INSTABILITY THRESHOLD CALCULATIONS FOR DIAMOND

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Abstract

DIAMOND, the UK third generation light source, will be optimised to produce high brightness radiation from a 3 GeV electron storage ring. To meet the needs of the user community it is planned to operate with various fill structures, including modes with few buckets filled. This will require high single and multibunch currents to satisfy specified brightness requirements, and estimates of instability thresholds are therefore an important part of the design process. We present here results of calculations of current thresholds for coherent instabilities in the storage ring, using established models of beam instability. We also comment on the influence of phase space diffusion caused by synchrotron radiation.

1 INTRODUCTION

Third generation light sources exhibit various types of instability that affect the quality of the beam or the brightness that can be achieved. The instabilities generally arise from impedance in the machine, as the motion of charged particles through the storage ring generates electromagnetic fields that act back on other particles. The thresholds at which various types of instability are seen depend on a number of lattice parameters. Here, we consider the most recent DIAMOND storage ring lattice [1], with relevant parameters given in Table 1. We have assumed that the RF is configured to give an energy acceptance of 4%, and the energy loss per turn resulting from insertion devices is 820 keV.

In this paper, we consider the effects of potential-well distortion, and single and mutlibunch instabilities. We give a brief discussion of each effect, and apply standard analytic formulae to find the relevant current thresholds for the present DIAMOND lattice. We also present the results of longitudinal tracking of coherent bunch oscillations in the presence of a higher harmonic RF system, using a model that incorporates quantum radiation.

2 SINGLE BUNCH INSTABILITIES

The short range wakefield of a single bunch, acting only over a time period of the order of a bunch length, is generally associated with low quality-factor resonating structures within the ring. We therefore consider only the action of broad-band and resistive wall impedances on a single bunch, which are expressed for the longitudinal direction as

$$Z_{//bb}(\omega) = \frac{R_s}{1 + \mathrm{i}\left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)}$$

and

$$Z_{//rw}(\omega) = (1 - \mathrm{i})\frac{Z_0 \beta}{2b\omega_0}\sqrt{\frac{2\omega}{\mu_0 \sigma}}$$

respectively [2]. In both these cases the transverse and longitudinal impedances are approximately related by [2]:

$$Z_{\perp}(\omega) = \frac{2c}{\beta b^2}\frac{Z_{//}(\omega)}{\omega}.$$

In these expressions, $\omega_r = \beta c/b$ is the beam pipe cut-off frequency, β is the relativistic velocity, μ_0 is the permeability of free space, and Z_0 is the impedance of free space (377 Ohms).

Table 1: Parameters used in instability calculations.

Broadband shunt impedance	R_s	6300 Ω
Beam pipe radius	b	25 mm
Vacuum vessel conductivity	σ	1.69 10^6 Ω^{-1}
Design current	I_0	300 mA
Design energy	E_0	3 GeV
Phase slip factor	η	1.60 10^{-4}
Synchronous phase	ϕ_s	2.56 rad
Revolution frequency	ω_0	3.85 10^6 s^{-1}
Machine circumference	C	489.24 m
RF harmonic number	h	816
RF amplitude	V_{rf}	3.34 MV
Natural bunch length	$\sigma_{\tau 0}$	9.08 10^{-12} s
Natural energy spread	$\sigma_{\delta 0}$	9.61 10^{-4}
Betatron tunes	Q_x, Q_y	28.9, 10.7
Synchrotron tune	Q_s	0.00440
Beta functions at RF cavities	β_x, β_y	10.0, 10.0 m
Transverse damping times	τ_x, τ_y	9.8, 9.8 ms
Longitudinal damping time	τ_s	4.9 ms

2.1 Potential Well Distortion

The wakefields of a single Gaussian bunch modify the single particle longitudinal equation of motion [2]. This results in a direct change in the synchrotron oscillation frequency, and also an indirect change through a variation of the synchronous phase. The equilibrium energy spread of particles is unaffected, so the bunch length remains inversely proportional to the synchrotron frequency. The ratio of the bunch length extended by this effect, σ_τ, to the natural bunch length, $\sigma_{\tau 0}$, is given by

$$\frac{\sigma_\tau}{\sigma_{\tau 0}} = \frac{\cos\phi_s}{\cos\left(\phi_s + \frac{I_b\omega_0}{V_{rf}\cos\phi_s}\Sigma_1\right)\cdot\sqrt{1+\frac{I_b\omega_0}{hV_{rf}\cos\phi_s}\Sigma_2}}$$

