EXTERNAL Q STUDIES FOR APT SC-CAVITY COUPLERS

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Abstract

Coupling coefficients for the APT superconducting cavity couplers have been predicted using an improvement of the method previously developed for the French Trispal project [1]. We here present the method and a proof of the formula used to compute the external Q. Measurements on a single-cell copper cold model exhibited a very good agreement against simulation. Then, we established that the original coupler design lead to an insufficient coupling in $\beta=0.64$ cavities. Different solutions were proposed to fix this problem, like combining impedance discontinuities in the line and an off-centered disc end tip. Finally, it was decided to increase the beam tube diameter though it has some influence on the cavity end-cell performance.

1 INTRODUCTION

The superconducting accelerating cavity prototype for the Accelerator Production of Tritium project and its associated power coupler have been intensively studied in Los Alamos [2]. Among the main characteristics is the external quality factor of the cavity. This parameter determines the coupling between the cavity and the RF line that feeds it.

Some methods to compute the external Q already exist. In 1990, Kroll and Yu [3] proposed one based on a fit on a branch of Slater’s diagram. Unfortunately, this method is limited to low $Q_{ext}$ values (less than a few hundreds) and is not suitable for our purpose. The author of the present paper recently proposed a new method [4]. It has been improved since, and the resulting method in fact is equivalent in its principle to another one described in 1993 by Hartung and Haebel [5]. But our method differs both in the proof and in the practical way to operate. Moreover, we derive from it a procedure to compute fields and local power losses under operation in a cavity and its power coupler.

2 METHOD

Let us consider a lossless cavity initially containing some RF energy $W$ at its resonant frequency $\omega$. If this cavity is weakly coupled to an infinite line, this line drives out a certain RF power $P$ and the energy stored in the cavity gradually decreases. The external $Q$ then is:

$$Q_{ext} = \frac{\omega W}{P}.$$

Only a single mode is assumed to travel along the line. The power transported by the travelling wave along the line may be computed either from the electric or the magnetic field amplitude:

$$P = \frac{1}{2} \int \int_{\text{line x sect.}} |E|^2 \, ds = \frac{\eta}{2} \int \int_{\text{line x sect.}} |H|^2 \, ds,$$

assuming that $\eta$ is the mode impedance. The stored energy in the cavity (assumed to be under vacuum) is:

$$W = \frac{1}{2} \int \int_{\text{cavity}} \varepsilon_0 |E|^2 \, dv = \frac{1}{2} \int \int_{\text{cavity}} \mu_0 |H|^2 \, dv.$$

We assume the line mode is a TEM, and that the dielectric is vacuum: $\eta=\varepsilon_0$. Then, the external $Q$ can be expressed as:

$$Q_{ext} = \frac{\omega \int \int_{\text{cavity}} |F|^2 \, dv}{c \int \int_{\text{line x sect.}} |F|^2 \, ds}, \quad (1)$$

$F$ being either the electric ($E$) or magnetic ($H$) field. (If the line is not under vacuum and/or the mode is not a TEM one, a coefficient taking the line mode impedance into account has to be introduced in equation (1)).

Unfortunately, computing the Qext with the formula (1) would require the use of a dissipative code. Though such codes now exist, they are more difficult to use and much slower than non-dissipative ones.

Inverting the sign of time gives a second solution of Maxwell’s equations that represents the same cavity slowly gaining energy from an incoming wave travelling in the line. According to the superposition theorem, we can add these two solutions (fig. 1).

![Fig. 1. Transforming a travelling-wave problem into a standing-wave one.](image)

Inside the line, the two added travelling waves drive the same RF power $P$ in either direction, and they interfere into a standing wave. Let us choose the reference plane at an electric field antinode: the standing wave field amplitude is there twice the one of the travelling waves. Inside the cavity, the two added fields have an arbitrary phase difference $\varphi$. Using the same formal expression as in equation (1), we can define the quantity $Q_1$ as:

$$Q_1 = \frac{\omega \int \int_{\text{cavity}} |E_1|^2 \, dv}{c \int \int_{\text{ref. plane}} |E_1|^2 \, ds} = \frac{1 + e^{i\varphi}}{4} Q_{ext},$$

where the suffix 1 indicates the resulting field after addition. This field is a pure standing wave in both the cavity and the line. The line can be terminated at the
reference plane with the appropriate boundary condition (perfect magnetic wall) without changing the fields, making this problem computable by MAFIA or any other cavity code.

Now let us use the superposition theorem again, but by subtracting the two solutions instead of adding them (the resulting fields will be noted with the suffix 2). At the same reference plane, we now have a magnetic antinode which field amplitude is twice the one of the travelling wave. Inside the cavity, the resulting field is now \(1-e^{i\varphi}\) times the original field. We define \(Q_2\) as:

\[
Q_2 = \frac{\omega}{4\pi} \int_{\text{ref. plane}} \left| H_2 \right|^2 dv = \frac{\left|1-e^{i\varphi}\right|^2}{4} Q_{\text{ext}}
\]

This problem can also be computed by MAFIA with the other boundary condition (perfect electric wall) at the reference plane. As for any value of \(\varphi\), \(\left|1+e^{i\varphi}\right|^2 + \left|1-e^{i\varphi}\right|^2 = 4\), we have then: \(Q_{\text{ext}} = Q_1 + Q_2\).

