplitude in arbitrary units. Horizontal scale is in samples. Sample rate is at 2 Gs/sec.](#)

![Fig. 5 Frequency domain signature of longitudinal oscillations near transition for the above time domain data. I = 7 mA. The main machine impedance occurs at h=1. Vertical scale is 10 db/div.](#)

![Fig. 6 Measured initial growth rate as a function of beam current. The associated beam momentum width is not measured.](#)
We conjecture that the rapid growth at the onset is the linear part of the instability growth, and should be comparable to the theoretical model presented in the first section. The low-frequency oscillation, we assert, is due to nonlinear wave overturning and self-trapping, and occurs at the lower synchrotron frequency of the trapped particles [2]. In Fig. 6, we show the scaling of the growth rates with beam intensity, and in Fig. 7, we show the associated scaling of the saturated fluctuation amplitudes.

Performing a beam transfer function measurement leads to the surprising result that the beam has bifurcated into several beamlets, as shown in Fig. 8. As the tune is moved away from transition, approximately 0.3 units, we find that these beamlets merge back into a single beam.

Fig. 7 Saturated longitudinal fluctuation amplitude as a function of beam current.

**DISCUSSION**

It is clear that the growth rate saturates with increasing beam current, as does the saturated fluctuation amplitude. It is found that there is essentially no mode growth if the tune is moved to 0.5 units from transition, qualitatively confirming our theoretical model. However, without a direct measure of the momentum distribution, we are unable to make a quantitative comparison.

We also note that while the saturated amplitude increases with beam current, the increase is only approximately linear, indicating that nonlinear behavior may render the instability benign. The importance of nonlinearity in the dynamics is underscored by the bifurcation of the beam into beamlets, suggested by the beam transfer function measurement, and the results of the simulation (Fig. 9). Further work is planned to quantify the stability properties with a simultaneous measurement of momentum distribution.

Fig. 8 Beam transfer function measurement within approximately 0.07 units of transition. Top trace is the magnitude of the BTF; bottom trace is the corresponding phase. 2 kHz/div centered at h=2.

Fig. 9 Longitudinal phase space from a simulation near transition with a resonator wakefield, Z/n = 10 ohms, Q = 100. Beamlets form on both the high and low energy sides of the core.

**REFERENCES**
