

## REFINED TRACKING PROCEDURE FOR THE SOLEIL ENERGY ACCEPTANCE CALCULATION

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### Abstract

In third generation light sources like SOLEIL, the inevitable strong sextupoles lead to significant non-linearities affecting the transverse beam dynamics. We already emphasized the large contribution of transverse and longitudinal non-linearities to the SOLEIL energy acceptance: energy dependence of Twiss parameters, non-linear off-momentum closed orbit and non-linear synchrotron motion. In order to face other effects as non-linear betatron motion, synchrotron oscillation, coupling, and diffusion processes, a refined tracking was performed using a 6D tracking code. This work confirms the strong effect of the non-linear synchrotron motion and suggests to choose the working point so as to avoid any crossing of the linear coupling resonance for off-momentum particles. The use of the Frequency Map Analysis (FMA) helps to understand the different kinds of particle losses.

### 1 INTRODUCTION

To account for the higher order effects in energy deviation ( $\delta$ ), a module for automatic calculation of energy acceptance ( $\epsilon_{acc}$ ) and Touschek beam lifetime ( $\tau_T$ ) had been introduced earlier in the BETA code [1]. We showed that when particles are Touschek scattered with large  $\delta$  in dispersive sections, the non-linear chromatic closed orbit as well as the variation of the optics with energy have to be taken into account. This algorithm is very helpful during a design phase because  $\epsilon_{acc}$  variation along the storage ring is obtained within 2 minutes, which allows us to test a large number of cases.

Nonetheless, in order to approach a more realistic modelling, we have to face other effects such as:

- non-linear betatron motion, i.e. deviation from phase space ellipses (non-linearity is worst inside the achromat),
- non-linear synchrotron motion, i.e. effects of higher order chromaticities and momentum compaction factor (already partly implemented in BETA [2]),
- coupling from horizontal to vertical plane (small vertical gaps limitations),
- synchrotron radiation: behaviour of the particles during the damping process (diffusion, resonance crossing).

The best way to take into account all these effects is to perform a 6D tracking, i.e. with synchrotron oscillation and radiation turned on.  $\epsilon_{acc}$  calculation problem is then reduced to the very simple question: is the particle with starting coordinates  $(0, 0, 0, 0, \pm\delta, 0)$  stable or not after a sufficient number of turns?

We present here the results of our calculations, using the 6D tracking code TRACYII [3]. In order to better understand these results and to know how the particles get lost we used the FMA [4] which has the advantage of providing a good understanding of the inner complex

structure of the dynamic aperture. The results are presented so as to highlight two important effects: a full betatron coupling which can be reached for given  $\delta$  and the effect of non-linear synchrotron motion.

### 2 TOUSCHEK TRACKING

The particle energy is changed from -6% to +6% (RF energy acceptance) by steps of 0.1%. For each  $\delta$ , particles are tracked over 500 turns. The calculations take into account the vacuum chamber limitation and particularly a limiting 5mm vertical aperture in the short straight section. By including some quadrupole rotation errors we generate some coupling in order to check whether the lost occurs in the horizontal (x) or the vertical (z) plane. The synchrotron oscillation period is roughly 140 turns and the damping times are 5600 and 2800 turns respectively in the transverse and longitudinal planes. This means that an integration over 500 turns will be mainly relevant for studying the variation of the working point with energy in the tune space (resonance crossing).