with

$$\Sigma_1 = \sum_{p=-\infty}^{\infty} p e^{-\frac{1}{2}\omega_0^2\sigma_\tau^2 p^2}\,\mathrm{Re}\left[\frac{Z_{//}(p\omega_0)}{p\omega_0}\right]$$

and

$$\Sigma_2 = \sum_{p=-\infty}^{\infty} p^2 e^{-\frac{1}{2}\omega_0^2\sigma_\tau^2 p^2}\,\mathrm{Im}\left[\frac{Z_{//}(p\omega_0)}{p\omega_0}\right],$$

where I_b is the single bunch current. Using the values presented in Table 1, we find the ratio of bunch length to the natural bunch length is 1.016.

2.2 Fast Head-Tail Instability

To model the effect of the wakefield from the head of the bunch on the tail, we split the bunch into two macroparticles executing synchrotron and betatron oscillations [3]. By causality, particles in the tail of the bunch cannot affect the head, whose transverse motion is simply represented by betatron oscillations; particles in the tail execute oscillations driven by wakefields from the head. A phase space map may be written down that takes into account the interchange of the roles of head and tail every half a synchrotron period. Applying a stability condition to the map gives the beam stability condition

$$I_b < \frac{4\pi\left(E/e\right)\omega_0 Q_\beta Q_s \omega_r b^2}{2R_s c^2}$$

where I_b is the bunch current. Above threshold, the driven oscillations become unstable, and current is lost from the bunch. Using the values from Table 1 we find the bunch current limit for stability against the fast head tail effect is 45 A.

2.2 Head-Tail Instability

If the effects of chromaticity are included in the model for the fast head-tail effect, we find a chromaticity-dependent stability condition:

$$I_b < \frac{2\pi\left(E/e\right)\eta Q_\perp \omega_r b^2}{\xi_\perp c^2 \sigma_\tau \omega_0 R_s \tau_\perp}.$$

Correction of the chromaticity to zero (achieved by use of sextupole magnets in the lattice) prevents the onset of head tail instability in the beam.

2.3 Longitudinal Microwave Instability

A coasting beam model is set up, with the assumption that the spatial density of particles is constant but for a small sinusoidal perturbation; the Vlasov equation then leads to a dispersion relation [2] that gives the coherent frequency shift for the perturbation. For a given beam current the perturbation will grow exponentially unless the complex impedance at the frequency of the perturbation is within certain bounds. The coasting beam result is applied to a bunched beam by replacing the total current with the peak current, which may be easily related to the bunch current. We thus arrive at the condition for stability against the longitudinal microwave effect

$$I_b < \sqrt{\frac{3}{8\pi}}\,\frac{C\left(E/e\right)\beta^2\eta^2\sigma_\delta^3}{bR_s Q_s}.$$

Using the values presented in Table 1 we find that the threshold bunch current is 16.6 μA. It is clear from the above expression that above threshold, the momentum spread, and hence the bunch length, varies as $I_b^{1/3}$. For a total current of 300 mA, the bunch length is increased by a factor 2.8. Taking account of SPEAR scaling raises the threshold peak current by a factor of $(b/c\sigma_\tau)^{1.68}$ [4] and no bunch lengthening would be expected at 300 mA.

3 MULTIBUNCH INSTABILITIES

3.1 Longitudinal Multibunch Instability

Our calculations are based on a model consisting of a number of rigid, identical bunches equally spaced around an accelerator [3]. The longitudinal equation of motion of a particular bunch in the presence of impedance is written down. Only the contribution of the higher order modes of the RF system to the impedance are considered, since wakefields from any other source will have damped to zero between the passage of two bunches. Solving the equation of motion reveals that the coherent oscillations will grow exponentially unless

$$I_0 < \frac{4\pi Q_s\left(E/e\right)}{R_{\mathrm{hom}}\omega_{\mathrm{hom}}|\eta|\tau_s},$$

in which case the oscillation is damped by the effects of synchrotron radiation. In this expression R_{hom} and ω_{hom} are the shunt impedance and frequency of a higher order mode of the RF cavities. Using the values presented in Table 1 and the most recent proposal for the DIAMOND RF system [5], where the maximum value of $R_{\mathrm{hom}}\omega_{\mathrm{hom}}$ for a single cavity is 1.15 10^{13} Ωs^{-1}, and assuming use of six cavities, we find a total current threshold of 3 A.

