ABSOLUTE MINIMUM EMITTANCE OPTICS FOR SUPER-ACO*

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Abstract

We have succeeded in optimizing an optics for the Super-ACO storage ring with the absolute minimum emittance of about 9.5 nm.rad. This limit is reached by satisfying some particular conditions on optical functions in the center of the bending magnets (dipoles). In this optics, the maximum betatron function values are almost the same as in the nominal one (38 nm.rad) and the dynamic aperture is very comfortable. We present in this paper the different steps of this study, as well as indications on expected brilliance and beam lifetime. Finally, the very first experimental results are also presented.

1 INTRODUCTION

Brilliance is often taken as the quality factor of a synchrotron light source. It represents the number of photons emitted per unit of time, per unit of solid angle, per percent of bandwidth and per unit of cross section of a source point. Apart from diffraction effects, the brilliance is inversely proportional to the particle beam emittance. Therefore, the smaller the emittance of the electron beam, the higher the brilliance of the produced radiation. In a storage ring, the natural horizontal emittance, $\varepsilon_{\theta x}$, results from the balance between the radiation damping and the quantum fluctuations. For an isomagnetic guide field, the emittance is given by:

$$\varepsilon_{\theta x0} = \frac{C_q \gamma^2}{J_x \rho} \langle H \rangle_{\text{dipoles}}$$

(1)

where $C_q = 3.84 \times 10^{13}$ m, $\gamma$ is the beam energy in mass units, $J_x$ is the horizontal damping partition number and $\rho$ is the bending radius. $\langle H \rangle$ is the average over the dipole length of $\gamma \eta_x^2 + 2 \alpha_x \eta_x \eta_x' + \beta_x \eta_x'^2$. $\gamma_x$, $\alpha_x$ and $\beta_x$ are the Twiss coefficients and $\eta_x$, $\eta_x'$ are the dispersion and first derivative respectively.

When the energy and dipole characteristics are fixed, the emittance can be kept small by keeping betatron oscillations, excited by photon emission, small. This is achieved by minimizing the expression $\langle H \rangle$.

For many reasons, the Double Bend Achromat (DBA) structure also called modified Chasman-Green (CG) is the most common lattice for synchrotron light sources. Twenty years ago, a comprehensive treatment [1] demonstrated that for this structure, with $N$ dipoles, there is a minimum achievable emittance given by:

$$\varepsilon_{mCG} = \frac{1}{4\sqrt{15}} \frac{C_q \gamma^2}{J_x} \left(\frac{2\pi}{N}\right)^3$$

(2)

This value is obtained under the achromatic condition, where the dispersion function together with its derivative are zero at opposite ends of the dipole pair. This provides achromatic straight sections for insertion devices. Relaxing the condition of vanishing dispersion in the straights gives the absolute minimum value of the emittance which is a factor of three lower than $\varepsilon_{mCG}$ [2]:

$$\varepsilon_m = \frac{1}{12\sqrt{15}} \frac{C_q \gamma^2}{J_x} \left(\frac{2\pi}{N}\right)^3 = \frac{\varepsilon_{mCG}}{3}$$

(3)

In our knowledge, no storage ring operates at its minimum emittance (nor at $\varepsilon_{mCG}$ value). There are several possible reasons for that. As it will be shown later, obtaining the minimum emittance requires a very low beta value in the dipole which could generate unacceptably high beta values in focusing quadrupoles leading to large chromaticities. Strong chromaticity correction sextupoles reduce the dynamic aperture and this has a negative impact on the beam lifetime. Moreover, betatron tunes selected to achieve the minimum emittance are not usually the ones required for good dynamic aperture. An other reason can be the difficulty to set specific values of beta functions in the straight sections to optimize the injection and the radiation source properties. Therefore, actual lattices result from a compromise. The general tendency is to increase the emittance by a factor of two to three above the minimum value.

In this paper, we will show that we safely achieve, theoretically and experimentally, an unprecedented level of low emittance for a given storage ring.

2 MINIMUM EMITTANCE

To obtain $\varepsilon_m$, we have to minimize the mean value of $H$ in the dipole with any constraints on $\eta$ and $\eta'$ at its entrance. This can be achieved when the following conditions are fulfilled in the center of the dipole [2]:

$$\eta = \eta_m = \frac{\rho \theta^2}{24}, \quad \eta' = 0.$$  \hspace{1cm} (4 - a)

$$\beta = \beta_m = \frac{\rho \theta}{2\sqrt{15}}, \quad \beta' = 0.$$  \hspace{1cm} (4 - b)

where $\theta$ is the bending angle of the dipole.

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2 APPLICATION TO SUPER-ACO

Super-ACO is a 0.8 GeV positron storage ring. It has a fourth order symmetry modified CG – DBA lattice with 8 dipoles (ρ = 1.7m) and 4 families of quadrupoles. Each quadrupole has extra coils which are powered to produce a sextupole field. Therefore, Super-ACO has 4 families of sextupoles. It is routinely operated in its standard modified CG configuration where the straight sections are alternately with (even sections) and without (odd sections) horizontal dispersion. The horizontal emittance is 38 nm.rad and the betatron tunes are 4.71 and 1.71 in horizontal and vertical plane respectively. Figure 1 shows betatron and dispersion functions over a half-cell of the standard lattice of Super-ACO.

The application of equation (3) to Super-ACO using $J_x = 1.04$ (calculation taking into account the edge effect), yields to a minimum absolute emittance of 9.5 nm.rad. This corresponds to $\eta_m = 0.043$ m, $\beta_m = 0.172$ m and $\eta' = \beta' = 0$, in the center of the dipole (equations 4). One can note the required low values of $\eta_m$ and $\beta_m$.

