HIGH LUMINOSITY MUON COLLIDER DESIGN

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Abstract
Muon Colliders have unique technical and physics advantages and disadvantages when compared with both hadron and electron machines. They should be regarded as complementary. Parameters are given of 4 TeV high luminosity \( \mu^+\mu^- \) collider, and of a 0.5 TeV lower luminosity demonstration machine. We discuss the various systems in such muon colliders.

Introduction
The possibility of muon colliders was introduced by Skrinsky et al. [1], Neuffer [2], and others. More recently, several workshops and collaboration meetings have greatly increased the level of discussion [3, 4]. A detailed Feasibility Study [5] was presented at Snowmass 96.

Technical Questions
Hadron collider energies are limited by their size, and technical constraints on bending magnetic fields. Lepton (\( e^+e^- \) or \( \mu^+\mu^- \)) colliders, because they undergo simple, single-particle interactions, can reach higher energy final states than an equivalent hadron machine. However, extension of \( e^+e^- \) colliders to multi-TeV energies is severely performance-constrained by beamstrahlung. The luminosity \( \mathcal{L} \) of a lepton collider can be written:

\[
\mathcal{L} = \frac{1}{4\pi E} \frac{n_\gamma}{2r_0 \alpha} \frac{P_{\text{beam}}}{\sigma_y} n_{\text{collisions}}
\]

where \( \sigma_y \) is the average vertical (assumed smaller) beam spot size, \( E \) is the beam energy, \( P_{\text{beam}} \) is the total beam power, \( \alpha \) is the electromagnetic constant, \( r_0 \) is the classical radius, and \( n_\gamma \) is the number of photons emitted by one bunch as it passes through the opposite one. If this number is too large then the beamstrahlung background of electron pairs and other products becomes unacceptable.

As the energy rises, the luminosity, for the same event rate, must rise as the square of the energy. For an electron collider, \( n_{\text{collisions}} = 1 \), and, for a fixed background, we have the severe requirement:

\[
P_{\text{beam}} \sigma_y \propto E^3
\]

In a muon collider there are two significant changes: 1) The classical radius \( r_0 \) is now that for the muon and is 200 times smaller; and 2) the number of collisions a bunch can make \( n_{\text{collisions}} \) is no longer 1, but is now related to the average bending field in the muon collider ring. For 6 T, it is 900.

In addition, with muons, synchrotron radiation is negligible, and the collider is circular. In practice this means that it can be much smaller than a linear electron machine. The linacs for the 0.5 TeV NLC will be 20 km long. The ring for a muon collider of the same energy would be only about 1.2 km circumference.

There are, of course, technical difficulties in making sufficient muons, cooling and accelerating them before they decay and dealing with the decay products in the collider ring. Despite these difficulties, it appears possible that high energy muon colliders might have luminosities comparable to or, at energies of several TeV, even higher than those in \( e^+e^- \) colliders.

Parameters
The basic parameters of a 4 TeV \( \mu^+\mu^- \) collider are shown schematically in Fig. 1 and given in Table 1 together with those for a 0.5 TeV demonstration machine based on the AGS as an injector. It is assumed that a demonstration version based on upgrades of the FERMILAB machines would also be possible.

![Figure 1: Schematic of a Muon Collider.](image-url)
Table 1: Parameters of Collider Rings

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>C of m Energy</td>
<td>TeV 4.5</td>
</tr>
<tr>
<td>Beam energy</td>
<td>TeV 2.25</td>
</tr>
<tr>
<td>Beam γ</td>
<td>19,000-2,400</td>
</tr>
<tr>
<td>Repetition rate</td>
<td>Hz 15-2.5</td>
</tr>
<tr>
<td>Muons per bunch</td>
<td>$10^{12}$</td>
</tr>
<tr>
<td>Bunches of each sign</td>
<td>2-1</td>
</tr>
<tr>
<td>Norm. rms emit. $\epsilon_N$</td>
<td>$\pi$ mm mrad 50-90</td>
</tr>
<tr>
<td>Bending Field</td>
<td>T 9-9</td>
</tr>
<tr>
<td>Circumference</td>
<td>Km 7-1.2</td>
</tr>
<tr>
<td>Ave. ring field $B$</td>
<td>T 6-5</td>
</tr>
<tr>
<td>Effective turns</td>
<td>900-800</td>
</tr>
<tr>
<td>$\beta^*$ at intersection</td>
<td>mm 3-8</td>
</tr>
<tr>
<td>$rms$ I.P. beam size</td>
<td>$\mu m$ 2.8-17</td>
</tr>
<tr>
<td>Luminosity</td>
<td>$cm^{-2}s^{-1}$ $10^{35}-10^{33}$</td>
</tr>
</tbody>
</table>

**Components**

**Proton Driver**

The proton driver is a high-intensity (four bunches of $2.5 \times 10^{13}$ protons per pulse) 30 GeV proton synchrotron, operating at a repetition rate of 15 Hz. Two of the bunches are used to make $\mu^+$'s and two to make $\mu^-$'s. Prior to targeting the bunches are compressed to an $rms$ length of 1 ns.

