Abstract

We propose a novel open resonator accelerating structure, estimate its effective shunt impedance and accelerating gradient, and discuss its prospects for implementation in hardware.

1 INTRODUCTION

Our open resonator accelerating structure can efficiently accelerate charged particles with a high gradient using stored electromagnetic (EM) field energy that can be provided by a variety of EM sources including lasers. In our technologically simple structure, field generation and particle acceleration can be combined in a single device, greatly simplifying accelerator design.

In a mirrored open resonator, the EM field amplitude varies slowly along a particle trajectory because diffraction limits the focused spot to a diameter greater than the EM wavelength. Thus energy is inefficiently transferred from the field to the particle, especially if the particle is slow [1]. So an open mirror resonator cannot generate EM fields and accelerate particles. However, the selective placement of perturbing dielectric or metal strips in the resonator cavity can cause the field to vary strongly. Particles passing through small holes in these strips will experience an efficient field-particle interaction that can be used to accelerate, as well as velocity modulate and bunch, charged particle beams [2].

2 OPERATIONAL PRINCIPLES

In our open quasi-optical Fabry-Perot resonator, the field variation along the beam path is produced by the special geometry of one of the two reflecting mirrors seen in Fig. 1. Here spherical (1) and plane (2) mirrors allow the standing wave, excited through a coupling slot (3), to be limited by a caustic surface (4). A pre-accelerated and pre-bunched electron beam (5) is injected parallel to the alternating electric field component and passes through holes in a sequence of strips (6) that are perpendicular to the electric field and to the beam.

We choose the strip thickness and spacing so that during acceleration the beam is in the gap while during deceleration the strips shield the beam. Thus for the limiting relativistic case, the strip thickness and spacing are λ/2, half the EM free space field wavelength. The mirror surface is also λ/2 from the base and the beam holes have diameters less than λ/2 and are located λ/4 from the base.

In the main resonator, the lowest EM field mode is a paraxial standing wave concentrated between mirrors whose amplitude is Gaussianly distributed in x and z. Since both the plane mirror and the gap bases are situated at standing wave nodes, the resonator field alone determines the gap field amplitude, phase, and spatial distribution. So the gap field amplitude along the particle path is almost constant and nearly that of the standing wave maximum above the gap. The field decays exponentially inside the strips.

There are several interesting variants of our open resonator. With multiple beam holes with different x, several beams can be simultaneously accelerated. Multi-beam acceleration can also be realized by making the gaps several half-wavelengths deep and by locating holes at the standing wave maximum in y. By appropriately shaping the hole geometry, we can obtain quadrupole focusing.

3 ESTIMATED PARAMETERS

To compare our device with other accelerating structures, we use the effective shunt impedance, \( R \), and the accelerating gradient, \( T \). \( \varepsilon = e_0 E/m_0 \omega c \) is the normalized maximum standing wave in the electric field, \( E \), where \( e_0 \) and \( m_0 \) are the electron charge and rest mass, \( \omega \) is the EM field angular frequency, and \( c \) is the speed of light. Neglecting space charge and beam loading, the particle energy gain per gap is \( \Delta W_p = 2eW_0 \), where \( W_0 \equiv m_0 c^2/e_0 \). So the electric field gradient dependence is

---

1 Department of Physics.
2 Institute of Nuclear Physics.
\[ T = \Delta W_0 / \lambda = E / \pi. \] (1)

### 3.1 Circular EM field spot

We use a rectangular approximation for our resonator Gaussian EM field distribution and bound the field energy by a cylinder of diameter, \( d \), and height, \( h = q \lambda / 2 \), where \( q \) is the number of half-wavelengths in the \( y \)-direction. The stored resonator energy is

\[ W = \frac{\varepsilon_0 \pi d^2 q \lambda^2}{8} \int_0^h E^2 \sin^2 \left( \frac{2\pi y}{\lambda} \right) dy = \frac{\pi}{32} \varepsilon_0 q \lambda d^2 E^2 , \] (2)

the EM power lost in the resonator is

\[ P_r = \frac{\omega W}{Q} = \frac{\pi^2}{16} \varepsilon_0 e d^2 q \lambda E^2 , \] (3)

and the effective shunt impedance is

\[ R = \frac{(\Delta W_0)^2}{P_r L} = \frac{16Z_0}{\pi^2} \frac{Q/q}{n \lambda} \frac{Q/q}{n \lambda (\Omega/m)} . \] (4)

