EMITTANCE DILUITION DUE TO 3D PERTURBATIONS IN RF PHOTOINJECTORS

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Abstract
The predictions from different simulation codes are compared to investigate the effects of fluctuations of quantum efficiency and other sources of inhomogeneities in the performances of a typical RF photoinjector. The layout includes a RF gun and a focusing solenoid in a configuration aimed at minimizing the emittance growth due to space charge effects.

INTRODUCTION
Many applications from X-ray Free electron lasers to high energy colliders require high brightness beams produced by photo-injectors. The final performances of these devices are strictly linked to the beam quality produced by the electron source. In the case of FELs the role played by emittance becomes crucial at sub-nm wavelengths where the emittance is related to the transverse coherence of the output radiation. Most of the emittance budget that characterizes the beam at the undulator is produced at the injector in the first stages of the beam acceleration. The emittance optimization procedure rely on the linear theory [1] which has been verified both experimentally and numerically. In this analysis we examine in these optimized conditions, the role played by a non uniform electron emissivity that may be induced by non-uniformities generated by quantum efficiency variation on photocathode and laser illumination non-uniformity. This study has been performed by using two different codes based on different algorithms: the Los Alamos version of PARMELA (PARMELA-LANL) [2] and TREDI [3]. TREDI has been used in “static” mode, i.e. ignoring effects associated to the finite velocity propagation of signals within the bunch.

PROBLEM DESCRIPTION
The aim of this work is to study the effect of charge in-homogeneities at the cathode surface, by decoupling in a transverse Fourier space, the in-homogeneities occurring at a specific wave-number \( k = 2\pi/R \), on a scale of the beam spot radius \( R \), and higher. We have considered a standard S-Band (2856 MHz), 1.6 cells, BNL type photo-injector configuration[4], in a set-up optimized at minimizing the emittance in terms of accelerating gradient, extraction phase, beam spot size, focusing solenoid strength. Space charge effects compensation is achieved assuming both transverse and longitudinal flat charge distribution at extraction. The gun starts at \( z=0 \) and the drifts ends at \( z=2 \) m. The peak electric field in the gun and the solenoid peak magnetic field have been set respectively to 120 MV/m and 2.73 kG. The longitudinal shape of the pulse is square with a length of 10 ps and the charge is 1 nC. The phase of the centre of the bunch is 35°. No thermal emittance is included. The beam spot radius \( R \) is 1 mm. The charge distribution extracted from the cathode has been modelled as a perturbation with respect to the ideal case with the following cosine function showing a maximum on the centre of the spot:

\[
\rho_p(x,y) = \rho_0 [1 + \delta \cdot \cos(k_n x)] [1 + \delta \cdot \cos(k_n y)]
\]  

(1)

for \( x^2 + y^2 \leq R^2 \) and \( k_n = \frac{2\pi}{R} \).

Assuming that the values of \( \delta \) and \( k_n \) are small perturbations, we may write in first approximation

\[
\varepsilon(k_n, \delta) \approx \varepsilon_0 + \sum_n a_n \delta^j \left[ \frac{\partial^j}{\partial \delta^j} \varepsilon(k_n, \delta) \right]_{\delta=0}
\]

(2)

where \( \varepsilon_0 \) is the value of the unperturbed emittance and the coefficients \( a_n \frac{\partial^j}{\partial \delta^j} \varepsilon(k_n, \delta) \) show the sensitivity of the emittance in this injector configuration to the charge in-homogeneities at the frequency \( k_n \).

The study has been performed by varying the two parameters \( \delta \) and \( n \) and estimating the effect on the normalized rms emittance at the location of the first minimum (fig.1). The parameter \( \delta \) has been varied with values comprised between 0 and 40% and \( n \) the values \( n = 1, 2, 3, 4 \).