Two working points have been studied :

- Optics 1:  $\nu_x = 18.30$        $\nu_z = 10.27$ 
  - can reach  $3\nu_x = 55$  at large amplitude
  - close to  $\nu_x - \nu_z = 8$  ( $\Delta\nu = 0.03$ )
  - tunes crossing at given  $\delta$  (Figure 1)
- Optics 2:  $\nu_x = 18.20$        $\nu_z = 10.30$ 
  - far from third order resonances
  - far from  $\nu_x - \nu_z = 8$  ( $\Delta\nu = 0.1$ )
  - no tunes crossing (Figure 1)

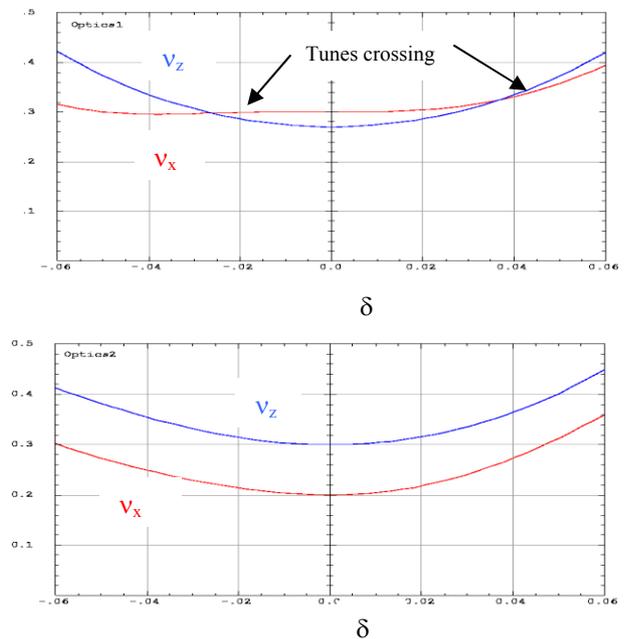


Figure 1: Zero amplitude tune shifts with energy.

### 3 IMPACT OF TUNES CROSSING AT GIVEN ENERGY DEVIATION

A 4D tracking taking into account coupling errors and including the vacuum chamber dimensions has been performed for the two optics in order to test the influence of tunes crossing (full coupling) at given  $\delta$ . Figures 2 and 3 show  $\epsilon_{acc}$  along one super-period of SOLEIL respectively for the optics 1 and 2. We define the local energy acceptance  $\epsilon_{acc}(s)$  as the maximum momentum deviation that keeps the particle having zero initial amplitude stable.

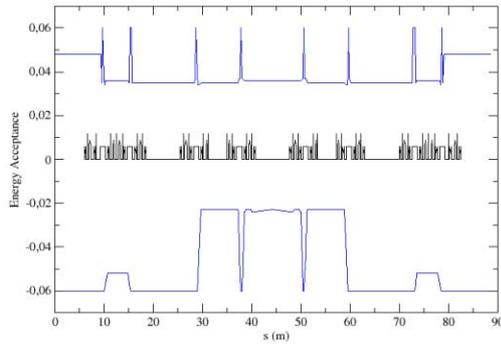


Figure 2 : Local energy acceptance for Optics 1.

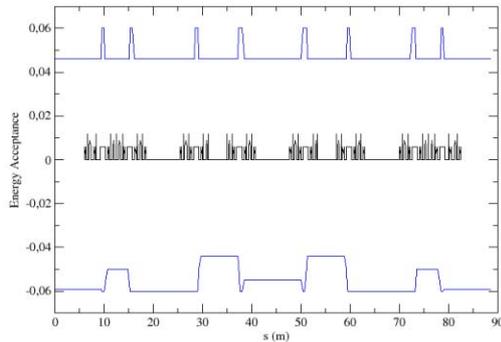


Figure 3 : Local energy acceptance for Optics 2.

For optics 1, the negative  $\epsilon_{acc}$  is very low at certain locations of the lattice and the positive one, except in the bending magnet, reach an upper limit of about +3.5%. Most of the particles are lost in the vertical plane. An illustration of this kind of loss process is given in figure 4. This is possible when some particles cross a region in tune space where their motion is resonantly excited to large vertical amplitudes. We have seen in figure 1 that at  $\delta=+3.5\%$  the particles cross the linear coupling resonance. This might be a contribution to the particle diffusion in the vertical plane. The situation for optics 2 is different. Negative  $\epsilon_{acc}$  is between -4.2% (particles are lost in the horizontal plane essentially in the achromat and short sections where the horizontal dispersion is the highest) and -6% (RF limit).

