

IMPEDANCES: MEASUREMENTS AND CALCULATIONS FOR NON-SYMMETRIC STRUCTURES

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Abstract

Coupling impedances are most often calculated for beam enclosures of circular cross-section, and resulting formulae are used to interpret beam measurements, sometimes even in the case of flat chambers. This leads to misunderstanding and inaccurate evaluations. Rigorous analytic formulae exist for smooth, resistive chambers of elliptical or rectangular cross-section, while computer programs allow one to evaluate more complicated, non circular-symmetric geometries. At low frequencies the impedance of thin resistive walls departs from classical formulae.

1 INTRODUCTION

The science of collective effects in high-energy accelerators is now mature and newcomers in this field are offered books, courses and handy formulae to facilitate their work. However, general formulae may not apply to all particular situations: one has to understand the simplifications made.

In this report we present a few cases where an additional effort may be necessary to form a correct view of the problem. They are concerned with space charge, flat vacuum chambers and thin resistive walls.

2 TRANSVERSE IMPEDANCE AND TUNE SHIFT

The transverse coupling impedance is generally defined as [1]

$$Z^T = \frac{-i \int_0^{2\pi R} [E + vB] ds}{\beta I \Delta} \quad (1)$$

This is the integral over the machine circumference of the Lorentz force on a unit charge per unit beam current I and unit displacement Δ , with $\beta = v/c$.

The betatron motion $x = x_0 e^{-iQ\omega_0 t}$ of a single particle in a coasting beam is modified by Z^T . The perturbed motion is given by

$$\ddot{x} + Q^2 \omega_0^2 x = i \frac{e\beta}{m\gamma} \frac{Z^T I}{2\pi R} \langle x \rangle \quad (2)$$

where $\langle x \rangle$ is the average, coherent motion, β and γ the relativistic factors, R the machine radius, Q the tune and ω_0 the revolution frequency.

By averaging over particles on the l.h.s., we obtain the collective(coherent) tune shift

$$\Delta Q = -i \frac{Nr_p}{2\pi\gamma} \frac{\beta^T Z^T}{Z_0 R} \quad (3)$$

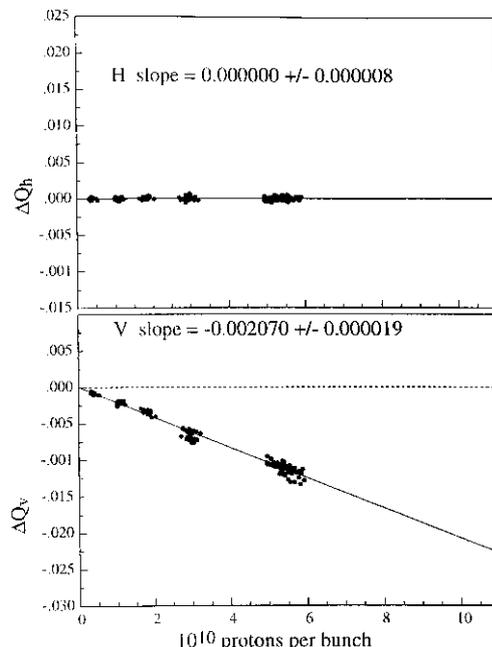


Figure 1: Single bunch tune shift measurements in the CERN SPS [2].

where $\beta^T \sim R/Q$ is the betatron function, Z_0 the impedance of free space, r_p the classical particle radius, and N the number of particles in the machine.

The impedance Z^T is evaluated at the frequency of the collective mode considered.

The real part of the impedance produces a growth rate and the imaginary part produces a frequency shift. In bunched beams, one has to use an effective impedance, averaged over the spectrum of the bunch head-tail modes.

The measurement of growth rates and frequency shifts of collective modes is the most straightforward way of measuring coupling impedances. In the CERN SPS, where a sustained effort is being made to reduce the coupling impedance in view of producing dense beams for the LHC, such measurements are made regularly [2]. An example is shown Fig. 1: the remarkable fact is that the tune shift in the horizontal plane is equal to zero to a high precision. This, as we will see, is explained by the fact that the majority of the vacuum chambers of the SPS are rather flat. The growth rates in the horizontal plane are half those in the vertical plane: this again is a consequence of the chambers' being flat.

These observations cannot be explained simply by invoking Eq. (3): this equation is valid only for round chambers, and even in this case does not apply to all situations. In fact what has been forgotten in Eq. (2) is that single particles may suffer a tune shift due to collective effects even when the beam as a whole is at rest: this is called the incoherent tune shift.

