We report on a study of the feasibility of a facility for $B$-meson resonances which are moving with a Lorentz boost of $\beta = 1.1$. Physics using the PETRA ring at 14 GeV together with a new technique, the boosted mean flight path of $C_T = 470 \mu m$ provides well separated decay vertices $[1]$. While the mean flight path of $B$-mesons originating from an annihilations would produce $\Upsilon(4S)$ produced at rest is too short to be resolved with present techniques, the boosted mean flight path of $cT/\beta = 470 \mu m$ provides well separated decay vertices $[1]$. The boosted mean flight path is extremely valuable for the reconstruction of $B$-mesons and to separate rare but important decays from exceedingly large backgrounds. Even more important, most of the primary goals in $B$-physics require the measurement of the time evolution of the $B$-mesons. In particular the effect of CP violation in neutral $B$-mesons originating from the $\Upsilon(4S)$ has a specific time dependence, $\sin \Delta m \left( t_1 - t_2 \right)$, where $\Delta m$ is the mixing frequency, $t_1$ is the lifetime of one $B^0$ meson which decays into a CP-violating channel, and $t_2$ is the lifetime of the other $B^0$-meson needed for flavour tagging. With a boosted $\Upsilon(4S)$ the variable relevant to CP-violation, $t_1 - t_2$, can be determined. Thus it appears that an heteroenergetic $e^+e^-$ facility is a promising tool to study CP-violation in $B$-mesons. In addition, at present $B$-physics studies are mainly directed towards a precise determination of the Kobayashi-Maskawa matrix. These investigations include measurements of the $B \to \tau$ transition, $B$-meson lifetimes, $B_d \bar{B}_d$ mixing and $B_s \bar{B}_s$ mixing. For all of these measurements the vertex information is essential. The optimum boost is given by the requirement to make the relative measurement error on the vertex position as small as possible. Depending on experimental details this optimum is close to $\beta = 1$. The boost $\beta = 1.1$, given by $e^+e^-$ beams colliding with 14 GeV on 2 GeV is within this optimum range.

2 Luminosity of an Heteroenergetic Collider

In an heteroenergetic two ring collider, the luminosity $L$

$$L = \frac{f_1 \cdot N_1 \cdot N_2}{2\pi \sqrt{\sigma_{x1}^2 + \sigma_{y1}^2 \cdot \sigma_{x2}^2 + \sigma_{y2}^2}}$$

(1)

(with $N_{1,2}$, the number of particles/bunch in the two beams; $\sigma_{x1,2} = \sqrt{\epsilon_{x1,2}/\beta_{x1,2}}$ the rms beam widths and heights; $\beta^*$, the amplitude function at the collision point (IP); $\epsilon$, the beam emittance and $f_0$, the bunch frequency) can be adjusted over a wider range than in a single ring $e^+e^-$ collider because the beam dimensions of the two beams need not necessarily be the same. However, the potential increase in luminosity due to free choice of relative beam sizes $r_2/r_1$ is limited to a factor of only two. The parameters in eqn(1) are restricted by the strength of the nonlinear beam-beam interaction which can be parameterized by the linear beam-beam tune shift

$$\Delta \nu_{x,y} = \frac{r_0 \cdot N_{2,1} \cdot \beta^*_{x,y}}{\sigma_{x,y,1} \cdot \sigma_{x,y,1} (\sigma_{x,y,2} + \sigma_{x,y,2})}$$

(2)

(with $r_0$ the classical electron radius, $\gamma$ the relativistic factor). Experience on $e^+e^-$-colliders shows that for the four tuneshift values we have to assume some maximum tolerable value of $\Delta \nu = 0.03 [2]$. If the beam sizes are different, the nonlinearity of the beam-beam force increases for the larger beam and decreases for the smaller beam so that the damage done to the larger beam outweighs the benefit for the smaller beam. This has been shown by theoretical investigations $[3]$ and by machine experiments at the SPS $[4]$. Therefore the optimum choice of the relative beam sizes is $r_2/r_1 = 1$. Taking into account these constraints, the luminosity formula may be expressed in terms of fewer parameters which themselves may be subject to further restrictions (which will be discussed below).

$$L = I_1 \frac{\Delta \nu \cdot \gamma_1 (1 + \kappa)}{2 \rho_0 \cdot \beta^*_{\rho}}$$

(3)

