INTRODUCTION

The Recycler Ring [1] located in the Main Injector tunnel at Fermilab is designed to store antiprotons from the Accumulator and the residual Tevatron stores as a part of Run II luminosity upgrade program. The Recycler Ring (RR) is expected to improve the luminosity by increasing the antiproton accumulation efficiency and recycling the residual Tevatron antiprotons after colliding stores [2]. Presently, the RR is being commissioned using protons as well as antiprotons. The successful operation of the Ring requires a beam lifetime of > 100 hours with stochastic cooling and high stacking efficiency. Therefore the study of beam evolution and emittance growth rate is essential for the RR to be an efficient storage ring. The basic parameters for RR are listed in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceptance (mm-mr)</td>
<td>40.0π</td>
</tr>
<tr>
<td>Average β (m)</td>
<td>42.0</td>
</tr>
<tr>
<td>Average beam pipe radius (m)</td>
<td>0.023</td>
</tr>
<tr>
<td>Beam energy (GeV)</td>
<td>8.89</td>
</tr>
<tr>
<td>Average beam γ</td>
<td>0.998</td>
</tr>
<tr>
<td>Maximum energy loss (GeV)</td>
<td>0.089</td>
</tr>
</tbody>
</table>

Table 1: The basic Recycler Ring parameters relevant for the computations detailed in this note.

BASIC FORMULATION

For most lifetime measurements, we introduce a thin approximately Gaussian beam of known intensity in the middle of RR aperture. In such cases, the beam lifetime and evolution are determined by two classes of processes: (1) Processes like single coloumb scattering, nuclear scattering or ionization by which the beam loses a particle abruptly at any given time and region of the beam; (2) Diffusion processes like multiple coloumb scattering, intrabeam scattering or some form of noise where the beam emittance grows and eventually the beam loses particles by hitting the aperture of the beam pipe. Assuming these two classes of processes are independent [6], we write the beam current at any given time as:

\[ I(t) = I_0 N_{ab}(t) N_{df}(t) \]

where \( I_0 \) is the initial beam introduced, \( N_{ab}(t) \) denotes time evolution due to abrupt loss of beam particles as in the first case, and \( N_{df}(t) \) denotes the time evolution due to diffusion processes as described in the second case. For most cases, we can write:

\[ N_{ab}(t) = e^{-\frac{t}{\tau_{ab}}} \]

where \( \tau_{ab} \) characterizes the lifetime due to processes belonging to the first case and assumed to be constant during the evolution. To describe the diffusive processes, we have to solve the Foken-Planck equation [3]:

\[ \frac{\partial f}{\partial \tau} = \frac{\partial}{\partial Z} \left( Z \frac{\partial f}{\partial Z} \right) \]

where \( f \) describes the particle distribution and subject to the boundary conditions:

\[ f(Z, 0) = f_0(Z) \]

\[ f(1, \tau) = 0 \]

\[ \frac{\partial f}{\partial z} \bigg|_{z=0} = 0 \]

where \( Z = \epsilon/\epsilon_a = \text{emittance/acceptance} \), and \( \tau = tR/\epsilon_a \) with \( R \), the diffusion coefficient. Here \( f_0 \) denotes the beam distribution at \( t = 0 \). In the case of pure multiple coloumb scattering phenomena, the diffusion coefficient \( R \) is given in terms of the mean scattering angle \( \theta \) by:

\[ R = \beta_{avg} \langle \dot{\theta}^2 \rangle \]

The general solution of the above equation can be written as:

\[ f(Z, \tau) = \sum_n C_n J_0(\lambda_n \sqrt{Z}) e^{-\lambda_n^2 \tau/4} \]
with coefficients $C_n$:

$$C_n = \frac{1}{J_1(\lambda_n)^2} \int_0^1 f_0(Z) J_0(\lambda_n \sqrt{Z}) dZ$$

where $\lambda_n$ is nth root of the Bessel function $J_0(Z)$. We obtain the total beam particles as a function of time:

$$N_{df}(t) = \int_0^1 f(Z, \tau) dZ = 2 \sum_n C_n \frac{J_1(\lambda_n)}{\lambda_n} e^{-(\lambda_n^2 R/4 \epsilon_a) t}$$

