LONGITUDINAL IMPEDANCE SIMULATIONS IN ORBIT: BENCHMARKING AND APPLICATION TO THE SNS EXTRACTION KICKER*

K. Woody, Tennessee Technological University, Cookeville, TN 38505, USA  
J. A. Holmes, V. Danilov, J. D. Galambos, ORNL/SNS, Oak Ridge, TN, 37831-6473, USA

Abstract
Longitudinal and transverse impedances have been incorporated into the ORBIT particle beam dynamics code. This paper deals with longitudinal impedance calculations. The model has been validated for low harmonic number ring stability calculations. Successful benchmarking with the ESME code, for a case without space charge, and with analytical instability thresholds, for coasting beams with and without space charge, has been carried out. The model was then applied to the SNS ring using the measured impedance of the proposed extraction kicker, and it was found that the instability threshold occurred at about four times the proposed current for 2 MW operation.

1 INTRODUCTION
Longitudinal and transverse impedances have been incorporated into the ORBIT particle beam dynamics code [1]. This paper deals with longitudinal impedance calculations. The longitudinal impedance model has been included in the ORBIT longitudinal dynamics package, which also includes space charge, RF cavities, magnets, and kinematic effects. Impedances are represented as longitudinal kicks at localized nodes that can easily be selected to satisfy the integration constraint of small length compared to synchrotron oscillation wavelengths in rings. The impedance kicks are carried out together with the longitudinal space charge forces using FFT methods and convolution with the beam current harmonics. The method and model are similar to those of the ESME Code [2]. In this paper we report the results of initial applications of the longitudinal impedance model. These include benchmarking the model both with ESME and with an analytic calculation for a low wave number case, and also application to the measured impedance of the extraction kicker in the SNS ring.

2 BENCHMARKS
In order to test the longitudinal impedance model, we selected a case for which analytic estimates of the instability threshold can be made. We chose a ring of length 90.261m and transition gamma $\gamma_T = 4.93$, a uniform coasting beam of protons with kinetic energy $E_0 = 800MeV$, intensity $4 \cdot 10^{13}$ particles, and truncated Lorentz energy distribution with halfwidth $\Delta E = 0.8MeV$ and truncation width $8MeV$. We selected an LRC resonator impedance given by $Z_m = \frac{R_S}{1 + iQ(\omega/m - \omega)}$ with quality factor $Q = 100$, resonant frequency $\omega = 3\omega_0$ (where $\omega_0 = 2.8MHz$ is the ring frequency), $\omega_m = m\omega_0$ are harmonics of the ring frequency, and $R_S$ is the shunt impedance. Although RF cavities were not included in these benchmark calculations, the calculations were carried out without and with space charge.

The benchmark strategy was to vary $R_S$ to find the numerical instability threshold and to compare that value with the analytic result of a Vlasov equation solution. The numerical results were also compared with ESME Code results for the same case. To insure numerical convergence, the calculations were carried out with various numbers of macroparticles up to $2 \cdot 10^6$ and for up to 128 in mode number $m$. These tests showed that, for this low frequency impedance, reasonable convergence was obtained with 16 Fourier modes and 128000 macroparticles.

Figure 1 shows the results of the benchmark calculations. The curves plot the values of the $m = 3$ harmonic of the evolved beam density divided by the

---

*Proceedings of the 2001 Particle Accelerator Conference, Chicago*
average value of all the harmonics as functions of the shunt impedance. Because of the assumption that $\omega_R = 3\omega_0$, the instability is expected to have this $m = 3$ structure. For the case without space charge (in red), the $m = 3$ harmonic becomes dominant essentially at the analytically predicted value of 240 Ohms. In the case with space charge, the dominance of the $m = 3$ harmonic occurs at 540 Ohms, about 20% above the predicted value of 450 Ohms. Because the point of dominance of the $m = 3$ harmonic is only an approximate measure of the instability threshold, we find that even with space charge the analytic and computational values of the threshold are consistent. For these same cases without space charge, but assuming uniform rather than Lorentz energy distribution, detailed comparisons of the evolving longitudinal phase space distributions in ORBIT and ESME were carried out, and excellent agreement was obtained. Thus, for long wavelength (low harmonic number) instabilities, the longitudinal impedance model in ORBIT agrees both with analytic predictions and with the ESME code.

3 SNS EXTRACTION KICKER

Application of the longitudinal impedance model was also made to the SNS extraction kicker using measured values of the impedance [3]. The results of these measurements are shown in Fig. 2. Because of the large beam currents in the SNS ring, the effects of the extraction kicker impedance are a concern.

The calculations were carried out for the SNS ring which has a circumference of 248m and a transition value of $\gamma_t = 5.25$. A proton beam of $E_0 = 1GeV$ kinetic energy distributed uniformly with 4MeV halfwidth and first harmonic phase length of 244 degrees was injected for 1060 turns. The beam intensity was varied to determine the limiting intensity for stability. A dual harmonic RF cavity of 40keV first harmonic and 20keV second harmonic was included, as were space charge forces, resulting in a careful simulation of the longitudinal dynamics for the SNS injection scheme.

Numerical assumptions were also carefully tested. Calculations were carried out with up to $2 \cdot 10^6$ macroparticles and as many as 64 modes $m$. For the highest numbers of macroparticles, mode-by-mode convergence up to $m = 16$ was observed.

The results of the calculations showed longitudinal stability up to about $8 \cdot 10^{14}$ protons, or about four times the SNS full intensity beam. Consequently, longitudinal instability due to the extraction kicker impedance is not expected to be a problem in SNS. An interesting effect observed in these calculations is that of the extraction kicker impedance on the longitudinal bunch factor of the beam. Figure 3 shows for a full intensity SNS calculation ($2.08 \cdot 10^{14}$ protons) that the effect of the impedance leads to greater beam bunching, and that bunching increases with increasing impedance. The effect of greater beam bunching is associated with larger peak beam intensities, and accordingly with larger transverse space charge effects. Such effects may lead to beam broadening if the enhanced transverse tune shifts excite lattice resonances, as pointed out in ref. [4].

Figure 2. Measured real (red) and imaginary (blue) impedances of the SNS extraction kicker versus frequency.

Figure 3. Bunch factor versus time in full intensity SNS calculation with measured impedance multiplied by 0 (navy), 1 (pink), 2 (green), 4 (orange), and 8 (burgundy).

One method of manipulating the bunch factor is through variation of the RF cavity voltages. Figure 4 shows the effect of RF voltage on bunch factor for three different values. For the values used here, varying the RF voltage produces an effect comparable to that of the extraction kicker impedance, with higher voltage values leading to greater bunching. Other methods of manipulating the bunch factor, such as varying the injection scheme or including an inductive insert, are also possible if necessary.
4 SUMMARY

A longitudinal impedance module has been included in the ORBIT Code and has been successfully benchmarked against both analytic Vlasov model calculations of instability thresholds and against results from the ESME Code. Calculations for the SNS extraction kicker using this model indicate stability up to four times the full SNS beam intensity. However, the impedance does enhance the beam bunching, which leads to greater local beam intensity. Consequently, this may give greater transverse space charge forces and possible excitation of transverse instabilities due to increased space charge tune shifts.

5 ACKNOWLEDGMENTS

We wish to thank Jim MacLachlan for comparing calculations with us using the ESME Code.

Research on the Spallation Neutron Source is sponsored by the Division of Materials Science, U.S. Department of Energy, under contract number DE-AC05-96OR22464 with UT-Battelle Corporation for Oak Ridge National Laboratory.

6 REFERENCES