PANCAKES VERSUS BEER-CANS IN TERMS OF 6D PHASE-SPACE DENSITY

S.B. van der Geer*, M.J. de Loos, O.J. Luiten, Eindhoven University of Technology, The Netherlands

Abstract

Uniformly filled ellipsoidal (waterbag) electron bunches can be created in practice by space charge blow out of transversely tailored pancake bunches [1]. Ellipsoidal bunches have linear self fields in all dimensions, and will not deteriorate in quality under linear transport and acceleration. There is a discussion if such a bunch is better than a conventional beer-can shape. This paper compares the two approaches in terms of usable phase-space density. Detailed GPT simulations of a simplified setup show that although the pancakes approach requires less charge, it is the application that is decisive.

INTRODUCTION

There is an ongoing discussion about the best method to create high-brightness electron bunches by photoemission in an rf-cavity [1, 2, 3, 4, 5].

A photo-emission laser with a top-hat temporal and a top-hat transverse profile creates a bunch resembling a uniformly filled cylinder, also known as a beer-can distribution. Such a distribution has linear (transverse) self-fields in the core of the bunch, and the core therefore does not suffer from deteriorating effects due to non-linear space-charge forces.

Recently it was shown that uniformly filled ellipsoidal (waterbag) electron bunches can be created in practice [1]. The method relies on space-charge blow-out of a photo-extracted pancake bunch, created by a femtosecond laser pulse with a half-sphere transverse intensity distribution. In theory, the resulting ellipsoidal bunch has linear self-fields everywhere, not only in the core.

It seems that the pancake method is superior to the beer-can approach because it creates bunches with linear self-fields everywhere, instead of only at the core. Furthermore, the initial phase-space density in the case of pancakes is orders of magnitude higher because of the shorter laser pulse duration. In practice however, both methods suffer from image charges, non-linear external fields and the fact that the desired profiles are not perfectly matched by the laser-system. Furthermore, the methods suffer from deformation due to path-length differences between on- and off-axis particles.

In this contribution we compare both approaches by detailed GPT [6] simulations in a simplified test setup. We abandon the notion of best approach, and focus on understanding the differences and how these play a role for different applications.

* Also at: Pulsar Physics, The Netherlands, s.b.van.der.geer@pulsar.nl.

SET-UP

When comparing pancakes to beer-cans one can easily define a test-case such that either of the two is the best. For example, beer-cans are intrinsically longer and suffer more from time-dependent external fields. Given a sufficiently high frequency of the accelerating fields, beer-cans will fail (although the resulting non-linearities can be corrected to some extent with a higher-harmonic cavity downstream). Pancakes on the other hand require a short photo excitation laser pulse. If one chooses the laser-pulse duration sufficiently long, and the accelerating fields too low, a bunch with a shape not resembling an ellipsoid is created. To avoid fruitless discussions about the test case, we use a uniform acceleration field of 100 MV/m, stretching from the cathode to infinity. The initial particle distribution is summarized in table 1.

Table 1: Initial particle distributions.

	Pancake	**Beer-can**
Transverse profile	Half-sphere	Top-hat
Radius	1 mm	1 mm
Emission profile	Gaussian	Top-hat
	30 fs FWHM	10 ps
Emittance	0.41 μm	0.46 μm

Defining Beam Quality

Apart from the test-setup, there is an other obstacle comparing pancakes versus beer-cans: How do we define the best bunch? Lower emittance is better for almost any application, but it is not a priori clear if a 10% reduction in energy spread is equally important as a 10% reduction in transverse emittance. Defining a universal measure for beam quality is not trivial because every application has its own preference for a selection of phase-space coordinates. However, because we want to compare beer-cans versus pancakes in general terms, in this paper we take all phase-space coordinate as equally important. We do this by calculating the minimal 6D phase-space volume of a certain amount of charge. This 6D phase-space volume is a Lorentz invariant *fundamental* measure for beam quality. The measure is robust as it is insensitive to outliers. In this paper we set the desired charge to a somewhat arbitrarily chosen value of 100 pC. Now the main question of this paper is reduced to the following:

Which approach gets 100 pC of charge in the smallest 6D phase-space volume, given uniform 100 MV/m fields?

SIMULATION RESULTS

With the aid of the General Particle Tracer (GPT) [6] code, we simulated the test setup described in section . Both the pancake and the beer-can scenario have been simulated repeatedly, where the initial charge has been varied from 100 pC to 1 nC. The average 6D phase-space density of the *smallest* ellipsoidal volume containing 100 pC is shown in figure 1.