Let us follow the approach of Mohl [3]. The force leading to the incoherent tune shift ΔQ_{ic} is proportional to the deviation of the particle from the beam centre

$$F_{ic} \sim \Delta Q_{ic}(x - \langle x \rangle),$$

whereas the force leading to the coherent tune shift ΔQ_c is proportional to the deviation of the beam centre from the centre of the vacuum chamber

$$F_c \sim \Delta Q_c \langle x \rangle.$$

The betatron equation for the particle is thus

$$\ddot{x} + Q^2 \omega_0^2 x + 2Q\omega_0^2 [\Delta Q_c \langle x \rangle + \Delta Q_{ic}(x - \langle x \rangle)] = 0. \quad (4)$$

The coupling impedance is by definition proportional to the wake force due to the displacement $\langle x \rangle$ of the beam centre, and thus

$$Z^T \sim \Delta Q_c - \Delta Q_{ic}.$$

From which we conclude that

$$\Delta Q_c = \text{effect of } Z^T + \Delta Q_{ic}. \quad (5)$$

We will illustrate this by considering the case of the space charge interaction.

3 SPACE CHARGE

We first consider a coasting beam in free space (Fig. 2a). The space charge force acting on a single particle results from a radial electric field E_r and an azimuthal magnetic field B_θ . The force is proportional to the particle amplitude and produces, for a beam of circular cross-section and uniform density, a tune shift

$$\Delta Q_{ic} = -\frac{Nr_p}{2\pi\gamma} \frac{\beta^T}{\beta^2\gamma^2} \frac{1}{a^2}. \quad (6)$$

Here a is the beam radius. The factor $1/\gamma^2$ comes from the partial cancellation of the effects of E_r and B_θ .

There is obviously no coherent space charge tune shift in free space, since there is nothing to which to refer the movement of the beam.

Now we consider the same beam at the centre of a perfectly conducting vacuum chamber of circular cross-section. The field lines have the same aspect as before, they are just truncated at the wall (Fig. 2b). The incoherent tune shift is the same as in free space. However, if now we displace the beam, the field lines distort as in Fig. 2c. There

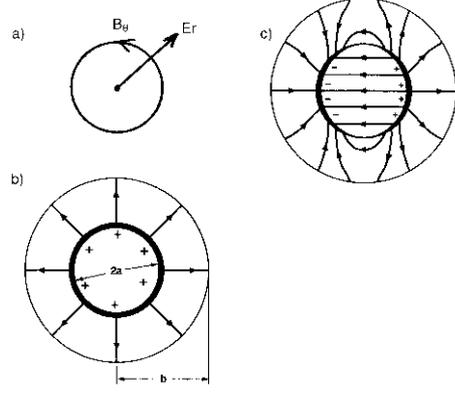


Figure 2: Field patterns for a beam in free space a) centered in a circular chamber b) and displaced c) [5].

results a coherent tune shift, which was calculated long ago by Laslett [4]

$$\Delta Q_c = -\frac{Nr_p}{2\pi\gamma} \frac{\beta^T}{\beta^2\gamma^2} \frac{1}{b^2} \quad (7)$$

where b is the radius of the vacuum chamber.

In order to calculate the space charge coupling impedance, we follow Chao [5], who solves Maxwell's equations with proper boundary conditions in the case of Fig. 2c. The source of the fields is a thin ring of charges at radius a , with a $\cos(\theta)$ distribution. The resulting transverse impedance is

$$Z^T = i \frac{Z_0 R}{\beta^2\gamma^2} \left[\frac{1}{a^2} - \frac{1}{b^2} \right]. \quad (8)$$

Since the beam radius a is in general much smaller than the chamber radius b , the dominant term in (8) is the first one. Let b increase to infinity, and use formula (3) to calculate the tune shift induced by the space charge impedance in free space: we find the same value (but with opposite sign) as that of the incoherent tune shift (6), which is obviously wrong, since we know that there is no coherent tune shift due to space charge in free space. Using (5) instead gives the correct result, the same as shown in (7).

This can be explained in the following way. The displaced, circular uniform beam is properly represented by the superposition of the centred beam and the two lunules of Fig. 1c. The lunules generate the dipole moment and the coherent fields. However, they oscillate in the static field of the quiescent part of the beam: their tune is depleted by ΔQ_{ic} . This is just cancelled by the $1/a^2$ part of the impedance.

