CALCULATION OF WAKE POTENTIALS IN GENERAL 3D STRUCTURES

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Abstract

The wake potential is defined as an integration along an axis of a structure. It includes the infinitely long beam pipe regions and in case of numerical evaluation leads to pipe wake artefacts. If the structure is cavity like one can position the integration path on the pipe wall and only the integration over the cavity gap remains. In case of axis-symmetric protruding structures it was proposed by O. Napoly et al. to deform the path such that the integration in the pipe regions is again on the wall. The present paper generalizes this method of path deformation to 3D structures with incoming and outgoing beam pipes. Its usefulness is verified with the code GdfidL and no artifacts were observed.

INTRODUCTION

An important problem in modern accelerators is the determination of the wakepotentials or impedances of metallic structures in the vacuum chamber or being part of the vacuum chamber. Normally, this has to be calculated with numerical codes by integrating the longitudinal component of the electric field along the direction of flight of the charges. The integration domain includes infinitely long beam pipe regions and leads to pipe wake artefacts. If, however, the beam pipes are equal and no part of the structure protrudes into the pipe region one can integrate along lines parallel to the structure axis and on the pipe contour. Then, the integration on the infinitely long pipe surfaces vanishes and only the integrations over a finite gap remain. The values of the wake potentials on the pipe contour define a potential problem[3] whose solution gives the wakepotential anywhere within the cross section. In all other cases, when the beam pipes are different, or when part of the structure protrudes into the pipes cross section, this method cannot be employed. However, for axis-symmetric structures it has been found[1], [4] that the path of integration can be deformed such that it is on the boundary of the metallic structure and that the part in the pipes give no contribution. Lateron[2] this approach has been generalised for any arbitrary path of integration spanning the structure longitudinally.

In this paper, we generalise the method once more allowing in that way the artefact-free calculation of the wake potentials for any 3D structure, the only restriction being the existence of a common infinitely long tube area of arbitrary cross-section.

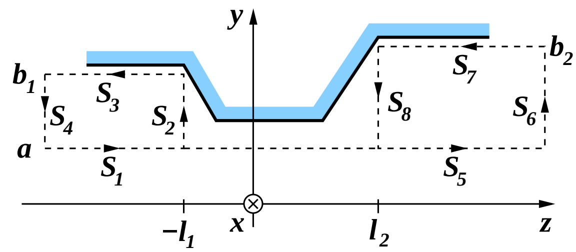

Figure 1: Upper part of the general 3D structure with different paths of integration.

DEFORMATION OF PATH OF INTEGRATION

Referring to Fig. 1 the longitudinal wake potential is defined by

$$W_z(x,y,s) = -\frac{1}{q}\int_{-\infty}^{\infty} E_z(x,y,z,t=(z+s)/\mathrm{c})\mathrm{d}z \quad (1)$$

In a first step, we show that the integral along S_5 equals an integral over TM-components along $-S_8$

$$\int_{l_2}^{\infty} E_z(x,y=a,z,t=(z+s)/\mathrm{c})\mathrm{d}z = \quad (2)$$

$$\int_{a}^{b_2} (E_y - \mathrm{c}B_x)^{\mathrm{TM}}(x,y,z=l_2,t=(l_2+s)/\mathrm{c})dy.$$

We introduce for any field component the definition

$$\bar{u}(x,y,z,s) = u(x,y,z,t=(z+s)/\mathrm{c}), \quad (3)$$

such that

$$\frac{\partial \bar{u}}{\partial z} = \frac{\partial u}{\partial z} + \frac{1}{\mathrm{c}}\frac{\partial u}{\partial t} \quad (4)$$

