DETERMINATION OF 100 pscc SHORT LINAC BUNCHES BY
BROADBAND PICKUPS AND RECONSTRUCTION TECHNIQUE

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Abstract
A new method for detecting short linac bunches is proposed and verified on a teststand.
With broadband pickups a voltage signal is obtained and Frieder analysis. From the amplitude at low frequencies we can reconstructed the rms bunch width. With two pickups the rms energy spread can be obtained without knowing the pickup properties explicitly. The bunch shape can be detected from amplitude and phase signals at high frequencies.
This method has been verified on a teststand for capacitative probes using a pulsar and Fourier analysis. The obtained smooth amplitude response up to 5 GHz allows the detection of 100 pscc relativistic bunches. The reconstruction method is insensitive against amplitude and phase fluctuation.

Introduction
At most accelerators the bunch shape is obtained directly from the measured voltage signal. But this correspondence of the two signals is limited at low and high particle energies. At high energies, the bunches are short, the pickup response in no longer flat up to the highest needed frequencies. Also cable attenuation has to be taken into account. At low energies even low frequency components can be damped due to the non-relativistic signal widening effect /4, 5/. If the linac operating frequencies are high, like in the case of the proposed Kaon factories /2, 3/, then the bunch shape determination is a problem.
By operating in the frequency domain these difficulties can be overcome. But here the analysis is more complicated because the voltage signal did not correspond directly to the beam pulse. In Chapter I it is shown how to get the rms bunch width and the particle distribution in this case. In Chapter II the method is verified on a teststand for capacitative probes with non-flat frequency response.

1. Analysis of the Pickup Response in the Frequency Domain

1. Fourier Analysis of Different Beam Pulses
In Fig. 1 the normalized Fourier components I(N)/f0 for three different beam pulses are plotted. Shown are the components of a Gaussian pulse, of a at Th = ± 2 Trms truncated Gaussian and of a parabolic pulse I(t) = (1/4)(t/Trms)²/². The parabolic pulse corresponds to an uniform filling of the longitudinal phase space area. The rms pulse width is \( \Delta \Phi_{\text{rms}} = \pm 100 \) for all three pulses.

For a Gaussian pulse the current per Schottky band I(N) is given by

\[ I(N) = \frac{1}{2} I_0 \exp \left(-N^2/2 \Delta \Phi_{\text{rms}}^2\right) \]  

(1)

I0: dc-current, N: frequency, \( \Delta \Phi_{\text{rms}} \): rms width whereas for the other two pulses no simple formula can be given.

From Fig. 1 it can be seen that up to harmonic number 6 all three curves are identical whereas they differ significantly above harmonic number 12. Therefore the rms pulse width can be determined from low frequency components without assuming anything about the distribution itself. For all longitudinal matching purposes the rms width is the important parameter. For getting the particle distribution high frequency components have to be added.

At low frequencies the Fourier components are identical only by keeping the rms and not the total width constant. The zero crossing of I(N) should not be used to determine either the rms or the total width because its value depends on the particle distribution /2, 3/.

2. Determination of the RMS Bunch Width
Let us assume a system where the beam pulse is travelling along the axis of a vacuum pipe and any kind of pickup is located at the pipe surface. The pickup is connected to a measuring device.

The so obtained voltage signal can be a complicated function in time depending on the pickup and beam parameters. The signal should not be affected by microwave contributions /4, 5/.

In frequency domain the pickup response is described by the complex transfer impulse or sensitivity S(f)/4, 5/.

\[ S(f) = Z(f) \exp(i\Phi(f)) \]

(2)

where Z(f) is the amplitude response and \( \Phi(f) \) is the phase shift. Both functions can be arbitrary and non-flat.

From the measured voltage signal \( U(f) \) the quantity \( T(f) = U(f)/Z(f) \)

(3)
can be obtained if Z(f) is measured before.

\( T(f) \) is the Fourier component of the to be known beam pulse. If necessary \( T(f) \) must be corrected for damping effects like cable attenuation and non-relativistic signal widening.

