4GLS BEAM-BREAK-UP INVESTIGATIONS

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Abstract

Beam Break Up (BBU) thresholds have been studied as part of the linac focusing scheme for the proposed 4GLS accelerator. A graded gradient focusing scheme, initially with a triplet of quadrupoles between each of the modules within the linac has been chosen. These quadrupoles are set-up in a defocusing – focusing – defocusing format with strengths of -1/2k, k, -1/2k. This value of k was altered and the BBU thresholds for the machine calculated using various BBU codes. Alternate cavity designs have also been investigated using CST’s Microwave Studio (MWS) to see how the effects of higher order modes (HOMs) can be minimised whilst maintaining fundamental field flatness across the accelerating cells. The number of cells have been parameterised and the corresponding BBU thresholds presented as a function of cavity geometry, with the intention of providing an optimum solution for 4GLS.

INTRODUCTION

This paper aims to discuss two methods for achieving the 100 mA current required for 4GLS. The first method is the adjustment of the focusing between the modules of the main linac. The linac contains 5 superconducting modules; each containing eight 7-cell cavities, where 77 pC bunches are accelerated from 10 MeV to 600 MeV. The correct choice of focusing scheme will be vital to avoiding BBU. Initial estimates of the threshold are made using the information for TELSA 9-cell [1] cavities before calculations are made for a basic 7-cell cavity model from MWS. The second method will look at altering the cavity shape to lower this threshold. The modelling examines the threshold as a function of the number of cells contained in the cavity. From this it considers making the cavities slightly elliptical, by creating a distortion of 3mm in one axis, with the aim of splitting the degeneracy of the dipole modes.

LINAC FOCUSING

Focusing Schemes

The graded gradient scheme [2] was chosen for 4GLS [3], in which the focusing magnets are always matched to the low energy beam, therefore the accelerating beam is matched for the first half of the linac and the decelerating beam for the second half. The focusing was investigated with a triplet, doublet and singlet of magnets in between each module. The triplet was set up with a defocusing – focusing – defocusing format with strengths of -1/2k, k, -1/2k. This option should give the greatest amount of flexibility. The doublet was set up in a –k, k fashion and the singlet option alternated between –k and k in between each module. The singlet option was investigated for completeness as a useful threshold was not expected. The doublet and singlet cases were investigated twice, once replacing the magnet removed with a drift space (the ‘long’ case) and the second time reducing the distance between the modules (the ‘short’ case). Both magnets and drift spaces were assumed to be 0.3 m long.

Calculating the BBU Threshold

The current threshold for a single cavity with recirculation has been calculated in many papers [4] and is given by the analytical formula below

\[ I_{th} = \frac{-2p_r}{e^{\left(\frac{R}{Q_m}\right)} Q_m k_m R_y \sin(\omega_m t_r)} \]

where \( (R/Q)_m \) and \( Q_m \) are the shunt impedance and quality factor for the transverse higher order mode (HOM) \( m \) with frequency \( \omega_m \), \( k_m = \omega_m/c \) is the wave number of mode \( m \) and \( p_r \) is the momentum of the recirculating beam. \( R_y \) is the transfer matrix for the entire recirculation from the cavity exit back to the cavity entrance. Longitudinal HOMs cause longitudinal BBU are not considered in this paper since this is believed to be a much weaker effect than transverse BBU. The equation cannot be used for a multi-cavity linac with many HOMs as the HOMs can interfere with each other. To obtain the threshold for such a linac a computer code is used. For the linac focusing section the thresholds are calculated using ‘bi’ from Cornell [5].

Scanning the Focusing Magnet Strength

The aim of this work was to understand the quantitative impact of different linac focussing schemes on the BBU threshold to see if an indication of whether the cheaper option of doublets between the modules could be chosen at this stage in the design study.

The effect of magnet strength on the threshold was investigated. The k value was altered between 0 and 5 m\(^{-2}\) and bi was run via a script, to facilitate processing of the output, and at each setting a threshold current was obtained. Once the best values for the current threshold was obtained, a further matching was carried out using MAD8. In this additional matching, the Twiss parameters were varied at the start of the linac in order to find the best values which kept the beam size small throughout the linac.

The singlet scheme produced the worst results with its threshold peaking at 16 mA for both the short and long cases. The triplet gave a broader spread or good threshold region than the other two with a threshold above 30 mA when \( 2.15 < k < 3.7 \), however the maximum current was found to be only 56 mA. The doublet has a slightly
narrower peak with a threshold above 35 mA over a range of $1.2 < k < 2.9$ and a maximum of 85 mA for the long case and 98 mA for the short case with the majority of the peak remaining above 35 mA for a range of $1.2 < k < 2.4$. The singlet, doublet and triplet results can be seen in Fig1.

**Figure 1: Current threshold vs. k value.**

From this plot the doublet short focusing appears to be the best option since it has relatively broad region of high current threshold. These simulations will need to be updated as the 4GLS cavity design progresses. It is expected that the calculated thresholds will be much greater in the final simulations as these will include the 7-cell HOM damped cavities recommended in the 4GLS CDR [3] which will minimise the HOMs through improved dampers and couplers. The possibility to further improve the threshold through the use of skew quadrupoles [6, 7, 8] will also be considered.