($\kappa = \frac{\beta^*_{\rho}}{\beta^*_t}$, $\rho = \frac{\gamma_2}{\gamma_1}$) Once these parameters have been determined, the beam current in the low energy machine $I_1$ depends on the ratio of $\beta^*$-values and the beam energies in the two rings:

$$I_1 = I_2 \frac{\gamma_1 \beta^*_{z1}}{\gamma_2 \beta^*_{z1}}$$

(4)

and then the beam emittances are fixed:

$$\epsilon_{z1} = r_0 I_2/(2 \pi f_0 \gamma_1 \Delta \nu (1 + \kappa)) = \epsilon_{z2} \beta^*_t/\beta^*_t$$

(5)

3 Interaction Region Layout

Under the assumptions discussed above, the luminosity in the two ring collider is inversely proportional to the achievable $\beta^*$-value of the high energy ring whereas the ratio $\beta^*_t/\beta^*_t$ determines the beam current in the low energy ring. Provided that the intensity limit in the low energy ring is not reached, only $\beta^*_t$ and $\kappa$ limit the luminosity. Thus the lattice design around the IP has to be optimized to achieve low $\beta^*_t$. The principle restriction of small $\beta^*$ arises from the large chromaticities $\xi'_t \sim 1/\beta^*$ produced in the low beta quadrupoles. The chromaticity has to be compensated with sextupole fields which introduce a
4 Maximum Beam Current

The limited maximum beam current in the high energy machine is the second serious restriction on luminosity. Whereas the low energy (small circumference) machine can be operated in single bunch mode the large circumference of the high energy machine has to be filled with many bunches. The choice of circumference of the smaller ring determines the number of bunches in the high energy machine. A reasonable choice is $C_1 / C_2 = 20$ which implies 20 bunches in the high energy machine. It is assumed that the rf power will be transferred to the beam by six superconducting 4-cell cavities. Using the theory of multibunch instabilities [5] and the measured parasitic mode parameters of the DESY superconducting cavity [6] it has been calculated that a total current $I_{\text{max}} = 70mA$ in 17 bunches (i.e. there is a 'gap' of three empty bunches) is below the multibunch instability thresholds. Assuming a maximum beam current of $I_1 = 60mA$ in the high energy ring, the single bunch current in the small ring is required to be $I_2 = 180mA$. With a bunch length of $\sigma_z = 3cm \approx \beta_z^* \rightarrow 20cm$. The total PETRA chromaticities with such an insertion would be about $\zeta_{\text{dyn}} = -45$, which are about half the values PETRA has been operated with as an $e^+ - e^-$-collider. The extra chromaticities $\zeta^* \simeq -5$ produced in the small ring are rather small. The chosen value of $\beta^*_y = 3cm$ is therefore not a limit but may be considered as a reasonable choice which helps to achieve small apertures of the low beta magnets so that they can be integrated easily inside the colliding beam detector. Thus a further decrease of the $\beta^*$ by a factor of 2 should be possible. With this ratio of $\beta^*$-values and for an energy ratio of $2\text{GeV} / 14\text{GeV}$, the beam current in the low energy ring has to be three times as large as in the high energy machine. The hard synchrotron radiation ($E_{\text{crit}} = 43keV$) produced by a 14 GeV beam in the separator magnet can be shielded by collimators at the low-$\beta$ quadrupoles and near the vertex chamber of the colliding beam detector. 

In order to obtain higher luminosity the current limit has to be overcome. It turns out that the beam emittances are already rather large. If the bunch current were to be further increased the emittance would very soon reach a practical limit regarding apertures of the magnets. Thus a substantial increase of luminosity can only be achieved by increasing the bunch frequency. Because the beams are completely separated already 4.2m from the IP, the minimum bunch distance is $d_B = 0.6m$. This allows 240 bunches in the high energy ring, corresponding to a factor of 12 increase in total beam current. The luminosity is increased by the same factor and then amounts to $L = 1.1 \cdot 10^{33}\text{sec}^{-1}\text{cm}^{-2}$. This is only possible if the very high beam current of $I_{\text{max}} = 720mA$ is compatible with multi-bunch stability. Furthermore one has to deal with multibunch instabilities in the low energy ring which is now filled with 12 bunches. The compensation of synchrotron radiation power loss of a 720mA PETRA beam ($P_{\text{synch}} = 12.22MW$) would require the installation of 88 superconducting 4-cell cavities. The calculation of the multibunch instability threshold current (as described above) with 100 cavities results in $I_{\text{thr}}^{\text{max}} = 40mA$ in 220 bunches. The multibunch instability calculations for the small ring, where three superconducting cavities are needed, result in a threshold current above the single bunch current limitation.

