FAST INSTABILITY OF POSITRON BUNCHES IN THE CERN SPS

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Abstract

LEP will be filled by repetitive injection of electron and positron bunches accelerated in the SPS from 3.5 GeV, the energy at which they are injected from the CPS, up to 20 GeV \(^{1}\). The nominal intensity per bunch has been chosen to be just below the predicted threshold for the transverse mode coupling instability, which correspond to a peak current of about 1A. Above this value fast beam losses are indeed observed after only 10 to 40 turns in the machine. Fast growing signals are detected in the high frequency region above 1 GHz both on longitudinal and transverse monitors. A description of the phenomenon is given in terms of both the mode coupling and the Beam Break Up theory. Computer simulations which reproduce well the observed features, are used to refine previous estimates of the SPS transverse coupling impedance.

Introduction

The transverse mode coupling instability has been observed in many e+e- storage rings \(^2\),\(^3\). These machines are filled by repetitive injection and accumulation of low intensity particle bunches; when the accumulated current reaches the threshold for instability a fast, usually vertical increase in the beam size is noticed, leading to partial beam loss and thereby limiting the intensity which can be accumulated in the machine. The explanation of this phenomenon, which had remained mysterious for a number of years, was given in 1980 by Kohaupt \(^4\) in terms of the coupling of head-tail modes. This was an extension to the transverse motion of a theory proposed by Sacherer \(^5\) to explain the longitudinal microwave instability through coupling of the longitudinal coherent bunch modes.

The head-tail modes of order \(m \pm 1\) are standing-wave patterns which oscillate at frequencies \(\omega_m \pm m \omega_{\text{s}}\), where \(\omega_m\) is the betatron and \(\omega_{\text{s}}\) the synchrotron frequency. The signal induced by mode \(m\) in a transverse monitor presents \(m\) nodes, so that its Fourier spectrum can be approximated by the spectrum of the bunch line density but displaced towards higher and higher frequencies as \(|m|\) increases. At higher beam intensities the wake fields change the oscillation frequency of the different head-tail modes by different amounts, so that adjacent modes \(m\) and \(m+1\) can merge and become coupled. Instead of two independent stable standing waves, the oscillation is then described by a travelling wave running along the bunch, and the amplitude of this travelling wave can grow exponentially under the influence of the wake fields.

In the SPS, when this machine is used as LEP injector, bunches of electrons and positrons are injected in one shot at 3.5 GeV/c from the CPS. While in e+e- storage rings the threshold for the mode coupling instability can only be approached from below, in the SPS it is possible to inject directly bunches with intensities up to four times the threshold value. In this case a fast beam loss occurs after 10 to 40 revolutions, that is within a fraction of a synchrotron period. Under these circumstances the concept of head-tail modes loses its meaning; instead of using the mode coupling model which is adequate when the threshold is approached from below, it seems more appropriate in this case to describe the interaction of the bunch with the accelerator structure in terms of the beam-break-up theory, which has been developed to calculate similar effects in linear accelerators. In the following both approaches are used in a complementary way to explain the observed threshold value. In addition the detailed features of the phenomenon are rather faithfully described by a computer simulation using multiparticle tracking.

Experimental observations

Fig.1 shows the evolution in time of the intensity of a single bunch after its injection into the SPS. One observes a sharp loss after 11 revolutions, while after about 30 revolutions only one third of the beam remains in the machine. The relevant beam parameters are shown in table 1. The synchrotron tune is \(Q_s = 0.015\), and therefore one expects a negligible influence of the longitudinal RF focussing on this fast phenomenon. Indeed switching the RF voltage on or off does not change the loss pattern. Fig. 1 has been recorded with RF off. The chromaticity was close to zero and slightly positive. Fig.2 shows the signal from a wide band vertical position monitor. The signal from the exponential directional coupler (band width 0 to 2 GHz) is fed to a battery of band-pass microwave filters, amplified and peak-detected. The bunch length is \(\Delta \eta = 64\) cm, and therefore the spectrum of the bunch line density extends to about 500 MHz. Indeed with filters tuned to [500 - 600] MHz one observes, starting from injection, a constant signal with small oscillations superimposed (the signature of residual injection oscillations). In contrast with this situation, when filters are tuned to [1.4 - 1.7] GHz one observes no signal on the first turn (there is no component of the bunch spectrum in this frequency range) but the signal grows on succeeding turns to reach a maximum at turn 11, which is where the first loss occurs in Fig.1.

