NONLINEAR BEAM BEHAVIOUR IN THE CERN ANTIPROTON COLLECTOR

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Introduction

The recently commissioned CERN Antiproton Collector (AC) Ring is part of the Antiproton Accumulator Complex (AAC) [1]. The function of this ring [2] and its present performance [3] are covered elsewhere in this conference. Critical, of course, is the number of antiprotons this Complex can deliver to the experimenters. A determining factor is the yield of antiprotons from the production target that are captured in the Collector ring. The antiprotons from the target have both a broad momentum distribution and a large amplitude distribution. This makes their efficient collection a formidable task. The large phase space distribution of the particles means that the nonlinear characteristics of the AC are important in determining the yield of p's. The results of studies using theoretical modelling plus experimental measurements of the nonlinear behaviour of the AC ring are presented in this paper.

Two types of nonlinear behaviour are investigated. First, there is the variation of the optical parameters with the momentum of the particles. This is reflected in the tune variation with momentum. Second, the nonlinear motion of the particles at increasing amplitudes introduces nonlinear resonant coupling behaviour in the envelope of the beam.

Tune variation with momentum

To make full use of the large momentum bite of antiprotons collected in the ring it is necessary that the tunes should not vary much with momentum. The particles should not cross low order resonances while their momentum spread is reduced during bunch rotation. Three families of sextupoles are included in the ring to ideally reduce the chromaticity and dispersion variation to near zero [4].

In small storage rings with short quadrupoles the nonlinear field terms in the quadrupole end-fields cannot be neglected. In fig. 1 the measured tune variation with momentum is plotted along with the theoretically predicted values with and without the inclusion of the end-field effects. Indeed, the horizontal tune does exhibit a quadratic dependence on momentum attributable to the pseudo-octupole terms in the quadrupole end-fields. The curves in fig. 1 include an additional arbitrary sextupole component compared to the predictions in ref. [5]. This is equivalent to an error of about 5% in the strength of the chromaticity correction sextupoles which cannot be trimmed out in the machine independently of other machine parameters.

It was possible to avoid that the extreme momentum particles cross the horizontal 1/2 integer resonance by moving the
tunes down from their design values. The amount of shift is limited by the present power supply configuration, but has so far proved adequate for avoiding a more tedious reshimming of the magnets.

Large amplitude behaviour

The necessity for strong chromaticity correction sextupoles brings with it the disadvantage that particles with large betatron amplitudes see strong nonlinear fields. This is particularly evident in the AC ring because of its large linear design acceptance. A numerical tracking program was used [6] to study these effects in the machine configuration as we now know it, fig. 2, where the motion in the horizontal and vertical phase planes is clearly smeared when the particle's amplitude is close to the 200f in both planes.

As a first step in deciphering this nonlinear motion it is useful to take the Discrete Fourier Transform of the tracking data, also shown in fig. 2 for each plane. This is a particularly powerful technique since the results can then be directly compared to the Fourier analysis of the measured beam response to a kick in the machine. The spectra reveal the frequencies of the harmonics of the nonlinear resonances present in the particle
motion. For example, the presence of resonances is seen with frequencies $Q_H - Q_V, Q_H + Q_V, 2Q_H + Q_V$. These are manifestations of nonlinear coupling resonances driven by the crossterms in the sextupoles, but do not necessarily correspond to lines with the same nomenclature in the tune diagram.

In order to compare the predicted amplitudes and phases of these resonances it is desirable that the model used for tracking accurately describes all the known nonlinearities in the machine. That is, not only the sextupole component of the ring should be correctly described, but also the contribution from the nonlinearities in the fringe fields of magnets, as was described in the preceding section.

