IMPLICATIONS OF INCOMPLETE ENERGY RECOVERY IN SRF-BASED ENERGY RECOVERY LINACS*

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Abstract

The choice of the loaded quality factor (Q_L) of a superconducting cavity is driven by many factors, including beam loading effects and microphonics. In accelerators with minimal beam loading, use of SRF cavities with relatively high loaded-Q allows one to employ lower power RF sources. Many individuals are therefore considering energy recovered linac designs making use of SRF cavities with loaded-Q values that are primarily limited by microphonic effects. While this is valid for machines which have near-ideal energy recovery, many applications do not necessarily fit this model. In some applications the second pass, energy recovered beam experiences a phase shift between one state of machine operation and a second state. One complication in this process is that the cavity resonance control algorithms are influenced by this phase shift. With respect to RF power requirements, this is a positive interaction inasmuch as the tuner partially compensates for the phase shift of the recovered beam. This work will go through the implications of partial energy recovery on the selection of the loaded-Q for cavity fundamental power couplers.

INTRODUCTION

Selection of the loaded-Q of a cavity is one of the major factors that is used when determining the RF power requirements for the system. This is especially true in energy recovered linacs where a substantial fraction of the decelerated beam energy is transferred to the accelerated beam through careful phase manipulation of the decelerated beam. Equations (1) and (2) are the general klystron power (P_{Kly}) and phase control requirements (\psi_{Kly}) for a cavity [1]

\[ P_{Kly} = \frac{(\beta + 1) L}{4 \beta Q_L (r/Q)} \left( E + I_0 R_c \cos \psi_s \right)^2 + \left( 2Q_L \frac{\delta f}{f_0} E + I_0 R_c \sin \psi_s \right)^2 \]  

(1)

\[ \psi_{Kly} = \arctan \left( \frac{2Q_L \frac{\delta f}{f_0} E + I_0 R_c \sin \psi_B}{E + I_0 R_c \cos \psi_B} \right) \]  

(2)

where \( P_{Kly} \) is the klystron power [W], \( E \) is the cavity accelerating gradient [V/m], \( r/Q \) is the shunt impedance [\Omega/m], \( R_c = Q_L (r/Q) \) is the coupling impedance [\Omega], \( \beta \) is the cavity coupling where \( \beta = (1 + Q_L) / Q_L \gg 1 \) for our purposes, \( I_0 \) is the magnitude of the vector sum of the beam currents in the cavity, \( \psi_B \) is the phase of the vector sum of beam currents in the cavity with respect to accelerating RF field (\( \psi_B = 0 \) is defined as beam being on crest and in phase with the cavity’s electric field), \( \psi_{\psi_0} \) is the phase difference between klystron power and the cavity electric field and \( \Delta f = \Delta f_n + \Delta f_s \) is the detune frequency of the cavity or the difference between the generator frequency and the cavity’s resonant frequency. There are two parts to this detuning. The first is the quasi-static detuning (\( \Delta f_n \)) due to static Lorentz force detuning, helium pressure fluctuations, mechanical tuners, etc. The second is dynamic detuning (\( \Delta f_s \)) due to microphonics and dynamic pendormotive effects. In both cases the root cause is a deformation of the cavity shape.

Under normal operation a cavity is tuned such that \( \Delta f = 0 \) by adjusting the mechanical tuners. Consider Equations (1) and (2) with no beam current. As \( \Delta f = 0 \) the klystron power is minimized and \( \psi_{\psi_0} \Rightarrow 0 \). The phase difference between cavity field probe signal and the klystron forward power signal measured under these conditions is then used as a reference signal for tuning a cavity. This signal is monitored by the low-level RF system and when it changes by more than a fixed amount, i.e. 3 to 5 degrees for JLAB machines, the mechanical tuners are operated until the phase error signal is corrected. For the sake of discussion in this paper we will call this phase difference when tuned the zero tune angle.