Combining MAXWELLS equations for TM- and TEM-fields only (H_z=0) gives

$$\frac{\partial}{\partial z}\left(\bar{E}_x + \mathrm{c}\bar{B}_y\right) = \frac{\partial}{\partial x}\bar{E}_z \quad (5)$$

$$\frac{\partial}{\partial z}\left(\bar{E}_y - \mathrm{c}\bar{B}_x\right) = \frac{\partial}{\partial y}\bar{E}_z \quad (6)$$

$$\frac{\partial}{\partial z}\bar{E}_z = -\frac{\partial}{\partial x}\left(\bar{E}_x - \mathrm{c}\bar{B}_y\right) - \frac{\partial}{\partial y}\left(\bar{E}_y + \mathrm{c}\bar{B}_x\right) \quad (7)$$

$$0 = \frac{\partial}{\partial x}\left(\bar{E}_y + \mathrm{c}\bar{B}_x\right) - \frac{\partial}{\partial y}\left(\bar{E}_x - \mathrm{c}\bar{B}_y\right). \quad (8)$$

Using (5), (6), (8) and $\nabla \cdot \vec{\bar{B}} = 0$ one can define an irrotational vector

$$\vec{G} = \vec{e}_x \left(\bar{E}_x + c\bar{B}_y\right) + \vec{e}_y \left(\bar{E}_y - c\bar{B}_x\right) + \vec{e}_z \bar{E}_z \qquad (9)$$

and apply STOKES' theorem over an area A delimited by S_5, S_6, S_7, S_8

$$\begin{aligned}\int_A \left(\nabla \times \vec{G}\right) \cdot d\vec{A} = 0 = \oint_S \vec{G} \cdot d\vec{s} \\ = \int_{l_2}^{\infty} G_z(y=a)dz + \int_a^{b_2} G_y(z=\infty)dy - \\ - \int_{l_2}^{\infty} G_z(y=b_2)dz - \int_a^{b_2} G_y(z=l_2)dy \\ = \int_{l_2}^{\infty} \bar{E}_z dz - \int_a^{b_2} \left(\bar{E}_y - c\bar{B}_x\right)(z=l_2)dy, \qquad (10)\end{aligned}$$

which gives exactly the equ. (2). Here, we have assumed small losses such that the integral over radiation fields at $z = \infty$ vanishes and we accounted for $G_z(y = b_2) = \bar{E}_z(y = b_2) = 0$. The important question of how the TM-fields are extracted will be addressed in the next chapter. In an analogous way we can show the equality of the integrals over S_1 and S_2 and obtain for the wakepotential (1)

$$\begin{aligned} qW_z(x,y,s) = \\ \int_a^{b_1} \left(E_y - cB_x\right)(z=-l_1, t=(-l_1+s)/c)dy \\ - \int_{-l_1}^{l_2} E_z(z, t=(z+s)/c)dz \qquad (11) \\ - \int_a^{b_2} \left(E_y - cB_x\right)(z=l_2, t=(l_2+s)/c)dy.\end{aligned}$$

The integrals over y go over TM-field components representing the radiation field and TEM-field components belonging to the solenoidal bunch fields. The latter are required if the two beam pipes are different. In the form (11), the infinitely long integrals over S_1 and S_5 are replaced by the integrals over S_2 and $-S_8$ and no numerical artefacts occur.

EXTRACTION OF TM- AND TEM-FIELDS

Since $\vec{G}$ (9) is irrotational it follows that $(\nabla \times \mathbf{G})_z = 0$ in the x,y-plane and $G_{x,y}$ can be derived from a scalar potential

$$G_x \mathbf{e}_x + G_y \mathbf{e}_y = \nabla \varphi. \qquad (12)$$

Taking the divergence and making use of (7) leads to a POISSON equation for φ

$$\begin{aligned}\nabla \cdot (G_x \mathbf{e}_x + G_y \mathbf{e}_y) = \\ = \frac{\partial}{\partial x}(\bar{E}_x + c\bar{B}_y) + \frac{\partial}{\partial y}(\bar{E}_y - c\bar{B}_x) \\ = \frac{1}{c}\frac{\partial}{\partial t}E_z - \frac{\partial}{\partial z}E_z = \nabla^2 \varphi. \qquad (13)\end{aligned}$$