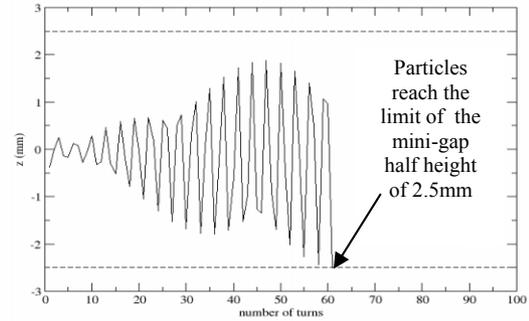


Figure 4 : Highly diffusive vertical motion.

The lowest value in the positive side is constant and equal to 4.5%. This clear-cut is the signature of non-linear resonances.

The FMA has been used in order to investigate the dynamics of the two optics. We combined the particle tracking code DESPOT [5] with the FMA. Particles are tracked with different initial conditions over 1026 turns. If the particle survives then the tunes and diffusion rate [6] are calculated. Coupling errors and vacuum chamber limitation (half horizontal aperture of 25mm (septum position) and half vertical aperture of 7mm) are considered but not synchrotron radiation and damping.

The optics 1 presents much information. The dynamics is dominated by the coupling resonance  $\nu_x - \nu_z = 8$ . Figures 5 and 6 exhibit frequency maps and dynamic apertures (given in reference to the centre of the machine and calculated in the long straight section where  $\beta_x = 10m$  and  $\beta_z = 8m$ ) respectively for particles having +4% and +5% energy deviation. The very dangerous node formed by 3<sup>rd</sup> order unallowed resonances ( $3\nu_x = 55$ ,  $\nu_x + 2\nu_z = 39$ ,  $3\nu_z = 31$ ) and the coupling resonance splits the stability domain and creates empty areas. At 4% the dynamic aperture is particularly affected. This agrees with the result of figure 1 where the crossing point is not far from +4%.

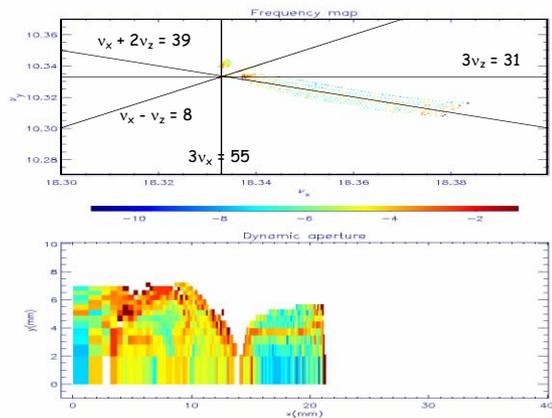


Figure 5 : Optics1 : Frequency map and dynamic aperture ( for  $\delta = +4\%$ ).

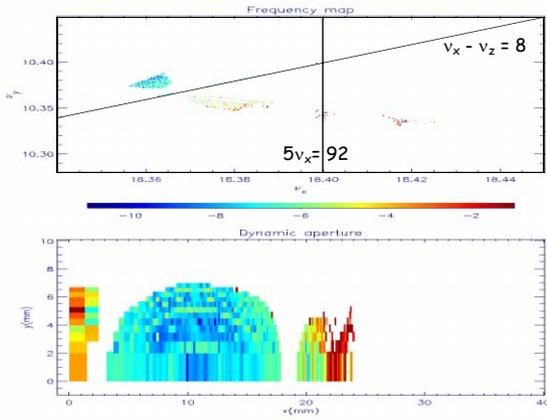


Figure 6 : Optics1 : Frequency map and dynamic aperture for  $\delta = +5\%$ .