The corrected quantity \( \tilde{T}(f) \) behaves like a Gaussian up to harmonic number \( N \), where \( N \) = 1(N) peaks for a Gaussian

\[ N = 0.32 \left( \frac{C}{\Delta \Phi_{\text{rms}}} \right) \]

or \( f_1 = Nf_0/0.16 \) for \( \Delta \Phi_{\text{rms}} \)

(4)

\[ I(N) = \frac{1}{2} I_0 \exp \left(-N^2/2 \Delta \Phi_{\text{rms}}^2\right) \]

The value for \( \Delta \Phi_{\text{rms}} \) is then obtained by doing a least square fit of \( T(f) \) to a Gaussian up to harmonic number \( N \). A much more simpler way is plotting the function

\[ \Delta \Phi_{\text{rms}} = \left(-N^2/(2 \ln (T(f)/Z(f)))\right)^{1/2} \]

(5)

which directly corresponds to \( \Delta \Phi_{\text{rms}} \) for the low frequency components, see Fig. 2. Here \( \Delta \Phi_{\text{rms}} \) is plotted for a square, a triangular and a Gaussian pulse. For a Gaussian \( \Delta \Phi_{\text{rms}} \) is identical to \( \Delta \Phi_{\text{rms}} \) for all frequencies, for the other two pulses only up to harmonic number \( N \).
The Fourier decomposition of a square and a triangular 
pulse are given by \( f/3 \). Also shown in Fig. 2 is the 
\( \Delta t_\text{rms} \) dependence of \( N_1 \).
For short bunches \( \Delta t_\text{rms} \) can be reliably determined
for \( N \sim 0.7 N_1 \). A relativistic bunch with a rms width of
\( T_\text{rms} = \pm 25 \) psec or a total width of \( T \sim 100 \) psec
is Gaussian like up to \( f = 6.4 \) GHz which means a
signal up to \( f = 4.5 \) GHz is enough. Such short bunches
are produced by the 1 GeV high frequency injector linac
of Kaon factories \( /2 / \).

\[ \Delta t_\text{rms} = \sqrt{\Delta t_2^2 - \Delta t_1^2}; \Delta t_1: \pm \text{rms-values} \]  

up to harmonic number \( N_p \), where \( N_p \) is the value of \( N_1 \)
for \( \Delta t_\text{rms} = \Delta t_2 \).
In Fig. 2, the quantity \( \tilde{t}_p \) (in Eq. (5) for \( \Delta t_2 \)
replace \( \tilde{t}_f \) by \( U_2(f)/U_1(f) \)) is plotted for a square pulse with
\( \tilde{t}_1 = \pm 11.2\) changing to a
triangular pulse with \( \tilde{t}_2 = \pm 150 \), which gives
\( \Delta t_\text{rms} = 100 \). For this extreme case the ratio
\( U_2(f)/U_1(f) \) is greater than 1 around harmonic number 9
due to the zero crossing of the square pulse amplitude.
But even here \( \Delta t_\text{rms} \) corresponds to \( \Delta t_\text{rms} \) up to harmonic
number 4 determined by \( \Delta t_2 = \pm 150 \). Therefore \( \Delta t_\text{rms} \)
can be detected in such a way if \( \Delta t_2 \) is greater
than \( 1.4 \Delta t_1 \).

For a low intensity beam or at high energies the
quantity \( \Delta t_\text{rms} \) is given by

\[ \Delta t_\text{rms} = \frac{L \cdot \Delta w_{\text{rms}}} {\gamma (\beta_y)^2 m c^2} \] (7)

\( \Delta w_{\text{rms}}: \pm \text{rms energy spread}; \gamma: \text{bunching wavelength} \)
The energy spread can be obtained without knowing the
bunch width and the pickup properties explicitly.
An elliptical phase space distribution and a
longitudinal waist position at the first pickup is
assumed for Eq.(7). By not starting from a waist
\( \Delta w_{\text{rms}} \) can be get by using 3 or more pickups.
At low energies, where space charge forces cannot be
neglected Eq.(7) is no longer valid. But here the use
of two pickups allows a test of the high frequency
corrections in \( Z(f) \). If we choose \( \Delta t_2 \geq 2 \Delta t_1 \), then
\( \Delta t_\text{rms} = \Delta t_2 \) and therefore \( \Delta t_\text{rms} \) can be determined without
knowing \( Z(f) \) explicitly.