**Modelling the 7-Cell Cavity with HOM Dampers**

An initial model of the 7-cell cavity with 2 solid ring HOM dampers, one of 76 mm and the other of 106 mm radius, made of TT2-112R ferrite were produced in MWS to obtain an initial estimate of the BBUs threshold current for 4GLS. Using the HOMs from this model with short doublet focusing in bi a threshold of 130mA was obtained, this is shown in the dotted blue line in Fig. 1 above. Although this value is not as high as desired it is very encouraging and refinement of this simulation model will provide the starting point for future work.

**ALTERNATE CAVITY DESIGNS**

**Alternating the Number of Cells**

Models of cavities with different numbers of cells were created in MWS [9] and their respective of $R/Q$ and $Q$ of the dipole HOMs in the first pass band, below 2.25 GHz for the 39mm beam pipe used in TESLA cavities, were recorded.

To ascertain the effect of the number of cells on the BBU threshold the HOM data was run in a one cavity BBU model in ERLBBU [4], the BBU code from Jlab. This model had short recirculation which would be a unit matrix if it were not for $R_{12}=1$. From Fig 2 below it can be seen that above 4 cells the difference in the number of cells makes only a small difference in the threshold. To check that this result is not just an effect of the recirculation chosen the threshold was calculated again with two other recirculations, one a unit matrix with $R_{12}=2.566$ and another recirculation similar to that of ERLP (the Energy Recovery Linac Prototype at Daresbury) [10] these can also be seen in Fig 2.

**Figure 2: Current threshold vs. number of cells.**

These plots show that there is a clear advantage in having less than four cells. Above three cells the difference in the threshold is small and the recirculation appears to be the dominant effect. Obviously cavities are not designed with only the BBU threshold in mind and cavities with three of fewer cells are unlikely to be used in ERLs.

**Distorting the Cavity Shape**

Deforming the cavities to split the degeneracy of some dipole modes may also lead to increases the threshold.

Three different options were investigated; the first involved making a deformation in the x plane of the cavity, Fig 3a. The second deforming the cavity...
alternately in x then the y plane, Fig 3b, before finally altering the deformation so that over the length of the cavity it will rotate 360°, figure 3c. There is a forth option of randomly orienting the deformation but this is not looked into here. The HOMs from the models in figure 3 were run in the same BBU model as was used above for the investigation into the number of cells. The results of this model are given in table 1. From this table it can be seen that the rotational deformation gives no improvement. The deformation in the x plane sees the threshold almost double. Alternately deforming the cavity in the x and y planes gives an order of magnitude increase.

![Table 1: Threshold of Alternate Cavity Designs](image)

<table>
<thead>
<tr>
<th>Type of Cavity</th>
<th>Threshold / mA</th>
</tr>
</thead>
<tbody>
<tr>
<td>7-cell cavity</td>
<td>17.9</td>
</tr>
<tr>
<td>7-cells as in Fig. 3a</td>
<td>33.1</td>
</tr>
<tr>
<td>7-cells as in Fig. 3b</td>
<td>254.9</td>
</tr>
<tr>
<td>7-cells as in Fig. 3c</td>
<td>17.8</td>
</tr>
</tbody>
</table>

Looking at the HOM Q for each of these models, in Fig 4, it can be seen that the cavity in 3a has significantly lower Qs for all the HOMs within the simulation.

![Figure 4: HOM Q for alternate cavity designs.](image)

A similar effect is seen for the HOM R/Q, in Fig 5. Although the R/Q of the HOMs of the cavity deformed in x and y is on average larger than that of the other cavity designs it does not have the peak values that the other designs have at the higher frequencies which dominate the threshold calculation.

![Figure 5: HOM R/Q for alternate cavity designs.](image)

The frequency difference of the degenerate modes was not found to increase with the squeezing of the cells. The frequency shift was found to be in the order of a few hundred kHz in the majority of cases, the greatest contribution to the increasing of the threshold for these cavities was found to be the lowering of the HOM R/Q. It is clear from these plots and table 1 the cavity deformed in x and y needs further investigation to prove it is not just an effect of that particular model. These results will need to be investigated using perturbation theory.

In addition to this to obtain an accurate threshold for 4GLS the cavity models will have to be expanded to include the couplers and HOM absorbers. Calculations will be needed to check that the bunches for the XUV FEL branch do not decrease the threshold for the bunches in the high average current loop. And the addition of skew quads to the recirculation arc along with other methods of suppressing the instability will be investigated.

**CONCLUSION**

A threshold of almost 100mA was found using the doublet focusing scheme for 4GLS. The basic 7-cell cavity model produced a threshold of 130mA and further refinement of the model should provide a more accurate value of the threshold. The model will need to include power and HOM couplers and far more accurate representation of the HOM dampers.

Deforming the cavities provided an increase in the threshold especially when the cavity was deformed alternately in the x and y planes, due to the HOM R/Q being reduced by a factor of 4. The x and y cavity requires extra investigation to confirm the substantial increase in the threshold.

**REFERENCES**