STUDY OF COHERENCE LIMITS AND CHIRP CONTROL IN LONG PULSE FEL OSCILLATOR

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Abstract
Electrostatic Accelerator FELs (EA-FELs) have the capacity to generate long pulses of tens microseconds and more, that in principle can be elongated indefinitely (CW operation). Since a cold beam FEL is by nature a "homogeneously broadened laser", EA-FEL can operate, unlike other kinds of FELs, at a single longitudinal mode (single frequency). This allows the generation of very coherent radiation. The current status of the Israeli Tandem Electrostatic Accelerator FEL (EA-FEL), which is based on an electrostatic Van de Graaff accelerator, allows the generation of pulses of tens microseconds duration. It has been operated recently past saturation, and produced single mode coherent radiation of record narrow inherent relative line width $\Delta f/f = 1 \times 10^{-6}$ at frequencies near 100 GHz. A clear frequency chirp is observed during pulses of tens of microseconds (0.3-0.5 MHz/µs). This is essentially a drifting frequency pulling effect associated with the accelerator voltage drop during the pulse. We report experimental studies of the spectral line width and chirp characteristics of the radiation, along with theory and numerical simulations, carried out using space-frequency model, matching the experimental data.

INTRODUCTION
The Israeli Electrostatic Accelerator FEL was relocated to Ariel and returned to operation a year ago. Peak power of 150W usable radiation was previously reported [1].

RADIATION MEASUREMENTS
The mm-wave radiation is transported to the user's rooms by means of a corrugated over-moded waveguide. The measurements were performed by two means: a) using W-band detector Millitech –DXP-10; b) using heterodyne mixer of Hughes-47496H-100 with local oscillator (LO) from a HP-8797D Network Analyzer. In both cases, Tektronix –TDS-784A oscilloscope was used to monitor the output. The input signal was attenuated in order to cope with the dynamic range of the detectors. The maximal peak power at the user location was measured to be 1200W.

SINGLE MODE OPERATION AND CHIRP
In most measurements single mode lasing was observed after a short mode competition period. This is expected because of the "homogeneous broadening" nature of FEL in the cold beam regime. The detector, operating at

![Figure 1(a)](http://www.JACoW.org/FEL/Experimental-Results/)

![Figure 1(b)](http://www.JACoW.org/FEL/Experimental-Results/)

Figure 1: Typical oscillograms of the radiation. Middle line: heterodyne (IF) output. Lower line: W-band detector output. The local oscillator (LO) frequency $f_{LO}$ is set very close to laser frequency: a) $f_{LO}=86,400$ MHz – the IF frequency decreases with time; b) $f_{LO}=86,402$ MHz – IF increases with time.
square-law detection scheme produces on the scope the difference (beat) intermediate frequency:

\[ f_{IF} = f - f_{LO} \]  (1)

there is no distinction in the measurement between negative and positive frequencies, and what is seen on the scope is a signal of frequency \( |f_{IF}| \).

In all measurements the IF signal exhibited either negative or positive chirp (Fig. 1). To determine the chirp direction of the laser signal, one should notice that if the laser signal has negative chirp (f drops down with time) the IF signal would also exhibit negative chirp only when \( f > f_{LO} \) and would exhibit positive chirp when \( f < f_{LO} \).

Fig. 1 shows typical oscillograms of the radiation. The heterodyne mixer frequency was set at two close frequencies: (a) 86,400MHz and (b) 86,402MHz, enabling the accurate determination of the single mode radiation frequency \( f_m = 86.401 \pm 1MHz \), and confirming negative direction of the laser chirp.

At heterodyne (local oscillator – LO) frequency 86,400MHz, the intermediate frequency (IF) decreases with time. At 86,402MHz – increases with time.

This behaviour of the IF signal indicates that the laser radiation exhibits a down shift frequency chirp. This effect is associated with the drift of the gain curve due to the beam energy drop during the pulse, and can be explained as time varying "frequency pulling" effect of the laser oscillator. A theoretical analysis of this effect was provided in an earlier publication [2].

**CHIRP ANALYSIS**

In order to analyze the chirp, we performed the so-called I/Q analysis (Fig. 3). The obtained oscilloscope signal (Fig 2.a) was multiplied by \( \sin(\omega_0 t) \) and \( \cos(\omega_0 t) \), with \( \omega_0 \) corresponding to an arbitrary chosen frequency (169.0MHz at Fig. 2a). These multiplied signals I and Q were subjected to slow-pass filters (LPF) of 15MHz bandwidth. Then, amplitude \( I^2 + Q^2 \) (Fig 2,b) and phase deviation were extracted, and the frequency deviation \( \Delta f(t) = f_f - f_0 \) Eq. (2) was obtained as the deviation time derivative of the phase deviation:

\[
\Delta f(t) = \frac{1}{2\pi} \frac{d}{dt} \arctan \left[ \frac{Q(t)}{I(t)} \right]
\]  (2)

To obtain the inherent spectrum width of the laser radiation (which we define as the linewidth of the wave when the spectral broadening due to the chirp is eliminated), we performed time-stretching. Namely, as the signal was approximated as \( \sin((\omega_0 + \omega_1) t) \), the time was transformed as \( t' = t(1 + \Delta f(t)/f_0) \), with the chirp rate \( \Delta f = 0.35 MHz/\mu s \) as follows from the data (Fig 2.d). The Fourier spectrum of the transformed signal is shown in Fig 2.c, exhibiting inherent spectral linewidth (FWHM) of 0.2MHz (in comparison with 2.0MHz FWHM of the original chirped signal spectrum).