\( \Delta W_c = n \Delta W_0 \) is the total energy gain in the resonator, \( n \) is the number of accelerating gaps, \( L = n \lambda \) is the length of the accelerating region, \( Q \) is the resonator quality factor, \( Z_0 = \sqrt{\mu_0 \varepsilon_0} = 120 \pi \) the characteristic vacuum impedance, and \( \varepsilon_0 \) and \( \mu_0 \) are the dielectric and magnetic constants. For simplicity we take \( L \) to be the caustic diameter, \( d = n \lambda \). The accelerating gradient in eV/m is

\[ T = \frac{\Delta W_0}{L} = \frac{4Z_0}{\pi^3} \frac{Q/q}{n \lambda} \sqrt{P_r} = 7.87 \frac{Q/q}{n \lambda (\Omega/m)} \sqrt{P_r(W)}. \] (5)

As \( \lambda \) decreases, \( R \) and \( T \) grow as \(-\lambda^{-1}\), however, mirror reflectivity is wavelength dependent as is \( Q \), so \( R(\lambda) \) and \( T(\lambda) \) are more complicated. \( n = 1 \) and \( Q/q = 100 \) \( R = 6.2 \) M\( \Omega \)/m at \( \lambda = 1 \) mm while \( R = 0.62 \) G\( \Omega \)/m at \( \lambda = 0.01 \) mm. For some arbitrary power \( P_r = 1 \) GW, the accelerating gradients are 2.5 and 250 GeV/m, respectively.

Because the EM field spot is circular on the spherical upper mirror, its area, the stored energy, and the power loss grow as \( n^2 = d^2 / \lambda^2 \) and so both \( R \) and \( T \) depend on \( n \).

### 3.2 Elliptical EM field spot

We eliminate this \( n \)-dependence by elongating the spot in \( z \), fixing it in \( x \), and having different upper mirror \((y,z)\) and \((x,y)\) plane curvatures. For a rectangular field distribution with a transverse \( z \) of \( d_z = n \lambda \) and \( x \) of \( d_x = \lambda \),

\[ R = \frac{4Z_0}{\pi^3} \frac{Q/q}{\lambda} = 48.6 \frac{Q/q}{\lambda (\Omega/m)} \] (6)

and

\[ T = 2 \sqrt{Z_0} \frac{Q/q}{\lambda \sqrt{n}} \sqrt{P_r} = 6.97 \sqrt{\frac{Q/q}{\lambda (m)/n}} \sqrt{P_r(W)} (\text{eV/m}) \] (7)

so that \( R \) is independent of \( n \) while \( T \) grows as \( n^{-1/2} \). Thus for any \( n \), \( R = 4.9 \) M\( \Omega \)/m and \( T = 2.2 / \sqrt{\pi} \) GeV/m at \( \lambda = 1 \) mm, and \( R = 0.49 \) G\( \Omega \)/m and \( T = 220 / \sqrt{\pi} \) GeV/m at \( \lambda = 10 \) \( \mu \)m.

### 4 PRACTICAL CONSIDERATIONS

To investigate if our open resonator accelerator can be realized in hardware, we consider the problems of (1) mirror surface damage by high power EM radiation and (2) forming and transmitting beam bunches through the small beam holes.

#### 4.1 Surface damage

We consider infrared-visible light and use the surface ablation threshold to limit the accelerating gradient. The maximum energy flux that materials can tolerate without substantial damage is pulse duration, \( \tau \), dependent. For \( \tau < 1-10 \) ps, the ablation threshold for the EM pulse energy absorbed in the resonator mirror is \( w_0 < 10^3 \) J/m\(^2\) [3]. Fixing \( \tau \) also fixes the resonator loaded quality factor, \( Q_l = \omega \tau \), and the number of accelerating periods, \( n = c \tau / \lambda \), with it the structure length, \( d_l = n \lambda \).