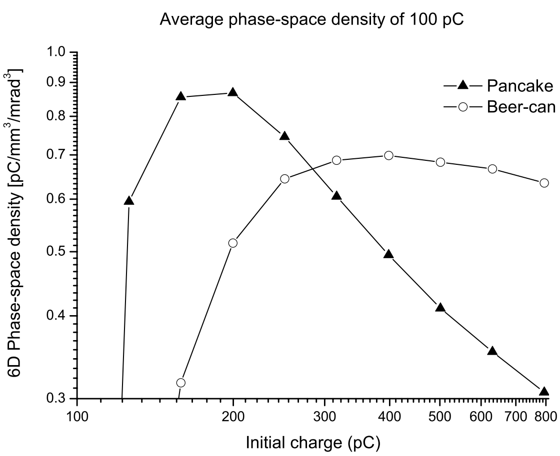

Figure 1: Average 6D phase-space density of the smallest volume containing 100 pC of charge, as function of initial charge. Shown are GPT simulation results at z=0.1 m in uniform 100 MV/m fields for the beer-can and pancake scenarios defined in table 1.

Both scenarios are able to reach the approximate same phase-space densities[1]. The crucial difference between beer-cans and pancakes is *how much charge* is needed to reach high phase-space densities. The beer-can approach needs to start with a few times more charge than is useful in the end, whereas the pancake approach requires only a few ten percent more charge. In other words, the beer-can approach pays a price in terms of additional initial charge because non-linear fields at the head and tail of the bunch renders a large fraction unusable.

Paradox

The question is: *why are pancakes not much better?* The femtosecond laser pulse duration of the pancakes is about three orders of magnitude shorter compared to the picosecond laser system of the beer-cans. Because the other five phase-space coordinates are similar, the initial phase-space density of the pancake approach is orders of magnitude higher. This difference in initial phase-space density is not reflected in the results previously shown. Clearly our simple test case is not able to capitalize on the higher initial phase-space density of the pancake approach.

[1]The fact that in this particular case the pancake is slightly better is due to the initial conditions. We have seen similar cases with the roles reversed, but in all cases the phase-space density was comparable. Furthermore, please note that a factor of 2 difference in 6-dimensional density translates to only a factor $\sqrt[6]{2} \approx 1.12$ in each of the phase-space coordinates.

Because we already know that in optimized systems the transverse emittance is more-or-less conserved [1], there must be a some dominant process in the longitudinal phase-space degrading the overall quality by orders of magnitude. Liouville dictates that local phase-space density is conserved, and our data analysis is insensitive to linear correlations between all six coordinates. Hence, the only possible way to reduce the core longitudinal phase-space is a non-linear correlation with an other phase-space coordinate.

Figure 2 shows the longitudinal phase-space for both a beer-can and a pancake for a typical initial charge of 400 and 125 pC respectively. Color-coded is the radius of the particles, and an almost perfect correlation appears: The shape in phase-space, for both scenarios, is a function of radius. One can visualize an ultra-thin curved surface by looking at figure 2 by interpreting the color as the 3^{rd} dimension. It is the thickness of this sheet that is conserved–if one assumes decoupling between the phase-planes–and this thickness is less in the pancake approach. However, in both cases the *apparent* thickness is fully dominated by the second-order dependency on the transverse coordinate.

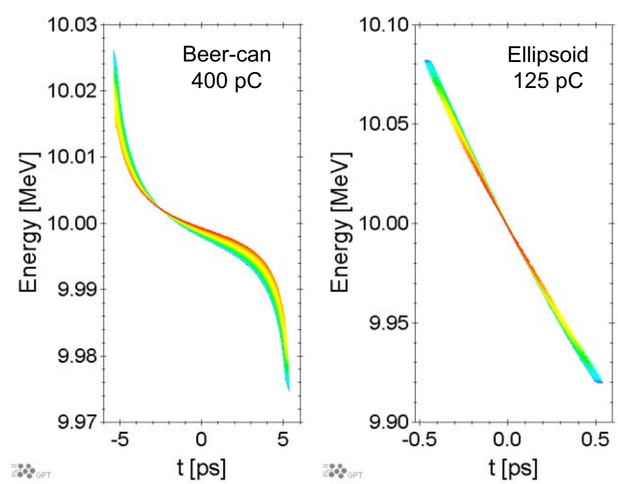

Figure 2: Longitudinal phase-space of the beer-can scenario at 400 pC, and the pancake approach at 125 pC. Color indicates particle radius, ranging from blue on-axis via green and yellow to red at the outer edge of the bunch.

DISCUSSION

The pancake approach requires less charge for similar phase-space density. The value of this advantage however is highly application dependent. In the case of a SASE-FEL for example, excess charge with low phase-space density simply does not contribute to the lasing process. Alternatively, there are applications where every part of the bunch contributes, such as ultrafast electron diffraction [7]. In that case excess charge will result in an overall blur and degrade signal to noise significantly.

