MEASUREMENT OF COMPLEX COUPLING DRIVING TERM OF LINEAR DIFFERENCE RESONANCE USING TURN-BY-TURN BEAM POSITION MONITORS

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Abstract
We have developed a method of measuring a complex coupling driving term \( C \) of linear difference resonance using turn-by-turn beam position monitors (BPMs) on the basis of the perturbation theory with the single resonance approximation. Since both amplitude and phase of \( C \) are obtained, we can uniquely determine strengths of counter skew quadrupole magnets for correction of the linear resonance coupling. We measured the coupling driving term \( C \) using the developed method at the SPring-8 storage ring where small skew quadrupole components are distributed as error magnetic fields. The strengths of the counter skew quadrupole magnets derived from the measured \( C \) were consistent with the strengths adjusted to minimize vertical beam size near the linear difference resonance.

INTRODUCTION
Betatron coupling is one of the essentials for machine performance in circular accelerators such as an electron storage ring for high-brilliance light source and a particle collider of high luminosity. A strong coupling may induce optics distortion, shifts of betatron tunes and excitation of resonances. In case a certain equipment installed in the ring causes the betatron coupling, a local compensation of the error field near the equipment can be an effective mean. On the other hand, if the coupling comes from small error skew quadrupole components distributed along the ring, the local compensation is not a practical method. The small error fields can be made by complicated factors such as alignment errors and field errors of magnets of the ring. In that case, a global compensation of the betatron coupling can be a practical way. Each error skew field is very small, but if the small errors are distributed all over the ring, their integration can make a non-negligible contribution to the coupling driving term \( C \). For the global coupling correction, two independent families of skew quadrupole magnets should be prepared to generate compensation terms to \( C \) in quadrature.

COUPLED BETATRON MOTION
We suppose small skew quadrupole magnetic components are distributed as error fields along the circumference of a circular accelerator. In such a case, the horizontal \((x)\) and vertical \((y)\) betatron beam motions can be coupled.

\[
x(s) = \left( A_1 e^{\frac{-2\pi i \nu_1 x}{L}} + A_2 e^{\frac{-2\pi i \nu_2 x}{L}} \right) \sqrt{\beta_x(s)} e^{2\pi i \phi_x(s)} + c.c.
\]
\[
y(s) = \frac{C}{2} \left( \frac{A_1}{\nu_2} e^{\frac{2\pi i \nu_2 y}{L}} + \frac{A_2}{\nu_1} e^{\frac{2\pi i \nu_1 y}{L}} \right) \sqrt{\beta_y(s)} e^{2\pi i \phi_y(s)} + c.c.
\]

where \( c.c. \) denotes complex conjugate of the preceding term, \( \beta_{x,y}(s), \phi_{x,y}(s), A_{1,2}, L \) and \( s \) are betatron functions, phase advances, integration constants, circumference of the ring and beam path length, respectively. The shifts \( \nu_{1,2} \) of unperturbed betatron tunes \( \nu_{x,y} \) are as follows,

\[
\nu_{1,2} = \frac{1}{2} \left( \Delta \pm \sqrt{\Delta^2 + |C|^2} \right),
\]

where \( \Delta = \nu_x - \nu_y - q \). The complex number \( C \) in eq.(1) is referred to the coupling driving term and is given by the following formula,

\[
C = \frac{1}{2\pi} \int_0^L K_{err}(s) \sqrt{\beta_x(s)\beta_y(s)}
\]
\[
\times e^{2\pi i [\phi_x(s) - \phi_y(s)] - \frac{i}{2} \Delta} ds
\]

\[
\equiv |C| e^{2\pi i \phi_c}
\]

where \( K_{err}(s) \) is the field strength of skew quadrupole errors.