Optimization of the loaded-Q for cavities operated with the beam phase at or near the crest of the electric field (without energy recovery) or when there is perfect energy recovery (i.e. where the decelerated beam is 180° out of phase with the accelerated beam) has been previously published [2,3]. In the former, the loaded-Q is optimized by insuring that all of the klystron power either goes into the beam or into the wall losses of the cavity, which for superconducting cavities is minimal. In such a case the matched loaded-Q is given by:

\[ Q_L = \frac{E}{I_0 (r/Q)} \]  

(3)

In the case of perfect energy recovery the optimal loaded-Q is generally driven by microphonics effects. In this case the ideal loaded-Q is given by:
\[ Q_z = \frac{f_0}{2\delta f_0} \] (4)

**EFFECTS OF INCOMPLETE ENERGY RECOVERY**

There are instances when it is necessary to operate an electron beam in a mode such that there is incomplete energy recovery, by which we mean the first and second pass beams are not 180° apart in phase. For instance in the Jefferson Lab IR Free Electron Laser (FEL) Upgrade we accelerate the first pass beam at 10° before crest in order to achieve bunch length compression and energy recover at 165° after crest to energy compress the large energy spread imposed on the beam by the FEL. This can lead to some interesting control issues.

Upon initial turn-on of the beam the klystron power and phase are given by the following.

\[ P_{kly} = \frac{(\beta + 1) L}{4\beta R_c} \left( \frac{(E + I_o R_c \cos \psi_s)^2}{2Q_c \delta f_0 f_0 E + I_o R_c \sin \psi_s} \right)^{1/2} \] (5)

\[ \psi_{kly} = \arctan \left( \frac{2Q_c \delta f_0 f_0 E + I_o R_c \sin \psi_s}{E + I_o R_c \cos \psi_s} \right) \] (6)

The cavity RF system resonance control algorithms typically adjust the tuner until the phase difference between the klystron power and the field probe signal is set to its baseline value. Thus the mechanical tuner will shift the cavity frequency, \( \delta f \), such that the average value of the numerator in Equation (6) is zero and the forward power is reduced. If one gives this added frequency shift the label \( \delta f_{12} \) then the equation for the klystron power becomes:

\[ P_{kly} = \frac{(\beta + 1) L}{4\beta R_c} \left( \frac{(E + I_o R_c \cos \psi_s)^2}{2Q_c \delta f_0 f_0 E + I_o R_c \sin \psi_s} \right)^{1/2} \] (7)

where

\[ 2Q_c \delta f_0 f_0 E = -I_o R_c \sin \psi_s \text{ or } \delta f_0 = \frac{-f_0 I_o R_c \sin \psi_s}{2Q_c E} \] (8)

\[ \delta f_{12} = \frac{-f_0 I_o R_c \sin \psi_s}{2Q_c E} = \frac{-f_0 I_o (r/Q) \sin \psi_s}{2E} \] (9)

which cancels out the effects of beam loading in the second term. But this tuning process takes time; tens of seconds for conventional motor driven tuners and several to tens of milliseconds for Piezo tuners. During this time the klystron must deliver sufficient power to maintain the gradient in the cavities at a level consistent with the energy acceptance of the machine.

After the tuning has completed the equation for the klystron power becomes:

\[ P_{kly} = \frac{(\beta + 1) L}{4\beta R_c} \left( \frac{(E + I_o R_c \cos \psi_s)^2}{2Q_c \delta f_0 f_0 E} \right)^{1/2} \] (10)

Consider the next situation. The cavity tuners have completed the process of compensating for the reactive power and the forward power is now back at the baseline phase difference with the transmitted power signal. Suddenly, the beam turns off. It can be easily shown that when the beam current is turned off, the cavity starts out detuned by \( \delta f_{12} \) and experiences an equal power transient and an equal, but negative, phase transient compared to the beam turn on transient. Again just as in the turn on transient the RF system must compensate for this transient until the cavity tuner has time to adjust the cavity length and reestablish the baseline phase difference between the forward and transmitted power.

**An Example of Incomplete Energy Recovery**

Consider the following scenario; first pass beam at a phase of 10° before crest, second pass beam at 165° after crest and a beam current of \( I_b \). In theory this is the situation when the FEL is lasing with 2.5% extraction efficiency [4]. Simple vector math can be used to determine that the resultant beam current is \( I_o = 0.087 \times I_s \angle 77.5° \). Figure 1 shows the power requirements for a 7-Cell JLAB Upgrade cavity as a function of average beam current. The cavity parameters are \( E = 10 \text{ MV/m}, \delta f_0 = 10 \text{ Hz}, \text{ and } Q_z = 2 \times 10^5 \).