Using BETA code [3], we adjusted the four families of quadrupoles to obtain the desired $\beta_m$ and $\eta_m$ values in the center of the dipole. At the same time, we constrained $\beta_z$ to be $\geq 2$ m in the injection straight section in order to not go beyond the kicker strength limits [4]. Moreover, we constrained, in the same section, the value of $\beta_z$ to be not far from the one of the standard optics ($\approx 12$ m). The best compromise resulted in an emittance of about 9.85 nm.rad which is very close to the absolute minimum value of 9.50 nm.rad. The optics over a half-cell of the minimum emittance point is shown in figure 2. In this case, the lattice has an eighth order symmetry, then the number of families of quadrupoles (Q1 = Q4 and Q2 = Q3) is reduced from four to two. The maximum $\beta_z$ values are very similar to those in the standard lattice. In the straight sections, $\beta_z$ values are not modified while $\beta_x$ values are three times lower. In the dipole, $\beta_z$ is decreased by more than a factor of two. The dispersion function is nonzero everywhere, and it is the same in both types of sections (0.39 m).

![Figure 1: Optics of the standard lattice.](image1)

![Figure 2: Optics of the minimum emittance lattice.](image2)

The strong focusing required to obtain the minimum emittance resulted in higher betatron tunes as compared to the standard lattice. They are of 5.86 and 2.86 in horizontal and vertical plane respectively.

Figure 3 is a zoom on the optical functions inside the dipole, the center being an axis of symmetry.

![Figure 3: Minimum emittance optics in the dipole.](image3)

The minimum values of $\eta$ and $\beta$ are respectively 0.040 m (compared to 0.043 m) and 0.233 m (compared to 0.172 m). In fact, to reach the minimal emittance taking into account the criteria listed before (which shall be more developed in [4]), we found that the optimization is the most efficient when the three conditions $\eta' = 0$, $\beta' = 0$ and $\eta_m = 0.043$ m are exactly fulfilled while the fourth condition on $\beta_m$ can be slightly relaxed.

3 DYNAMIC APERTURE

As the dispersion is nonzero everywhere, the correction of horizontal and vertical chromaticities is performed simultaneously with the minimization of the second order tune shift with amplitude, using the four families of sextupoles. The resultant dynamic aperture calculated in
the injection straight section (where $\beta_x = 2\ m$ and $\beta_z = 11\ m$) is shown in figure 4.

![Figure 4: Dynamic aperture of the minimal emittance lattice](image)

One can notice that it is very comfortable. It is larger than the physical aperture and exceeds 150 r.m.s of the horizontal beam size at this location. Calculated momentum dependence shows very low tune variation of less than 0.02 over $\pm3\%$ deviation, with large dynamic apertures over the same range. More details on nonlinear characteristics will be given in [4].

### 4 EXPECTED BEAM LIFETIME

When the emittance is so small for such low energy machine, one can ask what is the consequence on the beam lifetime? In fact, the beam dimensions being smaller, the Touschek effect is stronger. As compared to the standard lattice, we expect a Touschek beam lifetime ($\tau_T$) reduction of a factor of 4. Fortunately, there is a favourable consequence from this optics which can somewhat counteract this effect. As the dispersion in the dipole is lower, the momentum compaction is reduced by more than a factor of 2 ($0.0067$ instead of $0.0148$ for the standard lattice). For a fixed RF voltage, the longitudinal energy acceptance ($\varepsilon_{RF}$) is then increased by a factor of 1.5. Due to its strong dependence on $\varepsilon_{RF}$, the $\tau_T$ can be enhanced by a factor of 3 if $\varepsilon_{RF}$ is the machine energy acceptance, that is $\varepsilon_{RF}$ is smaller than dynamical and physical energy acceptances. Then, the reduction of $\tau_T$ should be small. The result of $\tau_T$ calculations taking into account the three limitations as well as gas scattering effect will be commented in [4].

### 5 EXPECTED BRILLIANCE

For the experiment where the brilliance is the figure of merit, the emittance reduction should be beneficial. Figure 5 shows an example for an undulator installed in Super-ACO. It can be seen that the gain in on-axis brilliance (photon/s/mm²/mrad²/0.1%b.w.) can reach a factor 6.

![Figure 5: On-axis brilliance in the standard (solid curve) and minimum emittance (dashed curve) lattices.](image)

### 6 FIRST EXPERIMENTAL RESULTS

Experimental studies of the minimum emittance lattice started few months ago and first results are very encouraging. At low current, the r.m.s horizontal beam dimension ($\sigma_x$) measured at the exit of a dipole is in very good agreement with the corresponding theoretical value. For example, when the beam is fully coupled we have $\sigma_{x,\text{exp.}} = 110\ \mu m$ compared to $\sigma_{x,\text{the.}} = 100\ \mu m$. The horizontal dispersion measured at BPMs locations has a mean value of 0.48 m with an r.m.s of $\pm0.012$ m. It is very close to the theoretical value of 0.50 m. Other important parameters have also been measured as the synchrotron frequency, the natural bunch length and the betatron functions in the quadrupoles. The results fit well the expected emittance and momentum compaction values. The complete results will be given in [4].

### 7 CONCLUSION

The Super-ACO minimum emittance optics looks promising. It is obtained with reasonable values of quadrupoles and sextupoles. Experimental studies are still in progress in order to achieve optimum performance. The final aim is to make this optics ready to be tested by users.

### REFERENCES