Such signals can be observed between 1 and 2 GHz (which is the upper limit of our monitoring system) with a broad maximum around 1.5 GHz. From this we infer that after entering the machine the bunch...
begins to distort vertically, this movement being amplified turn after turn until part of the beam hits the machine aperture and gets lost. The frequency limit 25 Hz suggest that there are about 3 "wiggles" along the bunch. This is what should be expected if the coupling impedance of the machine has its maximum around 1.5 GHz (the broad band model used up to now to describe collective phenomena in the SPS peaks at 1.3 GHz).

The directional coupler which has been used to obtain Fig. 2 comprises 4 strip lines which can be combined in different ways to measure as well the horizontal motion or the line charge density. Indeed signals comparable to Fig.2 can also be detected in these different configurations of the monitor, so that more work will be required to show whether instabilities occur really in all 3 degrees of freedom or whether there is cross talk in the monitor. For the time being the most probable explanation is that the phenomenon is mainly vertical, although a longitudinal microwave instability seems to develop at the same time. This conjecture is sustained by an experiment in which the beam loss was shown to occur earlier when the vertical aperture was limited by scrapers.

Fig. 3 shows how the number of turns after which the first loss occurs vary with the bunch intensity. It can be seen that below 1.3 $10^{10}$ particles the losses disappear. However, signals can be detected on the transverse monitor down to $10^9$ particles per bunch, so it is proposed to take this last value as the experimentally determined threshold.

The mode coupling approach

The first calculations concerning the mode coupling instability were relevant to small electron or positron bunches. In this case the spectrum of mode $m = 0$ extends into the GHz range, where the resistive component of the coupling impedance is large, and it is likely that modes $m = 0$ and $m = -1$ couple first and determine the threshold. A rough but handy criterion to evaluate the threshold is that the tune shift of mode $m = 0$ equals the synchrotron tune $Q_s$, since modes $0$ and $-1$ are separated in frequency by $Q_s$ at low intensity. For the long bunches of the SPS, it is the head-tail modes $-2$ and $-3$ which mainly overlap the resistive component of the transverse coupling impedance. The spectrum of the fundamental mode $m = 0$ extends only to 500 MHz, and in this range the coupling impedance is mainly inductive, producing a tune shift but negligible coupling to the adjacent modes, so that the basis on which the above criterion rested disappears. In this case an exact calculation or the evolution of a large number of head-tail modes is necessary. This has been done by Chin.

using a broad band impedance model consisting of a resonator with $Q = 1$ and resonant frequency 1.3 GHz, which was the model currently used in the SPS. The values of $R/Q$ is deduced from beam measurements, and varies over a wide range from 12.5 to 47.5 $\text{MeV}^{-1}$ depending on the measurement and the way it is interpreted. Taking the highest value of 47.5 $\text{MeV}^{-1}$ Chin finds the threshold for coupling of modes $-2$ and $-3$ around 0.7 $10^{10}$ particles per bunch. As will be explained later, simulation studies show that the observations are best reproduced by using an impedance $R/Q = 23$ $\text{MeV}^{-1}$. Scaling to this value brings the threshold determined by Chin to 1.4 $10^{10}$. A previous work by Zotter gives exactly the same threshold when scaled to the same impedance. This value is slightly higher than the threshold found experimentally or by computer tracking (around $10^{10}$). It is interesting to note that at threshold the shift of the fundamental head-tail mode calculated by Chin amounts to 2 $Q_s$.

The Beam Break Up approach

The theory of cumulative Beam Break Up (or B.B.U.) was developed to explain instabilities of single bunches or trains of bunches in Linear Accelerators, in which particles travel only once through the machine structure. If the head of the bunch or the first bunch of a train is displaced from the central orbit, it excites transverse deflecting modes in the accelerating cavities, which in turn deflect the tail (or the following bunches of a train) when it crosses the same cavities later on. A similar phenomenon takes place in circular machines like the SPS. The intense wake fields which are generated in bellows and cross section variations of the vacuum chamber decay fast after the passage of the bunch, and have completely disappeared when the bunch comes back after one turn: to a single bunch, the machine appears like a long succession of identical but uncorrelated structures, like in a Linac. There is however a very important difference, which is due to the existence of dispersion in circular machines: this makes particles of different momenta travel at
different circular velocities. As a consequence a single bunch will stretch in azimuth after a certain time in absence of RF focusing, or will undergo synchrotron oscillations (the head replacing the tail and vice versa) if captured in an RF bucket. However when dealing with phenomena which develop within a fraction of a synchrotron period this aspect can be neglected and the B.B.U. model can be applied to explain the fast loss of positron bunches in the SPS.