We note that the driving term of the equation has to be taken at $z = -l_1, t = (-l_1 + s)/c$ and $z = l_2, t = (l_2 + s)/c$ respectively. The TEM-fields are easily incorporated by adding a line charge to the driving term. The extraction of the TM-/TEM-fields requires therefore the solution

$$\frac{\partial \varphi}{\partial y} = G_y = \left(\bar{E}_y - c\bar{B}_x\right) \qquad (14)$$

of two potential problems (13). Then, the wakepotential (11) becomes

$$\begin{aligned} qW_z(x,y,s) = \\ = \left[\varphi(x, y=b_1) - \varphi(x,y)\right]_{z=-l_1} \\ - \int_{-l_1}^{l_2} E_z(x,y,z,t=(z+s)/c)dz \qquad (15) \\ + \left[\varphi(x,y) - \varphi(x,y=b_2)\right]_{z=l_2}. \end{aligned}$$

where the potentials φ represent the integrals over y of the TM- and TEM-fields.

EXAMPLES

The new procedure has been implemented in GdfidL[5]. For verification, we compare the impedances of a mode-matching approach with the impedances as computed by taking the FOURIERtransform of the wakepotentials, Fig. 2.

The next check examples are the wakepotentials, Fig. 4, of a cross section step-in and step-out, Fig. 3. Their superimposition is compared to the wakepotential of a long cavity excited below resonance, Fig. 5.

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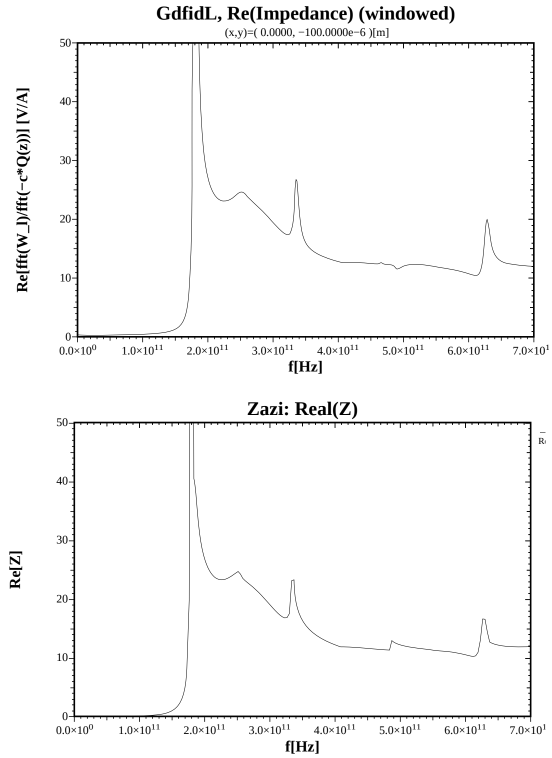

Figure 2: Real part of the longitudinal impedance of a round iris in a round beam pipe. Dipole modes dominate the result. Iris radius 1/3 and iris thickness 1/4 of the beampipe radius. Above: Derived from the wakepotential. Below: Mode-Matching result.

