FLEXIBILITY, TOLERANCES, AND BEAM-BASED TUNING OF THE
CLIC DAMPING RING

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Abstract

The present design of the CLIC damping ring can easily accommodate anticipated CLIC parameter changes. Realistic misalignments of magnets and monitors increase the equilibrium emittance. In simulations we study both the sensitivity to magnet displacements and the emittance recovery achieved by orbit correction, dispersion-free steering.

INTRODUCTION

The performance of the CLIC damping ring [1, 2] with shorter wiggler period was studied. The number and length of the wiggler modules remain the same as before. Their values are 76 units and 2 m, respectively. The wiggler period was reduced to 10 cm from 20 cm. Such wigglers provide smaller transverse beam emittances. Taking into account intra-beam scattering for a bunch population of \(4.2 \times 10^9\), the equilibrium transverse emittances are \(\gamma_{\varepsilon_x} = 570 \text{ nm}, \gamma_{\varepsilon_y} = 6.2 \text{ nm}\) for a betatron coupling of 1.1 \% and an RF voltage of 2.6 MV, as shown in Fig. 1. Moreover, the longitudinal emittance does not exceed the target value of 5000 eVm.

For the same RF voltage and betatron coupling, the transverse beam emittances achieved with the old wiggler period of 20 cm are \(\gamma_{\varepsilon_x} = 700 \text{ nm}, \gamma_{\varepsilon_y} = 8 \text{ nm}\).

The optics functions in the FODO sections were re-optimized to adopt the new wigglers to the damping ring. In principle, such wigglers can be produced from an alloy with permanent magnetization, since the wiggler field is 1.8 T and the wiggler full gap does not exceed 15 mm.

Up to now all lattice design work for the damping ring was based on the ideal lattice. Eventually the design must study the robustness of the lattice performance and develop cures against various misalignment error sources related to the imperfection of element placement over the damping ring. Even modern surveying technology does not provide initial misalignment errors of less than 50 - 100 \(\mu\text{m}\). Horizontal, vertical, longitudinal displacements and rotations of quadrupoles, sextupoles and field errors lead to betatron coupling, vertical dispersion, as well as vertical and horizontal orbit distortions.

The design betatron coupling for the CLIC damping ring is taken to be \(\epsilon_y/\epsilon_x \leq 1\%\). It is important to evaluate the residual coupling after beam-based correction.

CLOSED ORBIT AND BETATRON MOTION

The transverse particle motion can be expressed by \(x = x_c + x_\beta + D_x \delta\) where \(\delta\) is the energy deviation. Periodic closed orbit \(x_c\), betatron motion \(x_\beta\) and dispersion function \(D_x\) can be found from [3] the equations

\[
x''_c + (K_1 + G^2)x_c + K_{1t}y_c + \frac{K_2}{2}(x_c^2 + y_c^2) = G_{xc}
\]

\[
y''_c - K_1y_c + K_{1t}x_c - K_2x_cy_c = G_{yc}
\]

\[
D''_x + (K_1 + G^2)D_x + K_{1t}D_y + K_2(x_cD_x + y_cD_y) =
\]

\[
G - G_{xc} + (K_1 + G^2)x_c + K_{1t}y_c + \frac{K_2}{2}(x_c^2 - y_c^2)
\]

\[
D''_y - K_1D_y + K_{1t}D_x = K_2(x_cD_y + y_cD_x) =
\]

\[
-G_{yc} - K_1y_c + K_{1t}x_c - K_2x_cy_c
\]

and

\[
x''_\beta + (K_1 + G^2)x_\beta + K_{1t}y_\beta + K_2(x_cx_\beta - y_cy_\beta) = 0
\]

\[
y''_\beta - K_1y_\beta + K_{1t}x_\beta - K_2(x_cx_\beta - y_cy_\beta) = 0
\]

Here, \(G_{xc}\) and \(G_{yc}\) are the inverse bending radii of the horizontal and vertical dipole correctors. \(K_1, K_2\) and

![Figure 1: The transverse equilibrium emittances versus the betatron coupling in the presence of IBS.](image-url)
$K_{tt} = e\partial B_x / p_0 c \partial x$ are the normalized quadrupole, sextupole and skew field respectively. $G = 1 / \rho$ is the inverse bending radius.

The equations for the vertical emittance presented in the next section assume that the horizontal closed orbit after correction is close to zero.

**VERTICAL EMITTANCE**

When the beam is coupled by vertical dispersion or transverse betatron coupling, the normal modes of oscillation rotate from the pure horizontal, vertical and longitudinal planes into some hybrid directions.