The situation of the high energy ring filled with 240 bunches can be improved drastically by using a new multibunch feedback system proposed for PETRA as described in [8]. With such a feedback system improvement factors of up to 100 can be obtained [9]. Assuming an improvement factor of only 20, the current threshold would be pushed above the single bunch current limit of $240 \times 3mA = 720mA$. Therefore luminosities in the $10^{33}$-range are within the theoretical limit for a $14\text{GeV} \times 2\text{GeV}$-collider.
5 Design Example

The considerations of the previous section led to a set of main parameters for a 2GeV x 14GeV B-factory using the PETRA storage ring as the high energy machine. These parameters are listed in table 1. In the following we describe a design example for the 2GeV ring based on these parameters. The linear optics in the arcs consists of regular 90° FODO cells. Dispersion functions are matched in the straight section and at the IP. Fig 2 shows the linear optics of the 2GeV ring. The 2GeV ring geometry is shown in fig 4. The use of a mixture of two different bend radii

<table>
<thead>
<tr>
<th>Main Parameters of the 2GeV x 14GeV B Factory</th>
<th>PETRA</th>
<th>Small Ring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Energy</td>
<td>GeV</td>
<td>14.0</td>
</tr>
<tr>
<td>Circumference</td>
<td>m</td>
<td>2304</td>
</tr>
<tr>
<td>Harmonic number</td>
<td></td>
<td>3840</td>
</tr>
<tr>
<td>Bunches</td>
<td></td>
<td>20-240</td>
</tr>
<tr>
<td>Horiz. Emittance</td>
<td>mm</td>
<td>0.2</td>
</tr>
<tr>
<td>Emittance ratio</td>
<td></td>
<td>0.15</td>
</tr>
<tr>
<td>( \beta_x^* )</td>
<td>cm</td>
<td>47.2-23.5</td>
</tr>
<tr>
<td>( \beta_y^* )</td>
<td>cm</td>
<td>1.0-3.50</td>
</tr>
<tr>
<td>Beam Current</td>
<td>Amp</td>
<td>0.06-0.72</td>
</tr>
<tr>
<td>Beam-Beam Delta</td>
<td>Tesla</td>
<td>0.03</td>
</tr>
<tr>
<td>Bend Radius</td>
<td>m</td>
<td>192</td>
</tr>
<tr>
<td>Bend Field</td>
<td></td>
<td>2043</td>
</tr>
<tr>
<td>Synchr. Rad. Power</td>
<td>MWe</td>
<td>1.1-1.3</td>
</tr>
<tr>
<td>Sup. Cond. Cavity</td>
<td></td>
<td>6-88</td>
</tr>
</tbody>
</table>

Luminosity \( L = (0.09 - 2.2) \times 10^{31} \text{sec}^{-1} \text{cm}^{-2} \)

provides the correct geometry, gives the exact circumference, allows elegant dispersion matching, results in the required natural beam emittance and provides enough straight section to place injection elements, rf-cavities and diagnostic facilities.

The natural chromaticities are compensated with 3 sextupole families which also reduces the off energy \( \beta \)-beat at the collision point by a large fraction. The dynamic aperture as calculated by particle tracking is large enough to accommodate 20 standard deviations of a Gaussian beam (linear aperture is 11.3 standard deviations).

6 Summary

In the previous sections we have demonstrated that a 2GeV x 14GeV B-factory is an interesting concept which could provide excellent conditions for investigating B-physics. An heteroenergetic collider provides somewhat less luminosity than a conventional \( e^+ - e^- \)-collider [10]. This disadvantage is however overcompensated by the enhancement of detection efficiency up to two orders of magnitude leading to effective luminosities comparable with \( L = (1 - 10) \times 10^{34} \text{sec}^{-1} \text{cm}^{-2} \).

Another interesting feature is the low cost for such a project because existing facilities can be used with only a minimum of modifications.

For these reasons it is attractive to make further studies of the possibilities of heteroenergetic colliders for B-factories.

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[10] R.-D. Kohaupt, private communication