Up to now we have considered cylindrical vacuum chambers of circular cross-section. In fact, accelerators using classical, warm magnets usually have flat chambers. Back in 1963 Laslett [4] calculated the space charge effects in chambers of various shapes. He introduced the famous

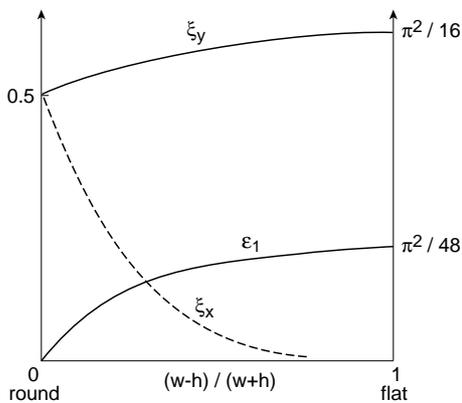


Figure 3: Rough sketch of Laslett's coefficients for an elliptical chamber of horizontal/vertical semi axes w/h [4, 6].

Laslett coefficients: ϵ_1 to describe the incoherent tune shift, and ξ for the coherent one. Figure 3 shows how these coefficients vary from a round chamber to a flat one made of two parallel plates. The coefficient ξ_x for the horizontal coherent effect was calculated later on by Zotter [6].

For chambers of general cross-section the coherent tune shift is obtained by multiplying Eq. (7) by 2ξ , while the incoherent one is given by Eq. (6) to which must be added a second term obtained by replacing $1/a^2$ by $2\epsilon_1/h^2$ (h is the half-chamber height). Thus the flat chamber contributes not only to the coherent tune shift, but also to the incoherent one: the wall images of the centred beam produce a static quadrupolar field superimposed onto the free-space field of the beam. Whereas the direct space charge effect is of the same sign in both x and y directions, the image effect has opposite signs in x and y . Notice how the horizontal coherent tune shift vanishes as soon as the chamber becomes flat: it is obvious that for two parallel horizontal plates, there cannot be any horizontal coherent tune shift.

4 THE GENERALIZED IMPEDANCE

The fact that the coherent space-charge tune shift in the horizontal plane of a flat chamber is zero is reminiscent of the SPS results. However, in the SPS the measurements are made with high-energy beams which are sensitive to the resistive-wall impedance (for multi-bunch beams) or high frequency impedances due to cavities and cross-section variations (for dense single bunches) and not to space charge.

In order to better understand in a general case the relation between the classical concept of coupling impedance and the beam measurements, let us consider separately a 'source' particle which generates the wake-fields, and a 'witness' particle which comes behind (since we consider ultra-relativistic beams, there are no fields ahead of the source particle). When the source particle travels on the axis of a cylindrical chamber of circular cross-section, the fields are as shown in Fig. 2a: there is no transverse force, since the effect of the electric field perfectly cancels that of

the magnetic field. Displacing the source particle generates a wake-field W which, provided the displacement is small enough, is predominantly dipolar. The witness particle is deflected by an amount which does not depend on its own transverse position, but is proportional to the excursion of the source particle: $\partial W_y/\partial y_s$ in the vertical plane, and $\partial W_x/\partial x_s$ in the horizontal plane correspond to the 'classical' coupling impedances.

In a flat vacuum chamber, the symmetry displayed in Fig. 2a is broken: the E and B fields no longer compensate each other. As a result, a source particle travelling on-axis generates a wake-field which is quadrupolar to first order. The witness particle is now deflected in proportion to its own excursion: $\partial W_y/\partial y_w$ and $\partial W_x/\partial x_w$ are the relevant quantities. These quadrupolar forces must be added to the dipolar forces arising from the displacement of the source particle.

This is beautifully demonstrated in Ref. [7]. Figure 4 reproduces results of computer calculations concerning the wake-fields of the LEP shielded bellows. The LEP vacuum chamber was elliptical with an aspect ratio w/h of about 2. The shielding of the bellows approximately prolongs the chamber walls.

Figure 4a shows the wake-field pattern for a centred 'source': it looks like that of a pure quadrupole. Figures 4b and 4c show the total wake-field seen by the 'witness' particle when the 'source' is displaced in the y or x directions, respectively. Figures 4d and 4e show that when the quadrupolar field (of Fig. 4a) is subtracted, a pure dipolar field (about half as large in the x direction as in the y direction) remains. We see that the quadrupolar and the dipolar field vectors add in the vertical plane, while they subtract in the horizontal plane: hence the coherent tune shift should be zero or small in the horizontal plane. However, the growth rates of instabilities depend only on the dipolar wake, and therefore we expect the horizontal growth rate to be about half that of the vertical one.