So, two MAFIA runs (with the same mesh) are sufficient to predict the external \(Q\). The reference plane position has no influence on the external \(Q\) and can be chosen anywhere in the line. Indeed, tests showed that the computed \(Q_{\text{ext}}\) was within a 0.5% variation when computed in double precision with different line lengths.

To prove the validity of this method, we tested it on a \(\beta=0.64\) single-cell copper mock-up (fig.3). The coupling could be changed by moving the electrical antenna more or less into the beam tube. The \(Q_{\text{ext}}\) has been computed and measured (by reflection and/or transmission) for various penetrations of the coupler antenna. The result (fig. 4) shows an excellent agreement between simulation and measurements. The discrepancy of measurements versus simulation is <20 % for reflection and <7 % for transmission.

3 THE \(\beta=0.64\) APT CAVITY

To cancel the reflected wave on the \(\beta=0.64\) APT cavity, the external \(Q\) must equal the internal \(Q\) which is:

\[
Q_{\text{int}} = \frac{\omega W}{P_{\text{beam}}} = \frac{\beta^{3/2} E}{2 \sqrt{\omega} I \cos \varphi},
\]

where \(W\) is the stored energy, \(P_{\text{beam}}\) the power gained, \(E=4.8\) MV/m is the accelerating gradient, \(r/Q\) (single-cell circuit-definition)=17.1\,\Omega, \(I=100\) mA is the beam current, \(\varphi=30^\circ\) is the phase angle between the proton bunch and the RF voltage. The goal value is then: \(Q_{\text{ext}}=0.22 \times 10^6\).

The cavity [6] has been simulated with the originally designed power coupler (fig.5). The computed \(Q_{\text{ext}}\) (assuming two couplers per 5-cell cavity) was: \(1.5 \times 10^6\), which is off by a factor 6.7. With such a design, the antenna would have had to be pushed 26 mm into the beam pipe in order to reach the goal: this is not acceptable in a real accelerator.

At first, we tried to increase the coupling, without modifying the cavity design, but only the coupler itself (fig. 6). Replacing the hemisphere antenna end tip with a disc was found to be efficient if the disc axis was shifted toward the accelerating cells. A further improvement was
obtained by introducing an impedance step in the coaxial line. Such a discontinuity generates a reflected wave that partially cancels the one at the antenna end, if the distance between the reflections planes is chosen suitably. It appeared that two such steps would be necessary: an impedance increase about \( \lambda/2 \) from the end tip, and an impedance diminution \( \lambda/4 \) further. These steps would also make the transition between the coupler dimensions and the feeding coaxial line, thus avoiding the use of a taper.

\[ Q_{\text{ext}} = 1.5 \times 10^6 \text{ (–8.3 dB from goal)} \]

**Fig. 5. Original coupler design**

\[ Q_{\text{ext}} = 0.20 \times 10^6; \text{ +0.6 dB more than goal} \]

**Fig. 7. Final geometry**

However, the above solution only barely gave the necessary \( Q_{\text{ext}} \) and would have let no margin in case of unexpected behavior. For this reason, we preferred another one that consisted in expanding the end-cell beam-pipe radius from 65 to 80 mm. This change was rather soft because 80 mm is already the beam pipe radius of the \( \beta=0.82 \) cavities. Computations showed that such a beam-tube expansion would almost give the required \( Q_{\text{ext}} \). In order to further improve the coupling, a 10-mm thick symmetrical disc was added at the antenna end tip (fig. 7). The \( Q_{\text{ext}} \) obtained (0.20\( \times 10^6 \)) is even a little lower than required.

Such a modification in the cavity geometry has of course some influence on the cavity performance. First of all, the end cell profile has been adjusted to keep a 700 MHz resonance frequency. As well, the transit time factor is poorer because 80 mm is already the beam pipe radius of the \( \beta=0.82 \) cavities. Computations showed that such a beam-tube expansion would almost give the required \( Q_{\text{ext}} \). In order to further improve the coupling, a 10-mm thick symmetrical disc was added at the antenna end tip (fig. 7). The \( Q_{\text{ext}} \) obtained (0.20\( \times 10^6 \)) is even a little lower than required.

The \( Q_{\text{ext}} \) can be easily and efficiently computed by lossless cavity codes in frequency domain. The losses in the coupler associated with the travelling wave can also be derived from the same simulation.

The desired \( Q_{\text{ext}} \) in APT \( \beta=0.64 \) cavities has been obtained by expanding the beam-pipe. If necessary, an asymmetrical end tip and appropriate impedance steps could further improve the coupling.

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**REFERENCES**