For very short bunches a good description of the B.B.U. can be given with a two particles model. However in the SPS the bunches are 64 cm long, and the variation of the wake fields over the bunch length must be taken into account. A simple way of doing this is to approximate the bunch by a train of short bunchlets and apply the theory developed by Yokoya for bunch trains in Linear Colliders: if the bunchlet at the head is displaced at injection by Y0, the bunchlet at the tail will be displaced after n turns by Yn such that

$$Y_n = t/2 \frac{c}{\omega_b} \frac{Q_e}{Q_1} \left( \frac{\omega_R}{\omega_b} \right)^{1/2} \exp \left( \frac{\epsilon E}{2} + \frac{\epsilon E}{\omega_b} \right) \left( \frac{\omega_R}{\omega_b} \right)^{1/2}$$

(1)

with

$$Q_e = \frac{\text{Net} \omega_R}{\text{Net} \omega_b} n$$

where t is the bunch length, \( \epsilon = \omega_0/2Q_e \) is the damping characteristic of the resonator model of the coupling impedance, and \( \omega_b \) the betatron frequency.

With \( R_1/Q_1 = 23 \text{ M}\Omega \text{ m}^{-1}, Q_1 = 1, N = 4.10^{10}, E = 3.5 \text{ GeV} \) one has \( Y_n/Y_0 = 29 \) after \( n = 13 \) turns, which means that if the injection error is \( Y_0 = 1 \text{ mm} \), the tail of the bunch will be lost on the vertical aperture. This is a good description of what we observe in the SPS.

Another feature which appears in Fig.3 can be qualitatively explained, namely the change of slope of the curve at about 20 turns: this is just the time it takes for the bunch length to double in absence of RF capture, due to the difference in circular velocity. For small bunch intensities the growth rate of the instability is too small to compete with this stretching of the bunch, and a threshold naturally appears, which can be pushed to higher values of intensities by increasing the momentum dispersion in the beam.

**Computer simulations**

The program SIMTRAC is used to simulate turn by turn the behaviour of 10^6 superparticles. These are submitted to wake fields which correspond to the broad band model of the coupling impedance used in the other chapters. The bunch intensity is increased until particles reach an amplitude of 20 mm (about 13 mm, the machine aperture) in 1500 machine turns (23 synchrotron periods), and this defines the threshold for instability. One can also calculate the number of turns before amplitudes reach 13 mm, the machine aperture, as a function of the bunch intensity. The continuous curve in Fig. 3 shows the result of this exercise: it fits rather well the measured points provided the coupling impedance is adjusted to be \( R/Q = 23 \text{ M}\Omega \text{ m}^{-1} \) with a \( Q_1 = 1 \). Under these conditions the threshold found by the program is \( N = 10^{10} \). The tune shift of mode \( m = 0 \) at the threshold is 1.5 \( Q_0 \).

It has been pointed out recently that a better fit to the actual SPS impedance could be a broad band resonance with \( Q = 6 \). Formula (1) shows that in this case a lower threshold has to be expected if \( R/Q \) is kept constant. This is also well borne out by the simulation: in order to obtain a good fit to the measurements one has to take \( R/Q = 17 \text{ M}\Omega \text{ m}^{-1} \) when \( Q = 6 \). The reason is that the wake fields are more efficient to drive the instability if they do not decay too much within a bunch length.

The program was also used to calculate the threshold intensity for the longitudinal microwave instability. It shows that with a longitudinal impedance \( Z/n = 6.7 \text{ M}\Omega \) and \( Q = 6 \) (values suggested in ref.12) the longitudinal threshold is 0.5 \( 10^{10} \) for \( \epsilon_0/E = 6 \times 10^{-3} \) and 1.6 \( 10^{10} \) for \( \epsilon_0/E = 10^{-3} \). It is therefore not surprising that a longitudinal instability is observed at the same time as the transverse one for the lowest values of the momentum dispersion.

**Conclusions**

The fast instabilities observed on the positron bunches injected into the SPS at 3.5 GeV/c are reasonably well explained by existing theories: the mode coupling theory below and up to the threshold, and the Beam Break Up theory well above threshold. Computer simulations allow a precise fit to the data and are useful for the determination of the coupling impedance.

At 20 GeV, the energy at which the beams are injected into LEP, the threshold for the mode coupling instability is a factor two higher than at injection into the SPS. A better understanding of the phenomenon may thus allow, through adequate changes of the injection parameters, to eventually speed up the filling of LEP by a factor two.

**References**

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