For a demonstration machine using the AGS [6], two bunches of $5 \times 10^{13}$ at a repetition rate of 2.5 Hz at 24 GeV could be used.

**Target**

Predictions of nuclear Monte-Carlo programs [7, 8, 9] suggest that $\pi$ production is maximized by the use of heavy target materials, and that the production is peaked at a relatively low pion energy ($\approx 100$ MeV), substantially independent of the initial proton energy.

Cooling requirements dictate that the target be liquid: liquid lead and gallium are under consideration. In order to avoid shock damage to a container, the liquid could be in the form of a jet.

**Pion Capture**

Pions are captured from the target by a high-field (20 T, 15 cm aperture) hybrid magnet: superconducting on the outside, and a water cooled Bitter solenoid on the inside. A preliminary design [10] has a Bitter magnet with an inside coil diameter of 24 cm (space is allowed for a 4 cm heavy metal shield inside the coil) and an outside diameter of 60 cm; it provides half (10 T) of the total field, and would consume approximately 8 MW. The superconducting magnet has a set of three coils, all with inside diameters of 70 cm and is designed to give 10 T at the target and provide the required tapered field to match into the decay channel.

**Decay Channel and Phase Rotation Linac**

The decay channel consists of a periodic superconducting solenoidal (5 T and radius = 15 cm). A linac is introduced along the decay channel, with frequencies and phases chosen to deaccelerate the fast particles and accelerate the slow ones; i.e. to phase rotate the muon bunch.

Figure 2 shows the energy vs ct at the end of the decay channel.

![Figure 2: Energy vs ct of muons at end of decay channel with phase rotation; muons with polarization $P > \frac{1}{3}$, $\frac{1}{3} < P < \frac{1}{3}$, and $P < -\frac{1}{3}$ are marked by the symbols ‘+’, ‘.’ and ‘−’ respectively.](image)

The selected muons have a mean energy 150 MeV, $rms$ bunch length 1.7 m, and $rms$ momentum spread 20% (95%, $\epsilon_L = 3.2 eV$s). The number of muons per initial proton in this selected bunch is $\approx 0.3$.

**Polarization Selection**

If nothing is done then the average muon polarization is about 0.19. If higher polarization is desired, some selection of muons from forward pion decays ($\cos \theta_d \rightarrow 1$) is required. This can be done by momentum selecting the muons at the end of the decay and phase rotation channel. A snake [11] is used to generate the required dispersion. Varying the selected minimum momentum of the muons yields polarization as a function of luminosity loss as shown in Fig. 3. Dilutions introduced in the cooling have been calculated [12] and are included. A siberian snake is also required in the final collider ring.

**Ionization Cooling**

For the required collider luminosity, the phase-space volume must be greatly reduced; and this must be done within the $\mu$ lifetime. Cooling by synchrotron radiation, conventional stochastic cooling and conventional electron cooling are all too slow. Optical stochastic cooling [13], electron cooling in a plasma discharge [14] and cooling in a crystal lattice [15] are being studied, but appear very difficult. Ionization cooling [16] of muons seems relatively straightforward.

In ionization cooling, the beam loses both transverse and longitudinal momentum as it passes through a material medium. Subsequently, the longitudinal momentum can be restored by co-
herent reacceleration, leaving a net loss of transverse momentum.

The equation for transverse cooling (with energies in GeV) is:

$$\frac{d\epsilon_n}{ds} = \frac{dE_\mu}{ds} \frac{\epsilon_n}{E_\mu} + \frac{\beta_\perp (0.014)^2}{2 E_\mu m_\mu L_R},$$

(3)

where $\epsilon_n$ is the normalized emittance, $\beta_\perp$ is the betatron function at the absorber, $dE_\mu/ds$ is the energy loss, and $L_R$ is the radiation length of the material. The first term in this equation is the coherent cooling term, and the second is the heating due to multiple scattering. This heating term is minimized if $\beta_\perp$ is small (strong-focusing) and $L_R$ is large (a low-Z absorber).

Energy spread is reduced by placing a transverse variation in absorber density or thickness at a location where position is energy dependent, i.e. where there is dispersion. The use of such wedges can reduce energy spread, but it simultaneously increases transverse emittance in the direction of the dispersion. It thus allows the exchange of emittance between the longitudinal and transverse directions.

**Cooling System** The cooling is obtained in a series of cooling stages. In general, each stage consists of three components with matching sections between them:

1. a FOFO lattice consisting of spaced axial solenoids with alternating field directions and lithium hydride absorbers placed at the centers of the spaces between them, where the $\beta_\perp$’s are minimum.

2. a lattice consisting of more widely separated alternating solenoids, and bending magnets between them to generate dispersion. At the location of maximum dispersion, wedges of lithium hydride are introduced to interchange longitudinal and transverse emittance.

3. a linac to restore the energy lost in the absorbers.