The beam lifetime at any time can be computed using:

$$\tau_{mc} = -\frac{N(\tau)}{dN(\tau)/d\tau}$$

The beam life time varies with time but reaches an asymptotic value:

$$\tau_a = \frac{4 \epsilon_a}{\lambda_1^2 R}$$

The emittance growth can be obtained from:

$$\frac{d\epsilon}{dt} = \frac{\pi \beta \gamma}{R}$$

Now combining the expressions for $N_{ab}$ and $N_{df}$, we cast the time evolution of beam current as:

$$I(t) = I_0 e^{-\tau_{ab}} \sum_n \frac{C_n}{\lambda_n} J_1(\lambda_n) e^{-(\lambda_n^2 R/4 \epsilon_a) t}$$

Since the beam current measurement as a function of time is one of the most accurate measurement we can make in the Recycler Ring, we can fit the measurements for two parameters - for the diffusion constant $R$ and the lifetime due to abrupt processes $\tau_{ab}$. From these, further information about the vacuum residual gas scattering or other processes causing emittance growth can be extracted. This method also provides an alternative to other emittance growth measurements such as using Schottky detectors or techniques based on beam scrapers. Or simply, this could be a cross check on understanding of the relevant physical quantities as well as systematics of other measurement methods.

**FITTING PROCEDURE**

To apply the above formalism to real Recycler Ring data, we develop a fitting procedure. From the above formalism, the beam current at a given time $t$ after $t = 0$ can be written as:

$$I(t) = I_0 e^{-\tau_{ab}} \sum_n Y_n e^{-R \alpha_n t}$$

with the coefficients $Y_n = Y_n(\epsilon_a, \epsilon_a)$, and $\alpha_n = \alpha_n(\epsilon_a)$. The coefficients $Y_n, \alpha_n$ can be generated numerically for most cases once knowing the initial beam distribution $\sigma$, the RR acceptance $\epsilon_a$ and half aperture $a$. We can obtain

**ANTIPROTON BEAM TIME EVOLUTION**

The antiproton beam in the RR is cooled using stochastic cooling methods [4]. The well cooled beam provides an ideal situation for the beam time evolution studies as it has a Gaussian distribution whose initial width can be determined using scraping techniques or a Schottky detec-
After 2003 January shutdown, the RR vacuum residual gases were unusually dominated by leaks and contamination. An initial Gaussian beam of $2.2 \times 10^{11}$ antiprotons (width 3.7 mm) was allowed to evolve in time when no Main Injector ramping was present [5]. The above figure illustrates the beam evolution where the dots denote the beam current measured for 2.5 hours. The data was fitted using the first five terms of the expansion (equation 1) for $I(t)$ for $\tau_{ab}$ and the diffusion constant $R$. The fitted values of $\tau_{ab} = 37.8 \pm 1.7$ hours, $R = 0.65 \times 10^{-10}$, $\frac{d\epsilon}{dt} = 9.75 \pm 2.2 \pi \text{mm-mr/hour}$ [7] is consistent with directly measured values of 40 hours and 9-11 $\pi$-mm-mm/hour.

REFERENCES


[5] As the Recycler Ring is located in the MI tunnel, the RR beam is affected by the stray fields while the Main Injector is ramping. More beam pipe shielding and ramping power supplies are installed to minimize this effect.

[6] This is only an approximation as the threshold scattering angle for single coloumb scattering depends on the size of the beam and beam pipe aperture.

[7] The emittance quoted here is the normalized 95%, i.e., $\epsilon_{N95} = 3.0\epsilon_{rmsN}$