![Figure 1. Klystron power requirements as a function of beam current, \( I_b \), initially after the beam current is turned on and after tuner restores the field probe to klystron phase to the nominal value.](image)

Figure 2 shows the transient and steady state phase shifts associated with the same beam loading and cavity parameters. It should be noted that for times long compared to the response time of the control
system, a cavity and its associated control system does not know the difference between detuning caused by microphonics and steady state detuning that are the results of the cavity not being properly tuned.

![Figure 2](image2.png)

**Figure 2.** Klystron phase shifts under the same conditions as that shown in Figure 1.

**Data From the JLAB FEL Upgrade**

During October 2006 beam operations, RF power and detune angle data were recorded which support the general trends in Figures 1 and 2. In this experiment two sets of data were taken. In the first, the tuner was allowed to operate and correct for the off-crest beam loading as the beam current was increased from 0 to 4.25 mA. In the second, the tuner was allowed to tune the cavity only when beam was off. The data was recorded from a JLAB 7-cell cavity which has a $Q_L$ of $2 \times 10^7$ and was operated at 5.6 MV/m. The results are displayed in Figures 3 and 4 and illustrate the effects of the initial turn on requirements. Unfortunately, the forward power signal is not well calibrated and the exact state of the second pass phase was not well known. Thus these figures are provided only to demonstrate the effect rather than to quantify it.

**An Example of Shifting From State to State**

One typical scenario in an FEL is to switch from one state of energy recovery to another when the FEL goes into and out of a lasing state. When the FEL lases energy is transferred from the electron beam to the optical beam, the energy spread of the exhaust beam is dramatically increased and the overall energy of each electron bunch is reduced. This is shown in Figure 5. The lower energy bunches couple with the nonzero momentum compaction of the recirculator lattice to generate a change in the path length (or equivalently, a phase shift).

![Figure 3](image3.png)

**Figure 3.** RF power as a function of beam current with the cavity tuners on to compensate for the reactive power (blue) and with the tuners off (red).

![Figure 4](image4.png)

**Figure 4.** Phase difference between klystron drive and cavity field probe as a function of beam current with the cavity tuners on to compensate for the reactive power (blue) and with the tuners off (red).

![Figure 5](image5.png)

**Figure 5.** Synchrotron light emitted at the second arc of the JLAB IR-FEL Upgrade under different lasing conditions: Not lasing (top), weakly lasing (middle) and strongly lasing (bottom). The colors indicate the intensity of the synchrotron light [5].

In order to extract the beam at the end of the linac the second pass beam must have a moderate energy spread and therefore must be energy compressed. Due to the large energy spread imposed on the beam by the FEL, the bunch is decelerated between 10º and 15º before trough of the RF waveform to insure that a high energy tail will not “spill” over the trough. Such high energy tails lead to poor transmission and/or beam loss.

**The Effect of Shifting From State to State**

Assuming that the beam is not in a state of complete energy recovery prior to lasing, i.e. $\delta f_{\text{st}} \neq 0$, the cavity starts with the dynamic power and phase requirements given in Equations (11) and (12)
respectively where $\delta f_{s1}$ is the detune frequency shift for the beam just prior to lasing and $\psi_{s2}$ is the phase

$$P_{s1} = \frac{(\beta + 1)L}{4\beta R_c} \left( \frac{\left(E + I_o R_c \cos \psi_{s2}\right)^2}{2Q_c \delta f_{s1}} + \frac{\left(E + I_o R_c \sin \psi_{s2}\right)^2}{2Q_c \delta f_{s2}} \right)$$

(11)

$$\psi_{s1} = \arctan \left( \frac{2Q_c \delta f_{s1} E + I_o R_c \sin \psi_{s2}}{E + I_o R_c \cos \psi_{s2}} \right)$$

(12)

of the resultant beam current when the first pass beam and the beam laser exhaust beam are summed in the linac. Assuming that the tuners run and the cavities are retuned to the zero tune angle necessary for lasing the static detune frequency, $\delta f_{s2}$, in this second mode will become

$$\delta f_{s2} = -\frac{f_{l1} R_c \sin \psi_{s2}}{2Q_c E}$$

(13)

and you are back in a situation where there is minimum power that is given by Equation (10). Just as in incomplete energy recovery when the machine stops lasing the RF responds with an equal power transient and equal but negative phase transient as the transition from beam on (no lasing) to beam on (with lasing). There is an added set of transitions that must be also considered this is the transitions that occurs when going from a lasing state directly to beam off or from beam off directly to beam on with lasing. Such a transition will follow the same math as incomplete energy recovery where $\psi_{s2}$ and $\delta f_{s2}$ are substituted for $\psi_{s1}$ and $\delta f_{s1}$ respectively.