COUPLING DRIVING TERM MEASURED BY TURN-BY-TURN BPM
We measured the coupling driving term \( C \) using a turn-by-turn BPM system of the SPring-8 storage ring. The \( x-y \) betatron oscillations expressed by eq.(1) were induced by pinging a single-bunch beam stored in the storage ring.
in the horizontal direction with a pulsed bump magnet for 
beam injection. The betatron oscillations were then ob-
served using the turn-by-turn BPMs located about every 
two cells of the storage ring. To increase sensitivity of 
measuring the vertical oscillation induced by the betatron 
oscillations, we chose an operation point at the vicinity of 
the difference resonance. Equation (1) can be reduced to 
the following formulas suitable for fitting with measured 
turn-by-turn BPM data: if \( \Delta > 0 \),

\[
x_n = e^{-nD}\left\{ A_u \cos 2\pi (nv_u + \Phi_u) \\
+ A_v \cos 2\pi (nv_v + \Phi_v) \right\},
\]

\[
y_n = e^{-nD}\left\{ B_u \cos 2\pi (nv_u + \Psi_u) \\
+ B_v \cos 2\pi (nv_v + \Psi_v) \right\},
\]

where the damping factor \( e^{-nD} \) was artificially added, \( n_u \) 
and \( n_v \) are tunes of the eigen-modes \((u, v)\), respectively, 
and \( n \) is the turn number. If \( \Delta < 0 \), \( n_u \) should be exchanged 
for \( n_v \) in eq.(4). Free parameters in the fitting process are 
\( A_u, A_v, B_u, B_v, \Phi_u, \Phi_v, \Psi_u, \Psi_v, \nu_u, \nu_v \) and \( D \). Figure 1 
shows an example of the turn-by-turn BPM data with the 
fitted curves. We can observe a modulation of oscillation 
amplitudes with frequency \( \frac{|n_u - n_v|}{T_{rev}} \) in the \( x \) and \( y \) 
directions, where \( T_{rev} \) is the revolution time. The modulation 
in the \( x \) and \( y \) directions has a phase difference of 
90 degree. This is a typical feature of the coupling resonance 
which exhibit the exchange of energy between the two planes.

The absolute value \( |C| \) is derived from the fitted eigen-
tunes \( n_u, n_v \) and amplitude parameters \( A_u, A_v, B_u \) and 
\( B_v \). We can obtain the expression for \( |C| \):

\[
|C| = |n_u - n_v|\sqrt{1 - \left( \frac{A_v B_u + A_u B_v}{A_u B_u - A_v B_v} \right)^2}. \tag{5}
\]

The phase \( 2\pi \phi_c \) of \( C \) is derived from the fitted phase 
parameters \( \Phi_u, \Phi_v, \Psi_u \) and \( \Psi_v \). We can obtain the expression 
\( |C| \) and \( \phi_c \) as follows:

\[
\phi_c = \begin{cases} 
\Psi_u - \Phi_u + \phi_x - \phi_y - \Delta \cdot s_i/L, & \text{or} \\
\Psi_v - \Phi_v + \phi_x - \phi_y - \Delta \cdot s_i/L, \end{cases}
\]

where \( s_i \) denotes the beam path length at the \( i \)-th BPM. We 
will design values for the phase advances \( \phi_x, \phi_y \) of the 
unperturbed system, which are defined as zero at the beam 
injection point \( s = 0 \).

The fitting was carried out for the data of each turn-by-
turn BPM, and so we obtained as many experimentally de-

erived \( C \) as the number of the BPMs. The results are shown 
in Figure 2, where the obtained \( |C| \) and \( \phi_c \) are plotted as 
a function of the path length \( s_i \) of the \( i \)-th BPM. We see that 
\( |C| \) and \( \phi_c \) in Figure 2 are almost independent of \( s_i \), 
and we adopted averages of these values at all the BPMs 
as measured values of \( |C| \) and \( \phi_c \). The averages were 
\( |C| = 0.01628 \) and \( \phi_c = 0.492 \). Consequently, the real 
and imaginary parts were \( \Re(C), \Im(C) = -0.01625, 
0.00085 \).