In a few of the later stages, current carrying lithium rods replace item (1) above. In this case the rod serves simultaneously to maintain the low $\beta_\perp$, and attenuate the beam momenta. Similar lithium rods, with surface fields of 10 T, were developed at Novosibirsk and have been used as focusing elements at FNAL and CERN [17].

The emittances, transverse and longitudinal, as a function of stage number, are shown in Fig. 4. In the first 10 stages, relatively strong wedges are used to rapidly reduce the longitudinal emittance, while the transverse emittance is reduced relatively slowly. The object is to reduce the bunch length, thus allowing the use of higher frequency and higher gradient rf in the reacceleration linacs. In the next 7 stages, the emittances are reduced close to their asymptotic limits. In the last 3 stages, using lithium rods, there are no wedges and the energy is allowed to fall to about 15 MeV. Transverse cooling continues, and the momentum spread is allowed to rise. The total length of the system is 750 m, and the total acceleration used is 5 GeV. The fraction of muons remaining at the end of the cooling system is calculated to be 55%.

**Acceleration**

Following cooling and initial bunch compression the beams must be rapidly accelerated to full energy (2 TeV, or 250 GeV). A sequence of recirculating accelerators (similar to that used at CEBAF) could be used but would be relatively expensive. A more economical solution would be to use fast pulsed magnets in synchrotrons with rf systems consisting of significant lengths of superconducting linac.

For the final acceleration to 2 TeV in the high energy machine, the power consumed by a ring using only pulsed magnets would be excessive and alternating pulsed and superconducting magnets [18] are used instead.

**Collider Storage Ring**

After acceleration, the $\mu^+$ and $\mu^-$ bunches are injected into a separate storage ring. The highest possible average bending field
is desirable to maximize the number of revolutions before decay, and thus maximize the luminosity. Collisions occur in one, or perhaps two, very low-\(\beta^*\) interaction areas.

**Bending Magnet Design** The magnet design is complicated by the fact that the \(\mu^+\)'s decay within the rings (\(\mu^- \rightarrow e^- \pi^0\nu_e\)), producing electrons whose mean energy is approximately 0.35 that of the muons. These electrons travel toward the inside of the ring dipoles, radiating a fraction of their energy as synchrotron radiation towards the outside of the ring, and depositing the rest on the inside. The total average power deposited, in the ring, in the 4 TeV machine is 13 MW. The beam must thus be surrounded by a \(\approx 6\) cm thick warm shield [19], which is located inside a large aperture conventional superconducting magnet.

The quadrupoles can use warm iron poles placed as close to the beam as practical, with coils either superconducting or warm, as dictated by cost considerations.

**Lattice** In order to maintain a bunch with rms length 3 mm, without excessive rf, an isochronous lattice, of the dispersion wave type [20] is used. For the 3 mm beta at the intersection point, the maximum beta’s in both x and y are of the order of 400 km (14 km in the 0.5 TeV machine). Local chromatic correction is essential. Two lattices have been generated [21, 22], one of which [22], after the application of octupole and decapole correctors, has been shown to have an adequate calculated dynamic aperture.

Studies of the resistive wall impedance instabilities indicate that the required muon bunches would be unstable if uncorrected. In any case, the rf requirements to maintain such bunches would be excessive. BNS [23] damping, applied by rf quadrupoles [24], is one possible solution, but needs more careful study.

**Muon Decay Background** Monte Carlo study [25, 19] indicated that the background, though serious, should not be impossible. Further reductions are expected as the shielding is optimized, and it should be possible to design detectors that are less sensitive to the neutrons and photons present.

There would also be a background from the presence of a halo of near full energy muons in the circulating beam. The beam will need careful preparation before injection into the collider, and a collimation system will have to be designed to be located on the opposite side of the ring from the detector.

There is a small background from incoherent (i.e. \(\mu^+\mu^- \rightarrow e^+e^-\)) pair production in the 4 TeV Collider case. The cross section is estimated to be 10 \(mb\), which would give rise to a background of \(\approx 3 \times 10^4\) electron pairs per bunch crossing. Approximately 90\% of these, will be trapped inside the tungsten nose cone, but those with energy between 30 and 100 MeV will enter the detector region.

**Conclusion**

- Considerable progress has been made on a scenario for a 2 + 2 TeV, high luminosity collider. Much work remains to be done, but no obvious show stopper has yet been found.
- The two areas that could present serious problems are: 1) unforeseen losses during the 25 stages of cooling (a 3\% loss per stage would be very serious); and 2) the excessive detector background from muon beam halo.
- Many technical components require development: a large high field solenoid for capture, low frequency rf linacs, multi-beam pulsed and/or rotating magnets for acceleration, warm bore shielding inside high field dipoles for the collider, muon collimators and background shields, etc. but:
- None of the required components may be described as exotic, and their specifications are not far beyond what has been demonstrated.
- If the components can be developed and the problems overcome, then a muon-muon collider could be a useful complement to \(e^+e^-\) colliders, and, at higher energies could be a viable alternative.

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**References**


