Abstract: Air core superconducting cyclotrons are characterised by the feature of the magnet structure that is formed entirely from the current carrying superconductor configurations. The isochronous average magnetic field is provided by a spherical-like superconducting main coil while the focusing power is produced by the active flutter configuration. This design approach allows practically mutually independent design procedure for:
- isochronous magnetic field configuration
- modulation magnetic field configuration and
- suitable accelerating system

I. Iron Cored Superconducting Cyclotrons

In the case of an ordinary AVF cyclotron the concerned ferromagnetic structure has a dominant role in production of average magnetic field and field modulation for the purpose of axial focusing. The operating limit of an ordinary AVF cyclotron at a smaller pole radii is determined by the magnet bending power. When conventional coil is replaced by a superconducting one, the bending limit becomes considerably higher than the focusing limit, the latter then becoming an operating limit for energy-per-nucleon achievements in the region of light nuclei [1,2].

The most important types of the operating limits in the case of the iron cored superconducting cyclotrons are:

- $f/A = k_f (Z/A)$ focusing limit [2]
- $v_r = N/2$ and $v_z = N/2$ stopband limit
- $v_r + 2v_z = 3$ coupling resonance limit [3]
- $v_r = 1$ and $v_z = 1$ resonances

The stopband resonance limit and focusing limit are of a fundamental character. The coupling resonance limit can be crossed if the beam is perfectly centered and/or has the possibility to jump over the resonance. Less severe but still important are: saturation limit at $B_{av} < 2.35$ T and orbit separation limit [4].

I.1. Proton window [4,5,6,7]

In the early phase of design of MSU K500 superconducting cyclotron it was recognised that the problems of proton acceleration cannot be efficiently resolved in the type of machine which in the same time has to be used as a powerful heavy ion accelerator requiring to be designed with a sufficiently large radius in order to produce a sufficiently high bending constant $K_b$ and large focusing constant to bending constant $(K_f/K_b)$ ratio. Analytical considerations [4] confirmed in the numerical analyses [5,6,7] indicate that protons may be accelerated without experiencing potential blow up catastrophe, caused by the presence of the focusing limit [2] and dangerous coupling resonance $(v_r + 2v_z = 3)$ phenomena [3], only in the narrow window of the operatinal region determined by relation: $F = (3-\gamma)/4 \geq \gamma - 1 < F$ where $F$ field flutter, and $\gamma$ is the relativistic factor. Taylor series expansion of the "zero flutter" isochronous field:

$$B_{iso} = 3.11 B/R (1.5 \frac{B^2}{r(R)^2} - 1.5 \frac{r(R)^2}{B^2})$$

where $B = (v/c)_{max}$ also shows that an increment of the magnet pole radius $R$ (what is the basic requirement for the achievement of sufficiently high values of $k_f$ and $k_b$ at $\beta = \text{const}$ (constant proton maximum energy)) implies the decreasing of the local values of isochronous magnetic field. This feature combined with the fact that iron cored superconducting cyclotrons are characterized by constant value of the modulation field amplitude, gives rise to the overdosing of focusing power at respective value of proton energy. The results of numerical examination [5] of these problems, using K500 magnetic field data are given in figure 1, confirming the predictions based on "smooth approximation" and "zero flutter" formulae. If as an acceptable set of values of the extraction radii are accepted only the values in close vicinity of the value $R=26.5''$, it appears that:

1. The window for acceleration of protons, free of problems with $v_r + 2v_z = 3$ resonance and the lack of focusing power can be found in the region of proton energies between 125 and 155 MeV.

II. Air Core Superconducting Cyclotrons [5,8]

The prominent features of an air core design are:
- the mutually independent current settings in the superconducting main coil and flutter configuration
- the mutual independence of the symmetry numbers for accelerating system and flutter field configuration
- a considerable degree of freedom in accelerating system design
- considerable cost savings.

The limiting phenomena are under full control in
sense that maximum achievable energies/nucleon depend only on the symmetry numbers used and the performances of superconducting material employed. The maximum achievable proton energy per nucleon for air core superconducting cyclotrons is given in fig. 2 as a function of the field-symmetry number and minimum extraction radius allowed at maximum superconductor field value of 7 T at the current density of 30 kA/cm². The analyses of the feasibility of the air core cyclotron design satisfying the technical requirements at present status of the art has been performed up to the energies of 1 GeV/nucleon.

Figure 2. The maximum proton energy achievable as a function of field symmetry number and minimum radius at allowed total field in superconductor of 7 T.

In particular beam dynamics and basic design considerations are given for the air core superconducting cyclotron for acceleration of protons up to 115 MeV and Z/A = .5 heavy ions up to 45 MeV/nucleon (as a type of machine which can respond to the demands of contemporary medical and industry applications and can also have the distinguished performances as an heavy ion accelerator).