The vertical dispersion leads to coupling between the vertical phase space and the energy deviation induced by the synchrotron radiation. A vertical dispersion results from misalignment errors and non-zero closed orbit. The main contributions to the residual vertical dispersion are the following:

- vertical closed orbit in the sextupoles or quadrupoles;
- vertical misalignment of the sextupole;
- tilted quadrupoles;
- vertical dipole kick.

Using a flat-beam approximation [4], the contribution to the residual vertical dispersion can be expressed as

$$\epsilon_{yd} = \frac{C_q \gamma^2}{J_y} \frac{\|G\|^3 \mathcal{H}_y ds}{\|G^2\| ds} = 2J_y \frac{D_y^2}{\beta_y} \sigma_y^2.$$  

Due to the misalignment errors distributed over the ring, the square of the vertical dispersion $\langle D_y^2 \rangle$ can be divided into five terms. The first term arises from quadrupole rotation, the second term from sextupole misalignment. Both of them do not depend on the closed orbit. The third term comes from vertical dipole kick. The fourth and fifth terms are functions of the closed orbit and they include the local chromaticity, which should be adjusted to a small value in order to limit these terms. The chromaticity in the present design of the CLIC damping ring is compensated by the sextupoles which are located only in the arcs. Thus, average chromaticity is zero, but the local chromaticity in the region of dispersion is positive to compensate for the negative chromaticity produced by the dispersion-free regions.

The betatron coupling results from the following sources:

- vertical closed orbit in the sextupoles;
- vertical misalignment of the sextupole;
- tilted quadrupoles.

The betatron coupling couples the vertical emittance to the synchrotron radiation through the horizontal dispersion. Using a perturbation approach, far from linear coupling resonances, the vertical emittance contribution due to weak betatron coupling can be expressed as

$$\epsilon_{yc} = \frac{C_q \gamma^2}{16J_y \|G^2\| ds} \int_0^C \mathcal{H}_x \|G^3\| x ds \left[ \sum_{\pm} \frac{|Q_{\pm}(s)|^2}{\sin^2 \pi \Delta \nu_\pm} + 2R e^{-\frac{Q_+(s)Q_-(s)}{\sin(\pi \Delta \nu_+) \sin(\pi \Delta \nu_-)}} \right] ds$$

where

$$Q_{\pm}(s) = \int_s^{s+C} (K_{ty} - K_{tt}) \sqrt{\beta_x \beta_y} x e^{i[\psi_x(s) \pm \psi_y(s)] - \psi_x(z) \pm \psi_y(z) + \pi(\nu_x \pm \nu_y)]} dz$$

Here, $C_q = 3.84 \times 10^{-13}$ m, the sum over $\pm$ denotes a sum over both the $n^+ + n$ term (sum resonance) and the $n^- - n$ term (difference resonance), $\Delta \nu_+ = \nu_x + \nu_y$ and $\Delta \nu_- = \nu_x - \nu_y$. It is to be noted that this equation is not valid near the coupling resonance.

Total vertical emittance is the sum $\epsilon_y = \epsilon_{yd} + \epsilon_{yc}$

**DISPERSION-FREE STEERING**

For the orbit correction, dispersion free steering (DFS) [5] is used. The principle of DFS consists of a simultaneous correction of the orbit and dispersion. BPM readings of the closed orbit distortion can be represented by N component vectors $\overline{u} = (u_1, u_2, \ldots, u_N)$. The orbit is corrected by a set of M dipole correctors. The kicks of the correctors are represented by an M component vector $\overline{\theta} = (\theta_1, \theta_2, \ldots, \theta_M)$. The corrector kicks $\overline{\theta}$ are the solution of the linear system:

$$\left( \begin{array}{c} (1 - \alpha) \overline{u} \\ \alpha \overline{D}_u \end{array} \right) + \left( \begin{array}{c} (1 - \alpha)A \\ \alpha B \end{array} \right) \overline{\theta} = 0 \quad (1)$$

A response matrix $A$ of $N \times M$ dimension relates the corrector kick and beam position changes. Dispersion-free steering includes the BPM reading of dispersion. It is the $N$ component vector $\overline{D}_u$ that comprises BPMs reading of dispersion and $B$ is the $N \times M$ dispersion response matrix. The weight factor $\alpha$ allows a balance of the correction for the orbit ($\alpha = 0$) to the dispersion correction ($\alpha = 1$). The solution of Eq. (1) is computed by a least-square algorithm that minimizes

$$|1 - \alpha| \| \overline{u} + A \overline{\theta} \|^2 + \alpha^2 \| \overline{D}_u + B \overline{\theta} \|^2 \rightarrow \text{min}$$

MICADO is a fast least-square algorithm which is frequently used for orbit correction. It executes an iterative search for the most effective correctors. For the correction of a small number of kicks, MICADO is very efficient. To correct a large number of misalignment errors, the SVD method may provide a more effective correction.