The above results help us to understand the SPS measurements. However, in the SPS the transverse coupling impedance is supposed to arise mainly from unshielded bellows and similar cavity-like objects. The unshielded bellows constitute cylindrical cavities with a diameter of 15 cm and a length of 10 cm. They have exit ports matched to the flat vacuum chambers which have an aspect ratio ranging from 2.5 to 4. One may conjecture that the largest deflecting fields are located at the exit ports, and therefore display the same behaviour as the LEP bellows.

5 THE RESISTIVE WALL IMPEDANCE

The generalized impedances of resistive walls of elliptical or rectangular cross-section have been calculated analytically [8]–[10]. Here we reproduce on Fig. 5 the results of Yokoya [9]. One recognizes the behaviour of the Laslett coefficients. For a chamber with an aspect ratio of 2.5 or more (as in the SPS), the horizontal tune shift ($\partial W_x/\partial x_s + \partial W_x/\partial x_w$) is very small and the ratio of ver-

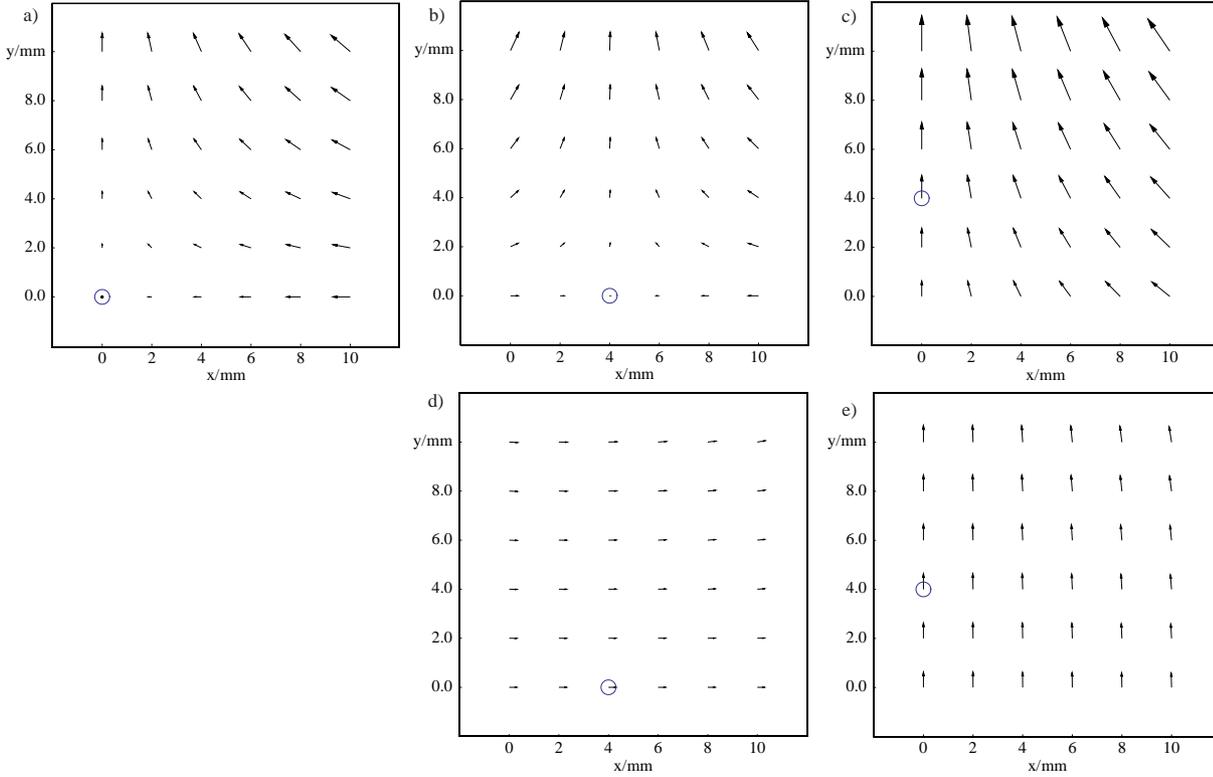


Figure 4: Effective transverse wake potentials of 10 mm bunches in LEP bellows for a centered beam a), and with x and y offsets b) and c). d) and e) are the same as b) and c), both with the quadrupolar field a) subtracted [7].

tical to horizontal impedance is close to 2. Using the vertical tune shift ($\partial W_y/\partial y_s + \partial W_y/\partial y_w$) to estimate the vertical impedance with Eq. (3) would lead to an overestimate of 50%.