The 115 MeV air core superconducting cyclotron consists of a spherically shaped superconducting main coil composed of four component coil sections, generating the average isochronous magnetic field of desired accuracy and the multipole axially symmetrical superconducting flutter configuration producing the modulation field of a sufficiently high amplitude to obtain necessary focusing power for acceleration of protons and heavy ions up to the energies determined by a bending power of the cyclotron magnet. The parameters of the cyclotron designed to accelerate 115 MeV protons and 45 MeV heavy ions with Z/A = .5 are given in table 1.

<table>
<thead>
<tr>
<th>Field symmetry number N</th>
<th>Outer coil radius</th>
<th>Inner coil radius</th>
<th>Extraction radius</th>
<th>Max. average current density</th>
<th>Max. frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>50 cm</td>
<td>13 cm</td>
<td>38 cm</td>
<td>30 kA/cm²</td>
<td>56.8 MHz</td>
</tr>
</tbody>
</table>

Table 1.

Actual average magnetic field compared to the isochronous magnetic field required to accelerate protons up to the energy of 115 MeV is shown in fig. 3a. This average field is produced with a spherically shaped main superconducting coil, composed of four independently powered sections shown in fig. 3b. The frequencies of radial and vertical oscillations together with the phase slip for 115 MeV protons are shown in fig. 4. The provided field accuracy permits sin of a phase slip to be retained below .5 without trim coils employment at an energy gain of .6 MeV/turn.

Figure 3. a) Actual average main coil magnetic field at proton energy 115 MeV b) Main coil magnetic structure and field lines from POISSON code calculations.

Figure 4. Frequency of radial (1) and axial (2) oscillations together with the phase slip (3) at proton energy of 115 MeV, based on the usage of 4 section in main coil, only.
III.1. Design considerations

1. Superconductor field calculations. The overall current densities applied to produce the required values of the average and modulation fields are less than 30 kA/cm². At the maximum superconductor field value of 7 T the required overall current density can be retained stable at 80% of the critical current and packing factor of 0.8 employing 2:1 copper to superconductor ratio.

2. Force calculations. The exerted magnetic force per unit volume $F_{mag} = JB$ at field level of 7 T gives rise to the characteristic length of 1 cm for the eventual point disturbance under the action of respective magnetic pressure. This value is smaller than the characteristic length of the point disturbances which may be stabilized by the Cu:Sc ratio 2:1 (1.5 cm). The results of GFUN3D calculations of azimuthal change of body-force-per-unit-length are shown in fig.5.

3. Stress calculations. One of the most important superconducting magnet design problems is that of minimizing thermal stresses which arise in magnet always when magnet is brought from the room temperature to 4.2 K. Thermal stress induced by a temperature gradient in the component of two-material ($i,j$) composite may be estimated from: $\sigma_{ij} = (dL)/L (E_j/A_i + E_i/A_j) A_j$, where $dL$, $E$, and $A$ are changes of the length, $E$ is the Young module value while $A$ is the component cross-section. The resulting value $\sigma_{ij} = 7$ MPa shows that at such a low copper to superconductor ratio the thermal stresses and induced changes in the component resistivities should be carefully analysed.

The Lorentz-force interaction between the current and the field results in stresses within the coil which tend to burst the coil radially outward and to crush it axially. The upper limits of tangential component of tensile stress of 78 MPa and axial compressive stress of 24 MPa in case of the self-field forces are estimated from the calculated axial and radial component of body force-per-unit-length. The samples of Stansol code numerical calculations of radial and hoop stresses, and radial displacements for the main coil section # 1 closest to the median plane, are given in fig. 5.

4. Quench calculations. The major objective of good magnet design is to avoid quenching due to conversion of e.m. energy into heat. The adopted design criteria was that energy density $B^2/2\mu_0 E/V$ should be always retained below the value which could generate the magnet coil temperature higher than 100-150 K (100 MJ/m³) able to induce the development of excessive thermal stresses. The basic process of quenching: temperature rise, voltage drop (which may be the source of arcing between the turns), spreading of the normal zone and derived protection techniques where the subject of the careful magnet stability analyses. Careful checkings of the different protection schemes were made employing numerical quench code calculations. The typical results are shown in fig.6.

5. The magnet heat inleak and cool-down calculations have shown that consumption of 9 TIL/h. of liquid He could be expected.

6. The effective screening of the magnetic field, if required to prevent the experimental set up, which cancels fringing field at $R_2/R_1$ ($R_1$ radius of main coil) could be obtained using the spherical like coil of radius $R_2$ at 8 lower current density than those employed in main coil.

Figure 5. Body force-per-unit-length and stress calculations 1) radial force; 2) axial force; 3) radial stress

Figure 6. a) Temperature and normal zone development; b) Voltage drop and current decay in coil # 1

References
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