The optics 2 which has been optimized in order to avoid the off-momentum tunes crossing does not present similar problems. As a comparison, figures 7 and 8 show the results for +4% and +5% energy deviations.

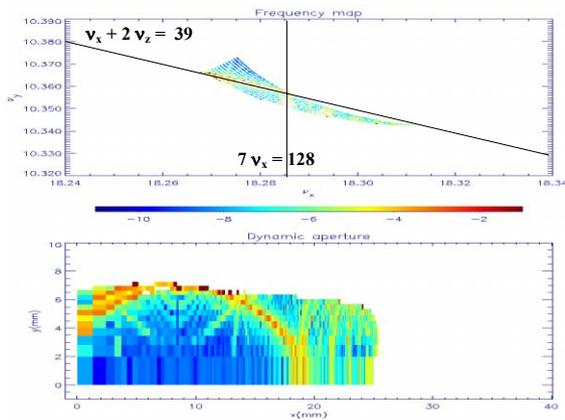


Figure 7 : Optics2 : Frequency map and dynamic aperture for  $\delta = +4\%$ .

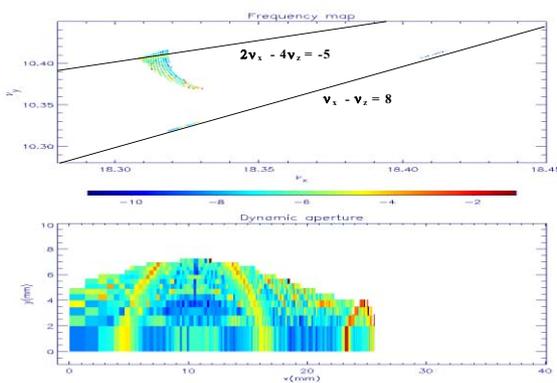


Figure 8 : Optics2 : Frequency map and dynamic aperture for  $\delta = +5\%$ .

#### 4 IMPACT OF NON-LINEAR SYNCHROTRON MOTION

A 6D tracking has been performed using TRACYII. As already shown [2], due to the high value of the second order momentum compaction factor, the RF-bucket is asymmetric in energy. For optics 2, a particle with a positive  $\delta$  of 4.5% (stable during a 4D tracking, see figure 3) for example can be lost because the corresponding energy deviation after one half synchrotron period is  $-8\%$ , which leads to loss.

Figure 9 shows the results for the  $\epsilon_{acc}$  in the case of optics 2. When comparing with the 4D tracking, one can note that the negative side is almost unchanged while the positive side has been shifted down to 4% even in the bending magnets where  $\epsilon_{acc}$  was of 6%. Using this full 6D tracking, including the vacuum chamber with vertical apertures of 10mm in medium straight sections and 5 mm in short straight sections, the calculated Touschek beam lifetime is about 36h at 2.75GeV, using the natural bunch length of 12ps (500mA in 416bunches) and a 1% coupling. The corresponding value using a 4D tracking (non-linear synchrotron motion not taken into account) was 55h for the same conditions.

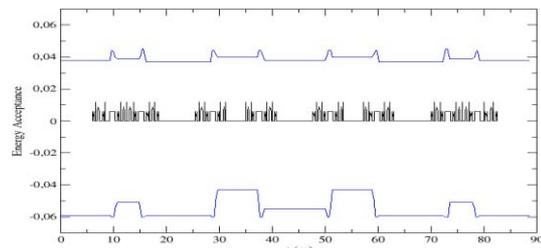


Figure 9 : 6D tracking local energy acceptance for Optics 2.

#### 5 CONCLUSION

The use of the Frequency Map Analysis has shown that it is important to avoid a crossing between tunes inside the energy deviations range of interest. The strong effect of the non-linear synchrotron motion has also been shown. The 6D computation of the Touschek beam lifetime becomes then a reliable criteria to tolerance magnetic errors, vacuum chamber dimensions and septum position.

#### REFERENCES

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