A tracking code MAD that includes the MICADO algorithm for the correction was applied to study the tolerance of the damping ring to misalignment errors. The displacement misalignment errors of $|\Delta x|_{rms} = 100 \mu m$ and $|\Delta y|_{rms} = 30 \mu m$ with Gaussian distributions truncated at $3\sigma$ were assigned to the quadrupoles and sextupoles respectively. The misalignment errors are set to the elements by a random generator. Many different "seeds" of random numbers have been averaged by the MAD code in order to obtain a statistically significant statement. As a result, the average horizontal and
vertical closed orbit distortion are $\langle \Delta x \rangle_{rms} = 6.7 \text{ mm}$ and $\langle \Delta y \rangle_{rms} = 4.5 \text{ mm}$ respectively. For the horizontal and vertical dispersion the average distortion is $\langle \Delta D_x \rangle_{rms} = 2 \text{ m}$ and $\langle \Delta D_y \rangle_{rms} = 1.2 \text{ m}$.

Also the DFS code developed by J. Wenninger was used. This code was used for LEP. To make the search for orbits yielding small vertical emittances fast and deterministic, a simultaneous correction of the closed orbit and the residual dispersion was implemented for LEP [5]. The principle of the correction is a simultaneous r.m.s. minimization of both orbit and dispersion with an appropriate relative weight factor. Usually the minimization was done with Singular Value Decomposition algorithms and the number of eigenvalues was limited to 25% of the total number of eigenvalues. The same approach was tested for the CLIC damping ring using the original LEP control system program that was designed to be easily adapted to any accelerator. Test results are shown in Fig. 2. Both orbit and dispersion can be easily minimized in a few iterations.

Two beam position monitors (BPM) were installed in each arc cell as it is illustrated in Fig. 3, and also near the quadrupoles in the FODO straight section. The total number of BPMs over the ring is 288 units. At the same BPMs the dispersion is monitored. The CLIC damping ring has 100 arc cells and two FODO straight section with wigglers.

In order to save space, the dipole correctors (small dipole magnet with variable field) are integrated as additional coils in the quadrupoles and sextupoles. X-dipole correctors are set as additional coils in the focusing qudrupoles where $\beta_x$ is maximum and the Y-dipole correctors are modelled as additional coils in the SD sextupoles of the arc. Dipole correctors are also located in the dispersion-free FODO straight section.

The above location of the correctors provides for an orbit and dispersion correction. After the correction the average horizontal and vertical closed orbit displacements become less than $\langle \Delta x \rangle_{rms} = 24 \mu\text{m}$ and $\langle \Delta y \rangle_{rms} = 50 \mu\text{m}$ respectively. The average kick of dipole correctors which are located in the arcs is 0.35 $\mu\text{rad}$ and 0.077 $\mu\text{rad}$ for the correctors located in the wiggler section. The damping ring comprises 226 horizontal correctors and 130 vertical correctors. After the closed orbit and dispersion correction, the values of the chromaticity are $\Delta \nu_x/\Delta p/p = 2.7$ and $\Delta \nu_y/\Delta p/p = 0.68$ at the sextupoles strength that was computed for the machine without misalignment errors. Nevertheless, after COD correction, the strong sextupoles excite spurious rms vertical dispersion $\langle \Delta D_y \rangle$ that is 4 mm. The rms vertical closed orbit distortion inside the sextupoles is 50 $\mu\text{m}$ which results in a betatron coupling of 4.5%. Due to betatron coupling and a vertical dispersion the vertical emittance increases up to 25 $\mu\text{m}$ (6 $\mu\text{m}$ comes from betatron coupling and note that IBS is not included) as shown in Fig. 4.

**Figure 2:** The dispersion correction of CLIC damping ring.

**Figure 3:** Location of the BPMs and correctors in the arc cell.

**Figure 4:** The dependance of betatron coupling, rms vertical dispersion and contribution to the vertical emittance due to vertical dispersion versus closed orbit distortion inside sextupoles.

**REFERENCES**