Figure 2: The absolute values (filled circles) and the 
phases (open circles) of \( C \) derived from the turn-by-turn 
BPM data measured at the operation point \((\nu_x, \nu_y) =
(40.3509, 18.3477)\). The obtained \( |C| \) and \( \phi_c \) do not almost 
depend on the path length \( s_i \) of the \( i \)-th BPM.
THE STRENGTH OF THE COUNTER SKEW QUADRUPOLE MAGNETS

The SPring-8 storage ring is composed of the 36 normal Chasman-Green cells (CGCs) and the four magnet-free long straight sections (LSSs) of 27m with matching sections [3]. Two-family counter skew quadrupole magnets for the coupling correction are installed at the achromatic arcs (ARCs) of the CGCs every two cells and at both ends of all the LSSs. Approximately, the phase $2\pi(\phi_x - \phi_y)$ in eq.(3) has a difference of $m\pi$ between the two ARCs or the two LSSs, and a difference of $(m + 1/2)\pi$ between the ARC and the LSS, where $m$ is integer. A compensation term $C_{SK}$ to the coupling driving term $C$ is replaced by replacing $K_{ARC}$ of eq.(3) with the strengths of the counter skew quadrupole magnets of the ARC and the LSS. The phases $2\pi(\phi_x - \phi_y)$ at the ARCs and the LSSs indicate that the compensation terms generated by the two-family correction magnets are in quadrature. Requiring $C_{SK}$ to be equal to $-C$, we can uniquely determine the total strengths $K_{ARC}$ and $K_{LSS}$ of the counter skew quadrupole magnets of the ARCs and the LSSs, respectively. Using designed values for the betatron functions $\beta_{x,y}$, the phase advance $\phi_{x,y}$ and $\Delta$ in the unperturbed system, we obtained $K_{ARC} = -0.0129$(m$^{-2}$) and $K_{LSS} = 0.0261$(m$^{-2}$).

To demonstrate the validity of our method, we also carried out another approach to determine the strengths $K_{ARC}$ and $K_{LSS}$, which were adjusted to minimize the vertical beam size $\sigma_y$ expressed as the following formula [1, 2]: if the betatron coupling makes a dominant contribution to $\sigma_y$,

$$\sigma_y = \sqrt{\frac{\frac{1}{2}|C|^2}{\Delta^2 + |C|^2}} \beta_y \epsilon_0,$$

where $\epsilon_0$ is a natural emittance in the unperturbed system. The vertical beam sizes measured by the x-ray beam imager (XBI) [4] and the two-dimensional interferometer (2D-interferometer) [5] using bending magnet radiation, and the absolute values of $C$ measured by the turn-by-turn BPM are shown in Figure 3 as a function of $K_{ARC}$ and $K_{LSS}$. We chose a operation point near the difference resonance in order to increase the response sensitivity of the vertical beam size to the skew fields. The strengths $K_{ARC}$ and $K_{LSS}$ giving the minimum vertical beam size were $-0.0115$(m$^{-2}$) and $0.027$(m$^{-2}$), respectively, where the measured $|C|$ also became minimum. The strengths $K_{ARC}$ and $K_{LSS}$ deduced from the measured $C$ are almost consistent with that from the vertical beam size, but we can see a slight discrepancy between these two results. The possible explanations of the discrepancy are as follows; when the effective coupling driving term becomes very small with the counter skew quadrupole magnets, 1) eq.(3) is not applicable due to the higher order terms of $C$, 2) effect of the counter skew quadrupole fields on the vertical dispersion contributing to $\sigma_y$ is not negligible.

REFERENCES


SUMMARY

We have developed the method of measuring the complex coupling driving term $C$ using the turn-by-turn BPM. $C$ of the SPring-8 storage ring was measured by this newly-developed method and the validity was demonstrated experimentally. The field strengths of the two-family coupling correction magnets of the ARC and the LSS were deduced from $C$ measured by our method. The determined strengths were almost consistent with that giving the minimum vertical beam size. The small discrepancy between the strengths obtained by those two approaches will not be important for the practical betatron coupling correction.

Figure 3: Vertical beam sizes measured by the XBI (open circles) and the 2D-interferometer (filled circles), $|C|$ (crosses) measured by the turn-by-turn BPM as a function of $K_{ARC}$ and $K_{LSS}$. For both of the beam size monitors, the minimum vertical beam sizes are given when $(K_{ARC}, K_{LSS}) = (-0.0115$m$^{-2}$, $0.027$m$^{-2}$), where the measured $|C|$ also becomes minimum. The operation point was chosen at $(\nu_x, \nu_y) = (40.3818, 18.3451)$ near the difference resonance.

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