6 THE THIN RESISTIVE WALL AT LOW FREQUENCY

For high enough frequencies or thick enough walls the transverse impedance is [11]:

$$Z^T = \frac{Z_0 R}{b^3} \frac{2\rho}{\mu\omega\delta} (1 - i \operatorname{sign}(\omega)) \quad (9)$$

where $\delta = \sqrt{\frac{2\rho}{\mu\omega}}$ is the skin depth, and ρ the metal conductivity.

Equation (9) describes the impedance when the thickness t of the wall is much larger than δ : the impedance then increases towards low frequency as $1/\sqrt{\omega}$.

When t is comparable to or smaller than δ we must replace δ in Eq. (9) by t , since the currents now flow in a metal layer of thickness t instead of δ before. In this regime the real part of the impedance increases towards low frequency as $1/\omega$ (this is not valid for the imaginary part, which stays constant). For this reason the mode of lowest frequency ($n - q$) ω_0 with the tune q just below an integer is strongly excited in large machines.

However, at very low frequency, when $t \ll \delta$, beam-induced currents partly flow outside the wall: they find there a return path with a smaller impedance than the thin wall itself [12].

This is modelled with an inductance L_1 in parallel to the wall resistance R_1 , giving

$$Z^T = \frac{Z_0 R}{b^3} \frac{2\rho}{\mu t} \frac{\omega - i\omega_c}{\omega^2 + \omega_c^2} \quad (10)$$

where $\omega_c = R_1/L_1$ is a resonant frequency.

Figure 6 from Ref. [13] shows how the real part of Z^T increases like $1/\omega$ until ω_c , where it reaches a maximum, then decreases like ω for $\omega < \omega_c$. The imaginary part is constant at low frequency.

In the LHC the beam screen is coated with a thin copper layer, for which $\omega_c/2\pi$ is about 100 Hz, and therefore this is of no relevance since the lowest-order mode frequency cannot be smaller than 3 kHz. However, for the Very Large Hadron Collider (VLHC) which is being considered, this mode can have a frequency as low as a few hundred hertz, and using Eq. (10) makes a difference.

There are other occurrences when such a treatment is in order: this concerns the kickers, which often have a ceramic chamber coated on the inside with a very thin metallic layer. Even at relatively high frequencies, beam-induced currents find complicated return paths outside the metallic layer. Computer programs exist to calculate the impedance

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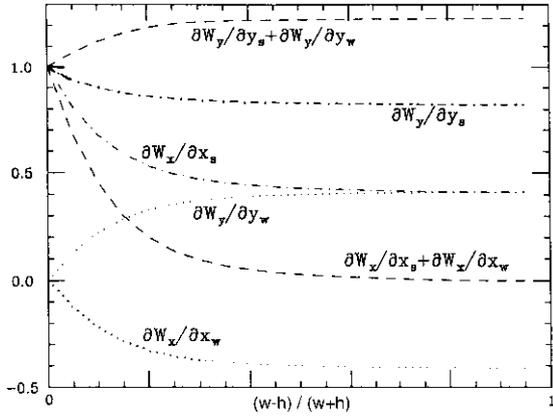


Figure 5: Wake functions for resistive elliptic pipes [9].

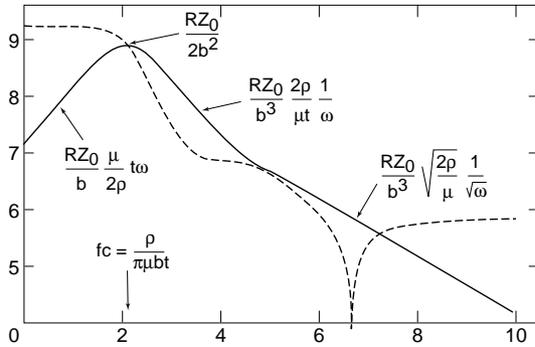


Figure 6: Decimal logarithm of real (full line) and imaginary (dashed line) parts of the transverse impedance in Ω/m of the LHC on a logarithmic frequency scale from 1 to 10^{10} Hz [13].

of multi-layered structures [13], while more complicated cases are measured on the bench [14].

The dip shown in Fig. 6 is due to the space-charge impedance [see Eq. (8)] which dominates at high frequency and changes the sign of the imaginary part (here the absolute value is shown). This feature can be misleading: it will not show up in a tune-shift calculation or measurement since the addition of the incoherent tune shift [see Eq. (5)] cancels it out.

7 CONCLUSIONS

The Laslett coefficients or the resistive-wall generalized impedance calculations can be used to evaluate the impedance of flat chambers. One can conjecture that they also approximately describe the case of cavity-like objects with flat entry ports.

The calculation of the impedances of thin and multilayered chambers are important for very large accelerators and for kickers with coated ceramic vacuum chambers.

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