From a technical point of view we note that the required femtosecond laser system for the pancake approach is not trivial, if possible at all, for the creation of pulse trains with high repetition frequency. Furthermore, pancakes rely on 'fast' cathode materials such as coper that have low quantum efficiency. On the other hand, pancakes only require transverse laser shaping, whereas the beer-cans need shaping in both space and time.

In our simple test setup presented in this paper, the usable phase-space density of both the beer-can and pancake approach is limited by non-linear transverse dependencies. No trace remains of the orders of magnitude higher initial phase-space density of the pancakes. Can this be corrected in a realistic setup? The correction of a similar effect, non-linear dependency of the slope in transverse phase-space as function of longitudinal coordinate, is well understood and commonly known as emittance compensation. We see no reason why the longitudinal phase-space dependency on the transverse coordinate couldn't be repaired as well. Based on figure 2 and promising results in [8] we speculate that the pancake approach is a better starting point for further improvement in the longitudinal phase-space. One can think of some predispersion in the injector such as a curved cathode or curved laser front [9]. An other option would be a downstream element tailor made for this purpose. If successful, it would make orders of magnitude higher phase-space density accessible for a variety of applications.

APPENDIX: PHASE SPACE VOLUME CALCULATION

Inspired by Ref. [10] the following procedure was used to calculate the minimal $k = 6$ dimensional phase-space volume of a fraction α of the beam. The input of the algorithm is the discrete set of *all* particle phase space coordinates $\mathbf{x}_i = (x_i, p_{x_i}, y_i, p_{y_i}, z_i, p_{z_i})$, where $i = 1, 2, \cdots, N$ with N the total number of particles in the beam. All coordinates are centered by subtracting the average, and the momenta are made dimensionless dividing them by mc, where m is the mass of the particles and c the speed of light[2].

The first step in the procedure is to fit a k dimensional hyperellipsoid through the entire distribution. This is conveniently done in terms of the $k \times k$ beam sigma matrix

$$\mathbf{\Sigma} = \begin{pmatrix} \langle x^2 \rangle & \langle x \cdot p_x \rangle & \cdots & \langle x \cdot p_z \rangle \\ \langle p_x \cdot x \rangle & \langle p_x^2 \rangle & \cdots & \langle p_x \cdot p_z \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle p_z \cdot x \rangle & \langle p_z \cdot p_x \rangle & \cdots & \langle p_z^2 \rangle \end{pmatrix}, \quad (1)$$

where $\langle \cdots \rangle$ indicates averaging over the entire distribution. The directions of the principal axes of the ellipsoid are given by the eigenvectors of the sigma matrix and the lengths of the principal axes follow from the corresponding eigenvalues.

The square root of the determinant of the sigma matrix is a measure for the root-mean-square volume of the distribution. This however is not the quantity we are interested in. We want the *minimal* phase space-volume of a *fraction* of the beam.

There are infinitely many hyperellipsoids just touching a sample point $\mathbf{x}_i$. There is however only *one* ellipsoid just touching $\mathbf{x}_i$ with the same relative shape and orientation as defined by the sigma matrix, but with different overall size. It is the volume of this scaled ellipsoid that we use to define the *per-particle* emittance ϵ_i. This volume is given by [11]:

$$\epsilon_i = g_k \sqrt{\det(\mathbf{\Sigma})} \left(\mathbf{x}_i^T \cdot \mathbf{\Sigma}^{-1} \cdot \mathbf{x}_i \right)^{k/2}, \quad (2)$$

where the geometric factor $g_k = \frac{\pi^{k/2}}{\Gamma(1+k/2)}$ and $g_6 = \frac{1}{6}\pi^3$.

Using Eqs. (1) and (2) the set $\{\epsilon_i | i = 1, 2, \cdots, N\}$ can be generated. By sorting the list of ϵ_i values and renumbering them in such a way that $\epsilon_1 < \epsilon_2 < \cdots < \epsilon_N$, the value $\epsilon_{\alpha N}$ is the k dimensional phase-space volume of a fraction α of the beam. Unfortunately, it is not necessarily the *minimal* volume: Although the above calculated $\epsilon_{\alpha N}$ defines the volume of the desired fraction, this volume is obtained by scaling down the shape of the entire distribution. The shape of the minimal volume of a fraction of the beam is not necessarily the same as the shape of the overall distribution. Particularly in the case of outliers and tails the mismatch can be significant. We therefore run the above procedure iteratively, with a gradual decrease in α towards the final desired value.

[2]Because GPT output in this paper output is analyized at fixed position instead of fixed time, we use ct and the Lorentz factor γ as equivalent longitudinal phase-plane coordinates.

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