A TOMOGRAPHIC TECHNIQUE FOR MAGNETIZED BEAM MATCHING *

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Abstract

To maintain low electron beam temperatures in the proposed RHIC electron cooler, careful matching of the magnetized beam from the source to the cooler solenoid is mandatory. We propose a tomographic technique to diagnose matching conditions. First simulation results will be presented.

INTRODUCTION

Electron cooling of heavy ion beams at γ ≈ 100 is foreseen to increase the luminosity of the Relativistic Heavy Ion Collider RHIC. This electron cooler consists of a superconducting energy-recovery linac (ERL) which accelerates electrons to energies up to 55 MeV, a beam transport system and bunch stretcher, and the cooling section [1]. To achieve sufficiently short cooling times, the cooling section of the proposed RHIC electron cooler is equipped with a solenoid. Figure 1 shows a schematic drawing of the RHIC electron cooler.

The fringe field of the solenoid produces a “rotation” of the electron beam, which in the thin-lens approximation is described as

\[
\begin{pmatrix}
  x' \\
  y'
\end{pmatrix}
\begin{pmatrix}
  x \\
  y
\end{pmatrix}
= S
\begin{pmatrix}
  1 & 0 & 0 & 0 \\
  0 & 1 & S & 0 \\
  0 & 0 & 1 & 0 \\
-1 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
  x' \\
  y'
\end{pmatrix},
\]

where \( S \) is the strength of the solenoid. To compensate the increase of the transverse momenta \( x' \) and \( y' \) and therefore the transverse electron beam temperature, due to the solenoid fringe field, it is essential to provide the electron beam with the appropriate correlation of phase space coordinates when entering the solenoid fringe field, namely

\[
\begin{align*}
\langle yx' \rangle_i &= -S, \\
\langle xy' \rangle_i &= S.
\end{align*}
\]

This can be achieved by equipping the electron gun with a solenoid. In this case, the fringe field of the gun solenoid provides a “rotation” of the electron beam which is subsequently compensated by the cooler solenoid, thus resulting in minimum electron beam temperature in the cooling section.

However, this correlation is in general only preserved in axially symmetric elements like drifts or RF cavities, but not in elements like dipoles and quadrupoles. In spite of the use of dipoles and quadrupoles in the beam transport section of the RHIC electron cooler, appropriate matching of the optics nevertheless ensures preservation of these correlation properties. In the easiest case with equal solenoid strengths for the gun and the cooling section and with zero acceleration between them, a unit matrix will trivially have the right properties.

To ensure proper matching between the gun and the cooling section solenoid, it is most straightforward to measure the correlation properties at the entrance of the cooler, \( y'(x) \) and \( x'(y) \). In this paper, we derive matching conditions from transverse phase space distributions of particles, and develop an appropriate tomographic phase space reconstruction technique.

DERIVATION OF MATCHING CONDITIONS FROM PHASE SPACE DISTRIBUTIONS

When the electron beam is generated at the cathode of the electron gun, phase space coordinates of individual particles are completely uncorrelated,

\[
\begin{align*}
\langle x_0 x'_0 \rangle &= \langle y_0 y'_0 \rangle = \langle y_0 x'_0 \rangle = \langle x_0 y_0 \rangle = \langle x'_0 y'_0 \rangle = 0.
\end{align*}
\]

The fringe field of the gun solenoid introduces a correlation according to Equation (2) without affecting the rms beam size, \( \sigma_x = \sigma_y \equiv \sigma_{y_0} \). For any given value of \( x_1 \) (or \( y_1 \)) the rms beam divergence \( \sigma_{y_1}(x_1 = \text{const.}) \) (or \( \sigma_{x_1}(y_1 = \text{const.}) \)) just outside the gun solenoid fringe field equals the rms beam divergence \( \sigma_{y_0} \) (or \( \sigma_{x_0} \)) on the cathode.

This property is applied to measure the betatron phase advance \( \psi \) between gun and cooling section, using the relation

\[
x'_2 = \frac{\sin \psi_x}{\sqrt{\beta_{x,1} \beta_{x,2}}} \cdot x_1 + \cos \psi_x \cdot x'_1,
\]

where the subscript 2 corresponds to the location just before entering the cooler solenoid fringe field. Squaring and averaging this equation for constant \( y_1 \), \( y_2 \) yields

\[
\begin{align*}
\langle x'_2 \rangle_{y_1=\text{const.}} &= \frac{\sin^2 \psi_x}{\beta_{x,1} \beta_{x,2}} \cdot \langle x_1^2 \rangle_{y_1=\text{const.}} \\
&\quad + \cos \psi_x \cdot \langle x'_1^2 \rangle_{y_1=\text{const.}},
\end{align*}
\]

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which can be solved for \( \sin^2 \psi_{x} \) as

\[
\sin^2 \psi_{1} = \frac{\langle x_{2}^{'2} \rangle |_{y_{2}=\text{const.}} - \langle x_{1}^{'2} \rangle |_{y_{1}=\text{const.}}}{\langle x_{1}^{'2} \rangle |_{y_{1}=\text{const.}} - \langle x_{1}^{'2} \rangle |_{y_{1}=\text{const.}}}.
\]

(6)

The Tomographic Reconstruction

To obtain information on phase space distributions at a certain location \( s \), a screen is installed downstream of that location to measure the particle density distribution in the \( x-y \) plane. A number of quadrupoles between the observation point \( s \) and the location of the screen is used to vary the phase advances between these two points independently, and without disturbing the Twiss parameters at the screen. With this setup, the phase-space distribution in one plane (\( x-x' \) or \( y-y' \)) can then be tomographically reconstructed from a set of beam profiles taken at different phase advances in that plane, while the phase advance in the other plane is kept constant. This reconstruction process is very similar to the one described for example in References [2, 3].

To reconstruct the phase space distribution \( x-y' \), the betatron phase advance in the \( x \)-plane is kept constant at an integer multiple of \( \pi \), while the phase advance in \( y \) is again varied in steps, spanning at least 180 degrees in total. The \( x \) axis is divided into \( N \) slices of width \( \Delta x \). For each of these slices, the phase advance \( y-y' \) is varied in steps, spanning at least 180 degrees in total, and the \( x \)-axis is divided into \( N \) slices of width \( \Delta x \). For each of these slices, we perform the reconstruction of the corresponding \( y-y' \) phase space distribution, which is then projected onto the \( y' \)-axis, thus giving the \( y' \) density distribution for each particular interval \( \Delta x \). Combining these, we therefore obtain the phase space distribution for the entire \( x-y' \) range. An analogous procedure leads to the \( y-x' \) phase space distribution. The reconstruction procedure is schematically depicted in Figure 2.

**CONCLUSION**

We have presented a matching technique for magnetized electron beams in electron coolers without a continuous solenoidal guiding field, like the RHIC electron cooler. This technique is based on measurements of the correlation between spatial coordinates \( x, y \) in one plane and the angular coordinates \( x', y' \) in the other. Since these correlations reflect directly the quality of the magnetized beam matching, this measurement is believed to be advantageous compared to other methods, like measuring the Twiss parameters and phase advances using dipole kicks.

We also described a tomographic phase space reconstruction technique that allows us to obtain the required phase space correlations. However, these properties may also be accessible in a more direct fashion, using, for example, a pepper pot.
Figure 2: Illustration of the tomographic reconstruction procedure. For each slice in $x$, the corresponding $y-y'$ phase space is reconstructed. Each of these is projected onto the $y'$ axis, before they are combined into the $x-y'$ phase space distribution.

It is planned to experimentally test this matching technique at the Brookhaven Accelerator Test Facility (ATF).

REFERENCES