A previous comparison between the two codes in the ideal configuration, i.e. at \( \delta=0 \), has shown a good agreement [5]. A final remark is in order about phase-space random generation: results obtained by Monte Carlo simulations of such low-emittance beams are very sensitive to the initial values of macro-particles used to describe the macroscopic charge distribution. As a matter of fact, the fluctuations associated with standard pseudo-random numbers generators translate in charge density gradients and fields obfuscating the emittance compensation mechanism at work. As a consequence, less than linear convergence is achieved as a function of the number of macro-particles. Quasi-random (or Halton or Sobol[6]) sequences yield usually much faster convergence because of the highly desirable capability of uniformly populating a given n-
dimensional box. This prevents macro-particles from getting too close one another, which induces a spurious collisional regime and unphysical space charge fields. For the study presented here, the quasi-random generator has been slightly modified by using the rejection method [6] to fit the distribution (1). The phase spaces were generated by TREDI and the particles set has been exported to PARMELA to ensure exactly the same initial conditions.

CALCULATIONS RESULTS

The behaviour of the transverse emittance as a function of the longitudinal coordinate at $\delta = 40\%$, for different values of $k_n$ is shown in fig. 1 as computed by PARMELA. The emittance undergoes a typical series of oscillations due to the changes in correlation between longitudinal slices along the bunch which are subject to different focusing as a function of the extraction phase. These oscillations exhibit the well known structure with a double minimum located at the places where the correlation is maximized. In the figure the effect of distributions with $n \neq 0$ on emittance is visible and compared to the optimal conditions.

Fig.1 RMS horizontal normalized emittance vs $z$ for $\delta=40\%$ and $n=1/2,1,2,4$ compared with the uniform case

As an indication of the emittance of the beam we have considered the first minimum, whose position may depend on the in-homogeneity parameter $\delta$ especially at the lower perturbation frequencies $k_n$. In figure 2 the value computed by PARMELA of the horizontal normalized rms emittance divided by the value obtained with a completely uniform distribution is plotted as a function of $n$. A qualitatively similar behaviour has been obtained with TREDI as it is shown in figure 3 where TREDI and PARMELA results relative to the emittance growth for $\delta=20\%$ are compared.

For $n=1/2$ TREDI indicates a reduction of the emittance not evidenced by PARMELA, but for values of $n \geq 1$ the two codes are in fairly good agreement and both give the maximum emittance increase for $n=1$.

This behaviour may be understood by looking at the $x$-$y$ space shown in fig.4, in three longitudinal positions: at the cathode ($z=0$), near the minimum of emittance ($z=1.25$ m) and near the maximum of emittance ($z=1.5$ m). The non-linear space charge forces induced by the non uniform transverse distribution at the cathode gives a deformation of the beam shape. The distortion is stronger when the non-uniformities are more localized respect to the cases in which they are more diffused and tend to a partial re-compensation along the drift.

In fig 4 the action of the solenoid focusing is also visible as a rotation of the distribution around the axis. The emittance degradation increases with the modulation depth $\delta$ as expected. An analysis of the data has shown a repeatable dependence of the emittance growth with $\delta$ of the following functional form,

$$\varepsilon(\delta,k_n) = \varepsilon_0\left(1 + a_\delta \delta^2\right)$$

suggesting that the first two coefficients of the series expansion (2) are negligible. In table 1 the values of the coefficients derived from the analysis of TREDI and PARMELA data are reported.
CONCLUSIONS

The preliminary analysis presented in this paper has been based on the observation of the projected emittance at the location of the first minimum along the propagation, as a function of the frequencies associated to the cosine perturbation defined in (1). A scaling law of this effect in function of the perturbation amplitude has been derived and some indications of the dependence of the effect with the transverse frequency have been obtained. In the future we plan to have further verifications of these scaling laws by extending the analysis to sine-like perturbations and to higher transverse oscillation frequencies. While with the sine-like perturbation we might expect similar results at high frequencies \( k_n \), the situation might be substantially different at low \( n \) where the phase has a significant influence on the distribution symmetry. Furthermore we plan to continue this study by extending the observation to the beam slice emittance, which is not affected by correlations between slices and is probably a better indicator of the influence of cathode inhomogeneities on the beam quality. This work will require a significant computational effort since the number of macroparticles and the transverse mesh fineness for the evaluation of the fields grow non-linearly with the frequency associated to the transverse mode.

REFERENCES