**Measured coherent oscillations**

In the AC small emittance proton test beams can be reverse injected via the accumulator ring. Firing the injection or ejection kickers at reduced strength allows one to give the beam a coherent kick with amplitudes as great as the acceptance limit in the ring, if desired. In the vertical plane there is no kicker but the beam can be missteered at injection to produce coherent vertical oscillations with amplitudes up to about 100π, or half the acceptance in the AC. The signal from a beam position monitor is digitized at each turn of the bunched beam in the machine. This is a useful diagnostic and has been successfully used at a number of other accelerators [7,8] as well as at CERN [9,10].

Some features of the measurements made in the AC are that the test beam has a very small emittance of about 2π compared to the 200π acceptance of the machine. Second, since coupling phenomena are dominant it is necessary to record the motion in both planes simultaneously. Third, only one pickup is used per plane so that the phase of the oscillations is found by reconstructing the motion from the Inverse Fourier Transform of the data.

The acquired information can be Fourier analyzed on-line in the control computer which accesses the digitized data [11], or the data can be further analyzed off-line on a larger computer. The amplitude and phase information from a Fourier analysis of the measured coherent oscillations of a bunch are shown in fig. 3. An example of a measurement where large coherent oscillations are generated in both planes simultaneously was chosen here to illustrate the nonlinear coupling that occurs when the amplitudes are large horizontally and vertically. The same harmonics are present in the spectra of the experimental data as in the tracking data in fig. 2, corresponding to nonlinear coupling resonances. As with all coupling phenomena in oscillatory motion there is an alternating transfer of energy between the two planes so that the oscillation amplitude grows in one plane at the expense of the other plane. Unfortunately for the β's this means that they hit the aperture limit sooner and are lost from the machine. In other words the dynamic acceptance of the ring is reduced. How this occurs is made more evident in the following section. A more detailed treatment of the data analysis, in particular the use of Inverse Fourier Transforms and noise filtering techniques is made elsewhere [12].

Measurement of the coherent beam oscillations has also been used to analyze and compensate linear coupling in the machine and also to study resonances for off-momentum particles, though limited space prevents analysis in detail here.
Analysis of nonlinear motion

Since it is the apparent emittance of the beam that is influenced by the nonlinear coupling we turn our attention to the behavior of the invariant of the particle motion. After each revolution of a particle, or bunch, in the ring it is possible to calculate the well-known Courant-Snyder invariant from its phase space coordinates and a knowledge of the linear lattice functions at the observation point. The invariant has the dimensions of emittance so plotting this quantity as a function of beta function phase advance, for example, over many turns shows the amount of apparent blowup in the beam emittance, or alternatively, reduction in the dynamic acceptance. Furthermore, taking the Discrete Fourier Transform of the computed invariant data [12] reveals the harmonics that are modulating the beam envelope, as opposed to the single particle resonances. Modulations of the beam envelope are also more relevant to the dynamic acceptance of the ring.

Since the nonlinear motion is due to coupling between the two planes the next step is to plot the invariant as a function of the horizontal and the vertical phase advance of the particle as a 3-dimensional surface. This gives the surface of section of the 4-dimensional phase space of the particle [13]. In the case of the horizontal invariant, fig. 4, the motion is seen to be influenced by the proximity of both sum and difference nonlinear coupling resonances. The fuzziness, or “smear”, in the tracking data in fig. 1 is now clearly resolved into regular motion on a single valued surface. A program has been written for the Fourier analysis and computing of the invariant surfaces from an arbitrary set of tracking data [14]. Both the horizontal (shown) and the vertical invariant surfaces are computed by the program as well as new invariants of the motion of the form \( mJ_H + nJ_V \) or \( mJ_H - nJ_V \).

Acknowledgements

Thanks to H.-O. Kuylenstierna for his help with coherent oscillation measurements [11]. As always, the comments and criticisms of my colleagues in the AR Group have been most thought provoking.

References

1. E. Jones, these proceedings.
2. B. Autin et al, these proceedings.
3. F. Pedersen et al, these proceedings.
9. E. Asseo, J. Bengtsson and M. Channel, these proceedings.
10. R. Cappi, CERN/PS/87-48 (PSR).