In Table 1 we estimate our open resonator accelerator parameters for \( \lambda = 0.5 \) and 10 \( \mu \)m for various pulse durations. \( P_r = W_r / \tau \) and \( W_c = W_r \omega \tau \) are respectively the power loss and energy loss per pulse in the resonator. \( W_b \) and \( P_b \) are the output bunch energy and power calculated assuming beam loading, \( W_b = W_r \). \( N_b \) and \( Q_b \) are the number of particles in the bunch, calculated as \( N_b = W_b / \Delta W_c \), and their charge. \( P_b \) is the average output beam power for the 100 kHz pulse repetition frequency.

<table>
<thead>
<tr>
<th>( \lambda ) (( \mu )m)</th>
<th>0.5</th>
<th>0.5</th>
<th>0.5</th>
<th>10</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_r ) (MW)</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>( \tau ) (ps)</td>
<td>0.1</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>( Q_l )</td>
<td>380</td>
<td>3,770</td>
<td>37,700</td>
<td>190</td>
<td>1,890</td>
</tr>
<tr>
<td>( N )</td>
<td>60</td>
<td>600</td>
<td>6,000</td>
<td>30</td>
<td>300</td>
</tr>
<tr>
<td>( d_l ) (( \mu )m)</td>
<td>0.03</td>
<td>0.3</td>
<td>3</td>
<td>0.3</td>
<td>3</td>
</tr>
<tr>
<td>( Q )-factor</td>
<td>1,130</td>
<td>11,300</td>
<td>113,000</td>
<td>570</td>
<td>5,660</td>
</tr>
<tr>
<td>( \omega \tau / \lambda ) (G( \Omega )/m)</td>
<td>85.0</td>
<td>30.8</td>
<td>9.8</td>
<td>24.6</td>
<td>9.8</td>
</tr>
<tr>
<td>( T ) (MeV)</td>
<td>0.8</td>
<td>3.0</td>
<td>9.4</td>
<td>2.4</td>
<td>9.3</td>
</tr>
<tr>
<td>( W_b ) (GeV)</td>
<td>0.3</td>
<td>3</td>
<td>3</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>( P_b ) (GW)</td>
<td>0.03</td>
<td>0.3</td>
<td>3</td>
<td>6</td>
<td>60</td>
</tr>
<tr>
<td>( N_b ) (x10(^6))</td>
<td>2.3</td>
<td>6.4</td>
<td>20</td>
<td>160</td>
<td>400</td>
</tr>
<tr>
<td>( Q_b ) (pC)</td>
<td>0.4</td>
<td>1.0</td>
<td>3.2</td>
<td>25.6</td>
<td>64.4</td>
</tr>
<tr>
<td>( A ) (pm)</td>
<td>520</td>
<td>52</td>
<td>5</td>
<td>20,800</td>
<td>2,080</td>
</tr>
</tbody>
</table>

#### 4.2 Beam emittance

With a \( \lambda / 4 \) beam hole diameter, our accelerating structure geometrical acceptance is \( A = \lambda^2 / 16 d_l \) which is tiny compared to conventional low energy accelerator beam emittance. Only at beam energies of \( \sim 10 \) GeV could the conventional geometrical emittance fit our...
acceptance for $\lambda = 10$ $\mu$m. The longitudinal acceptance is even more limiting to the injected beam acceptance since it decreases as $\lambda_{inj}/\lambda$, where $\lambda_{inj}$ is the RF injector wavelength.

These emittance problems can be addressed by designing the accelerator injector to have the same wavelength as the resonator. Providing a low emittance beam with sufficient charge suggests a field emission cathode [4] be placed at the first accelerating gap wall. For our $-10^{10}$ $\text{V/m}$ electric field amplitude, the autoemission current density can be as high as $10^9$ $\text{A/cm}^2$ which will provide sufficient bunch charge. The injector must be a $\beta$-graded accelerating structure so that the accelerating gap length can change with the electron velocity change.

5 CONCLUSION

Our open resonator particle accelerator powered by a conventional laser permits an electron beam energy to gain $\sim 10$ MeV per several mm. By combining resonators in series and in parallel, we should be able to realize a compact, high energy, high average power electron beam device.

ACKNOWLEDGEMENTS

We thank E.A. Knapp and W.P. Trower for useful comments.

REFERENCES