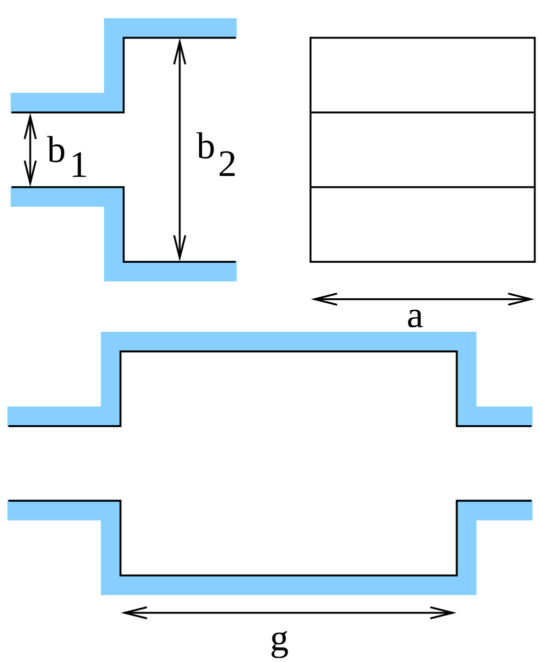

Figure 3: Above: Cross-section of step-out. a=10cm, b_1=5cm, b_2=10cm. Below: Rectangular cavity, g=60cm.

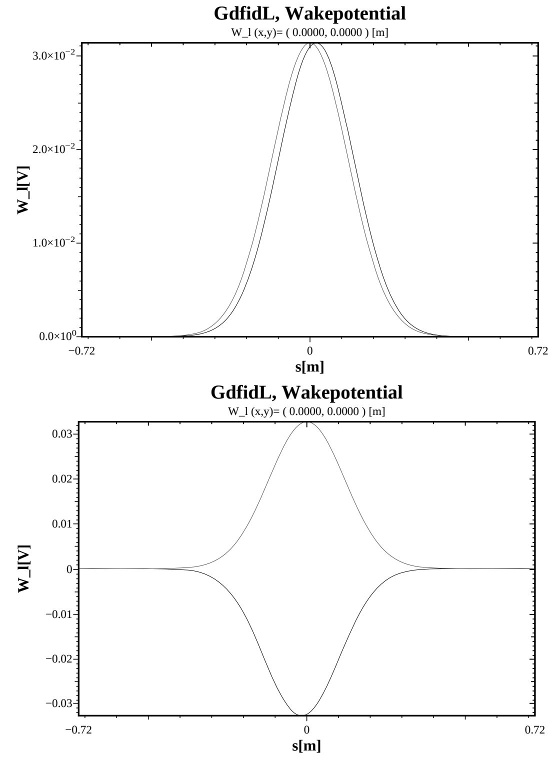

Figure 4: Above: Wakepotential of step-in. Below: Wakepotential of step-out. The dashed line is the exciting linecharge, bunch length 12cm.

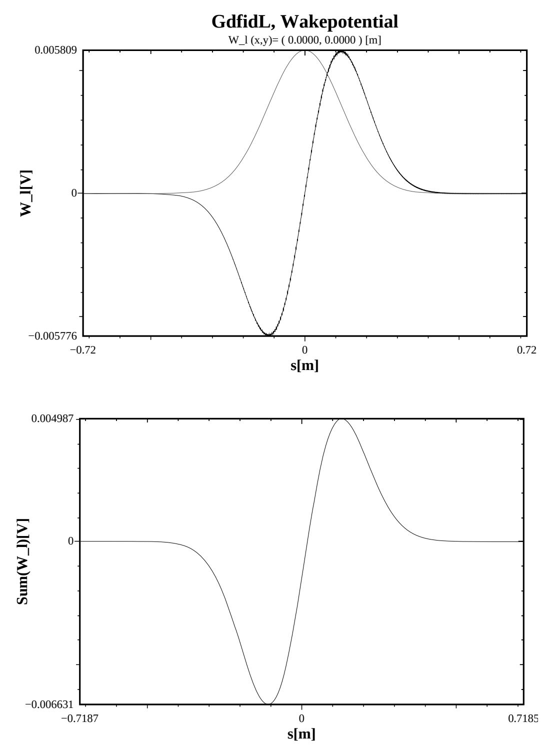

Figure 5: Above: Wakepotential of the cavity in Fig. 3. Bunchlength 12cm. The dashed line is the exciting linecharge. Below: Sum of the wakepotentials of the step-out and the step-in.