![Figure 2a, 2b, 2c, 2d](image)

Figure 2: I/Q analysis of the oscilloscope signal – low-pass filter is set to 15 MHz band width. Fig. 2a: original oscillogram. Fig. 2b: mode amplitude (blue line-experimental data, red line FEL 3D calculations). The mode frequency is 83,669.0 MHz. Fig. 2c: inherent spectral width, nearly pulse-time-limited (the limit is 0.1 MHz). Since the actual chirp is non-linear in time, the spectrum is wider (0.2 MHz). Fig. 2d: frequency deviation obtained as phase deviation time derivative (blue line-experimental data, red line FEL 3D calculations). The chirp rate is approximately 0.35 MHz/\( \mu s \).
This value is in excellent correspondence with the experimental data, taking into account accuracy of the parameters involved in the calculation.

The experimentally measured chirp behaviour (Fig. 2d) also agrees well with results of simulation of FEL3D.

The chirped instantaneous frequency is calculated by evaluating the rate of change of the computed phase accumulation in each round-trip traversal of the oscillation build-up[2].

We then eliminated the chirp mathematically and used a window of 10µs to obtain the Fig. 4b. In this case, bandwidth is window limited to 0.1MHz, enabling the determination of the inherent mode linewidth as 0.27 MHz.

<table>
<thead>
<tr>
<th>Figure. 4(a)</th>
<th>Figure 4(b)</th>
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<td>Figure 3: I/Q analysis block-scheme of the intermediate frequency oscillograms.</td>
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<td>Since the actual chirp is a little non-linear in time in the ends of the pulse, the spectrum width is somewhat higher than the pulse-duration-limited value (0.1MHz). We analyze here the chirp effect in terms of the basic theory of frequency-pulling in laser oscillators [3]. In our case this frequency pulling shift varies with time (chirps) due to the drift of the gain curve associated with the accelerator voltage drop during the pulse. Namely, for resonator eigenmode frequency $f_m$, resonator mode linewidth (FWHM) $\Delta f_{1/2}$, maximum gain frequency $f_{max}$ and gain bandwidth $\Delta f$ the pulled oscillation frequency $f$ is [3]:</td>
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<td>$f - f_m = (f_{max} - f_m) \cdot \frac{\Delta f_{1/2}}{\Delta f}$ (3)</td>
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<td>During the pulse, $f_0$ drifts towards lower frequencies (due to the accelerating voltage drop) with the rate</td>
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<td>$\frac{df_{max}}{dt} = C \frac{dV}{dt}$ (4)</td>
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<td>where:</td>
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<td>$C = \frac{e}{mc^2} \frac{df_{max}}{d\gamma} \frac{\Delta f_{1/2}}{\Delta f}$ (5)</td>
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<td>is the maximum gain frequency sensitivity to voltage drop (see e.g. [4]).</td>
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<td>$f_0 = \frac{\gamma^2 c^3 \beta_{co} C}{2\pi} \left( k_u + \frac{\bar{\theta}_{max}}{L_w} \right)$</td>
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<td>$\left[ 1 + \left( \frac{\beta_{co}^2}{\gamma^2 k_u + \bar{\theta}_{max}/L_w} \right)^2 \right] $ (6)</td>
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<td>where:</td>
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<td>$\bar{\theta}_{max}$ - the maximal gain of detuning parameter</td>
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<td>$f_{co}$ - the cut off frequency of the resonator waveguide</td>
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<td>For our operating regime we evaluated $C=156$MHz/kV. Synchronism width $\Delta f$ (FWHM of the FEL gain) was calculated using FEL3D software [5] and yielded $\Delta f = 6.0$GHz. The resonator eigenmode linewidth $\Delta f_{1/2}$ was measured (in “cold” resonator) to be $\Delta f_{1/2}=16$MHz (Q-factor of $5\cdot10^5$ at 86GHz). The voltage drop rate was measured to be 0.7kV/µs. The chirp rate $df/dt$ is therefore $df/dt = 0.3$MHz/µs.</td>
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FEL Experimental Results
MODE HOPPING AND OSCILATOR RELAXATION OSCILATION

Mode hopping was observed in many cases, and is associated with accelerating voltage drop, which was measured to be in the range of 7 – 30kV (depending primarily on pulse duration) during the observed pulses. A typical case is shown in Fig. 5 (arranged like Fig. 2). The mode frequencies were uniquely identified by taking several oscillograms with different heterodyne frequencies (usually by steps of 100MHz). One can see that the first mode (85,021.5 MHz) decays when the second (83,676.5MHz) rises.

One can also see a clear trend (both modes at Fig. 5, also the mode at Fig. 1): chirp rate is significantly higher at the rising stage of each mode, and this rate is larger as the corresponding mode rise is steeper. This phenomenon is probably connected to damped relaxation oscillations and is well reproduced by the FEL3D [5].

The chirp direction in both modes is the same (frequency decreases with time). The opposite chirp sign of the IF signal of the 83,676.5MHz mode (Fig 5c) is because \( f_m \), the mode frequency, is below that of the local oscillator \( f_{LO} \), and therefore the IF frequency \( f_{IF} = f_m - f_{LO} \) is negative (aliasing effect).

REFERENCES


Figure 5(a)

Figure 5(b)

Figure 5(c)

Figure 5: Demonstrating of mode hopping apposite with the accelerating voltage drop: (a) IF signal (b) signal growth and decay of the numerically filtered modes (c) isolated IF frequency chirp measurement of the two modes.