**Considerations for Operations at 100 mA**

Next we look at the application of these considerations at a higher current. Two types of machines will be considered. In all cases the 15 Hz of microphonics effects are applied as so as to increase rather than reduce the required power. We will consider a design where incomplete energy recovery and switching from one state to another is part of the machine design. In the second case we will look at the effect of a minor, 640 µm, shift in path length on a machine that is designed for perfect energy recovery.

In the first example the beam will shift dynamically from incomplete energy recovery with a 177º phase offset between the accelerated and decelerated beams (State 1) to perfect energy recovery (State 2). In both states the accelerated beam is at -15º with respect to the crest. The energy recovered beam is at 165º and 162º for states 1 and 2 respectively. The cavity is a 5-cell structure at 748.5 MHz with an $(r/Q) = 1000\Omega/m$. The operating gradient is 16.7 MV/m.

First we will look at the power requirements as a function of loaded Q. After that we will examine the transients that occur when switching the beam on and off and when switching from one state of energy recovery to another. Figures 6 and 7 show the power and phase requirements for the cavities described above as a function of loaded-Q.

![Figure 6. Required RF power as a function of loaded-Q showing the transient effects, no tuning changing state to state, and the steady state response after the tuners operate.](image)

![Figure 7. Phase shift due to detuning beam loading and microphonic effects.](image)

If one were to select the loaded-Q on the basis of minimum steady state power with perfect energy recovery, you would select $Q_L = 1.2 \times 10^6$. Doing so means that the actual power required to survive (e.g. the cavity gradient and phase remains sufficiently stable so as to maintain the beam energy within the acceptance of the machine) the full current state to state transients until the tuners respond is 120 kW. Looking at the worst case transient requires that the loaded-Q be set to $3 \times 10^6$. Figures 8 and 9 show the power and current requirements for the different operating modes and transitions required of the system with the loaded-Q set to $3 \times 10^6$.

As indicated in Figure 8 one would need at least 60 kW of RF power in order to operate the cavities in this mode. However, other operation modes could be...
considered. For example if State 1 is the turn on transients associated with turning the beam current on and State 2 is achieved by modifying the machine operation mode. One could turn the machine on at 75% of the operating current and ramp it up coincident with tuning the cavity, thus saving 20% of the RF power. Alternately, one could design a system using IOTs rated above 60 kW and design a DC power supply capable of maintaining that power level for several seconds and 40 kW for CW operation.

For the second example we consider a machine that is designed to operate with perfect energy recovery but has acquired a path length error, and subsequently, a phase shift. For this example, the cavity is an ILC style 1300 MHz cavity operated at 20 MV/m with a microphonics allowance of 15 Hz. Assuming perfect energy recovery, the optimal loaded-Q should be $7 \times 10^8$. Such a machine under ideal conditions would require 8 kW of RF power to function properly. However, if the second pass beam is shifted by as little as 1° the turn on and turn off transient power requirements would be 49 kW while the steady state requirement would increase to 12.6 kW. This is shown in Figure 10.

**SUMMARY**

Dynamic loading due to incomplete energy recovery is an issue for all machines that make use of energy recovered linacs. In some machines it is due to unintentional mismatch of the energy recovered beam with respect to the accelerated beam. In other machines such as those where a significant amount of energy is extracted from the beam between passes, it is intentional. In cases where there is the potential for rapid changes in the relative phase of the energy recovered beam, dynamic loading would be difficult to completely control using fast tuners. In such cases adequate headroom in the RF power would have to be designed into the system. Additionally, many other systems will be affected by this phenomenon. Examples include low-level RF systems, RF-power couplers and cavity tuners. For all these reasons proper machine design will require careful consideration of pass to pass phase parameters under the different operating scenarios. Additionally, tools need to be implemented to measure the phase difference between accelerated and decelerated beam at low or pulsed current as well as high current so that machine path lengths can be set and maintained to a high level of accuracy as the beam current is increased.

**REFERENCES**