3.2 Transverse Multibunch Instability

A similar model is used to that applied to the analysis of longitudinal multibunch instabilities, and in this case leads to the stability condition

$$I_0 < \frac{4\pi\left(E/e\right)}{\tau_\perp \omega_0 R_{\text{hom}} \beta_\perp}.$$

Using the values in Table 1 and the same RF system as for the longitudinal case, with the maximum value of R_{hom} at 48.2 kΩ m^{-1} per cavity, and again assuming six cavities, this gives a threshold of 346 mA. However, because of the simple model used to derive the above expression, it is usual to allow a factor of 2 to 3 as a safety margin. Inspection of the RF system impedance, shown in Figure 1, suggests that transverse feedback operating between 0.85 and 2.2 GHz may be required to ensure a stable beam at 300 mA.

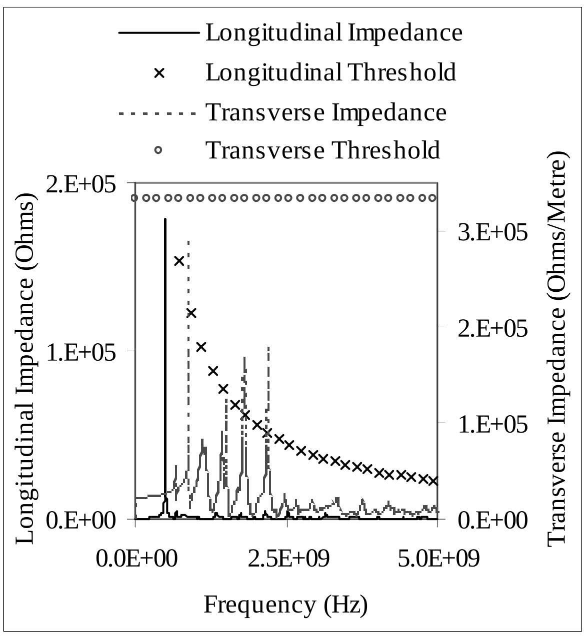

Figure 1: Impedance of the DIAMOND RF system and threshold impedances for multibunch instability [5].

4 LONGITUDINAL DAMPING

It is possible that higher harmonic RF cavities will be used in DIAMOND to introduce bunch lengthening and thus increase the Touschek lifetime. Longitudinal tracking has been carried out to study the damping of coherent oscillations in the presence of 702 kV of third harmonic RF (this increases the bunch length by a factor of 4.5). A bunch of 10^4 electrons was placed with its centre 0.25 ns from the synchronous point and tracked through many turns. The decay in the amplitude of the oscillation of the bunch centre about the synchronous point is plotted in Figure 2.

Without harmonic RF, the offset bunch executes synchrotron oscillations with exponentially decaying amplitude about the synchronous point. In the presence of harmonic RF the synchrotron oscillation frequency becomes amplitude dependent. Thus the offset bunch becomes stretched into a spiral about the synchronous point. As the entire bunch is wrapped around the origin the mean offset is greatly reduced. Phase space diffusion eventually blurs the structure of the spiral. A harmonic RF system increases the damping of coherent oscillations and must therefore raise the threshold current for longitudinal coherent instabilities.

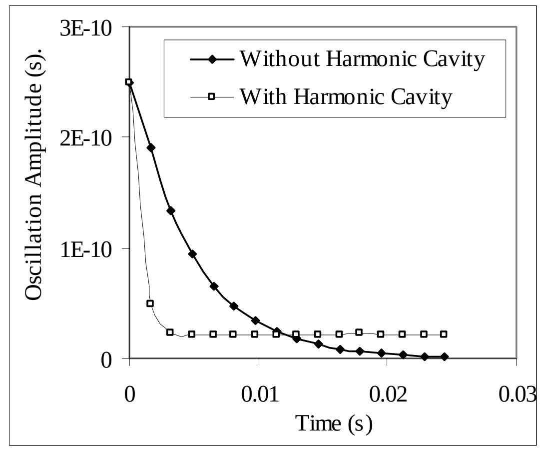

Figure 2: Graph showing the decay of a coherent oscillation both with and without higher harmonic RF.

5 CONCLUSION

The calculations presented in this paper give confidence to the performance of DIAMOND, though it may be that transverse feedback will be required to counter transverse multibunch instabilities. Use of higher harmonic RF to cause bunch lengthening is expected to significantly raise the thresholds for longitudinal instability. Further modes of instability have yet to be investigated (e.g. transverse fast blow-up), and it is intended that numerical simulations will be carried out to verify the predictions of analytic formulae.

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