4. Asymmetric Particle Distribution
All above given arguments for getting the rms bunch
width are also valid for an arbitrary asymmetric
particle distribution as long as each nonsymmetric

\[ \Delta t_\text{rms} = 0.68 (\Delta t_{\text{rms}} - \Delta t_1) \] (8)

or \( \Delta t_2 = 12 \) for \( \Delta t_\text{rms} = \pm 100 \)

In Fig. 3 a normalized parabolic beam pulse with
\( \Delta t_\text{rms} > 100 \) is compared with the reconstructed one
\( N = 12 \). The agreement is quite reasonable.
Fig. 5: Measured amplitude response of the 'small' pickup (scale div.: 10 dB, upper curve: 180 MHz, lower curve: 320 MHz)

Fig. 6: Measured amplitude response of the 'large' pickup (scale div.: 10 dB, 200 MHz)

Fig. 7: Measured phase shift of the 'large' pickup can be predicted at high frequencies and especially how sensitive is the bunch shape reconstruction to pickup noise. Here you expect some difficulties because at the needed high frequencies the beam components are small (Fig. 1) and amplitude and phase response are fluctuating.

In Fig. 5 the measured amplitude response $Z(f)$ is shown for a 'small' pickup ($d = 15$ mm, $C_p = 4.3$ pF) up to $f = 8.4$ GHz. Below 5.2 GHz the pickup behaves exactly like a RC-highpass with $f_c = 700$ MHz. The notch at $f = 5.6$ GHz is caused by a non optimal connection of the electrode to a 50 Ω cable. Afterwards microwave oscillations can be seen caused by the too large pipe radius $b = 3.5$ cm of the testbox. For the used coaxial 50 Ω transmission line the lowest TE11 mode occurs at $f = 1.9$ GHz.

The obtained smooth amplitude response up to 5 GHz allows the detection of short bunches with $T_{rms} \leq 25$ psec or a total length $T \sim 100$ psec (see Chapter 1). A new design is going on with a smaller radius of the testbox ($b = 2$ cm) and with a better connection of the electrode to the 50 Ω cable in order to improve the response up to 8 GHz. This will allow a bunch detection up to $T_{rms} = 4.125$ psec or a total length $T \sim 50$ psec.

In Fig. 6 and 7 the amplitude response (up to $f = 5.6$ GHz) and phase shift (up to $f = 1.3$ GHz) are shown for a 'large' pickup ($d = 35$ mm, $C_p = 10.6$ pF). This pickup behaves like a RC-highpass with $f_c = 230$ MHz.

In Fig. 8 and 9 a nonideal input pulse and the reconstructed one (dotted line) are shown together with the voltage signal of the 'large' pickup. The pulse repetition rate is 200 MHz and the rms pulse length is about $\pm 750$ psec. According to Eq. 8 an approximate reconstruction can be done with about 3 harmonics. With more than 10 harmonics you get a detailed reconstruction. The shown almost identical reconstruction is done with 15 harmonics ($f_0 = 3$ GHz). For amplitude and phase response the theoretical values of a RC-highpass with $f_c = 230$ MHz are used. The reconstructed pulse shape is insensitive to the fluctuations in the pickup amplitude and phase response.

The shape of the voltage signal does not correspond to the form of the input pulse. But the peak to peak distance of the voltage signal is given by the F.W.H.M. value of the input pulse signal. This behaviour is typical for all highpass devices where the bunching frequency $f_0$ is smaller or equal than the lower 3 db frequency $f_c$.

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