MODELIZATION OF THE THERMO-MECHANICAL STRUCTURE OF THE LHC MAIN DIPOLE AND INFLUENCE ON FIELD QUALITY

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Abstract

The mechanical structure of the main LHC dipole is analysed. A finite element model is used to estimate the loads and the deformations at cryogenic temperature. The correct setting of the model parameters is crucial to obtain a reliable model to forecast the influence of design and tolerances on field quality. We discuss how the prestress loss from room to cryogenic temperature experimentally observed in the prototypes can be predicted using the finite element model. An estimate of the influence on field quality of deformations and tolerances due to manufacturing is given.

1 INTRODUCTION

In the main Large Hadron Collider [1] dipoles, coil deformations are sources of field errors that must be estimated and, when possible, controlled and corrected. They are induced by assembling prestress, thermal shrinkage during cool-down and mechanical tolerances of components. Field-shape errors may produce instabilities in circulating particle trajectories, limiting the collider performances.

In this paper we discuss analytical and numerical models to describe the thermo-mechanical stresses [2, 3] in the LHC dipoles [4]. These models are used to derive a sensitivity table to evaluate the impact on field harmonics of deviations with respect the nominal design. Such sensitivities are used to analyse the data of the first three final prototypes, whose collared coils have been measured during the last year [5]. Moreover, we perform a MonteCarlo analysis to estimate the impact on random errors of component tolerances, giving a comparison with measured values.

2 MODELING COIL DEFORMATION

In this section we modelize the thermo-mechanical behaviour of the dipole coil using both an analytical formalism and available experimental data.

The thermal effect on azimuthal coil stress was measured in several 1 m long dipole models, with single or double aperture design, and in two 15m long dipole prototypes, MBP2N2 and MBP2O1. In Fig. 1 we show these data for all magnets made with austenitic steel collars. The loss of prestress induced by cool-down is fitted by:

$$\sigma_c \approx 0.5(\sigma_w - 15)$$

with a dispersion of the data around 10 MPa. Note that a similar dispersion was observed in the SSC dipole prototypes [2].

![Figure 1: Prestress loss for LHC dipole prototypes](image)

The prestress loss can be described through models of increasing complexity. We first assume infinitely rigid collars; the variation with temperature of the coil strain from $\epsilon_w$ (at 300K) to $\epsilon_c$ (at 1.9K) equals the different thermal contraction of the coil $\alpha_b$ and of the collars $\alpha_c$:

$$\epsilon_w - \epsilon_c = \alpha_b - \alpha_c.$$  

We also assume, in first approximation, that at constant temperature the strain-stress curve is linear. The slope is the Young modulus $E_w$ at 300 K, and $E_c$ at 1.9K:

$$\sigma_w = E_w \epsilon_w \quad \sigma_c = E_c \epsilon_c.$$  

One can compute the prestress loss due to cool down:

$$\sigma_c = \frac{E_c}{E_w} \sigma_w - E_c (\alpha_b - \alpha_c);$$

in this linear relation the slope is smaller than one (we have 0.5 from experimental data, see Fig. 1) only if $E_c < E_w$, that is an unphysical assumption.

Indeed, one has to take into account that the stress-strain relation for the coil is nonlinear, and, for $\sigma > 10$ MPa, can be well-approximated by a parabola tangent to the displacement axis $d$ (see for instance [2]):

$$\sigma = a(d - d_0)^2 \quad \sigma = F' \epsilon \quad F \equiv ad_0^2,$$

where $d_0$ is the length of the unloaded coil and $a$ is a constant. The Young modulus is defined as the tangent:

$$E = \frac{d\sigma}{d\epsilon} = 2F \epsilon = 2\sqrt{F\sigma}.$$
These relations hold at any temperature, and $F$ depends on the temperature. Substituting Eq. (5) in Eq. (1), one has

$$\sqrt{\sigma_w/F_w} - \sqrt{\sigma_w/F_c} = \alpha_b - \alpha_c, \quad (6)$$

and therefore the prestress loss is

$$\sigma_c = \frac{F_c}{F_w} \left( \sqrt{\sigma_w - 2\sqrt{\sigma_w \alpha_b - \alpha_c} + F_w \alpha_b - \alpha_c} \right)^2. \quad (7)$$

The slope of the warm-cold relation is given by

$$\frac{d\sigma_c}{d\sigma_w} = \frac{F_c}{F_w} \left( 1 - \frac{\sqrt{\sigma_w \alpha_b - \alpha_c}}{\sqrt{\sigma_w}} \right). \quad (8)$$

Indeed, one can have a slope $d\sigma_c/d\sigma_w < 1$ (as measured experimentally) and a physical ratio $F_c/F_w > 1$, due to nonlinearity.

An additional contribution to prestress loss comes from the collar deformations $\epsilon_{cw}$ and $\epsilon_{cc}$ (at 300 K and 1.9 K respectively), that are proportional to the prestress $\sigma$ through an equivalent Young modulus $E_{cw}$ and $E_{cc}$, that is related to the Young modulus of the material and to the collar geometry. Using finite element models and analytical estimates we evaluated $E_{cw} = E_{cc} = 26000$ MPa. Thus one has:

$$\epsilon_w - \epsilon_{cw} = (\epsilon_c - \epsilon_{cc}) = \alpha_b - \alpha_c \quad (9)$$

$$\sqrt{\sigma_w/F_w} - \sqrt{\sigma_{cw}/E_{cw}} = \sqrt{\sigma_c/F_c} + \sqrt{\sigma_{cc}/E_{cc}} = \alpha_b - \alpha_c \quad (10)$$

This effect on prestress loss is not negligible (of the order of 30%).

Using finite element models, one can implement the nonlinear stress-strain curve, or use a linearized model that assumes a constant Young modulus for the coil. In this second case, one can set such modulus equal to the tangent of the stress-strain curve at the average pressure (working point), both at 300 K and at 1.9 K. Indeed, one has to use a fictitious thermal contraction coefficient $\alpha_{cw}$ for the coil if one wants to recover the correct prestress loss and sensitivity of the complete nonlinear model.

Using the developed analytical model, that agrees well with the result of a finite element model, we try to recover the prestress loss from the known material properties. Using the current estimates, i.e. 1.5 ratio cold to warm elasticity modulus (for a fixed stress), and thermal contraction set to $6.5 \times 10^{-5}$, we do not obtain the experimental slope shown in Fig. 1. For this reason we also developed a fitted model, optimized to recover the right prestress loss at cold. This leads to material properties in contrast to present knowledge: we use a hardening ratio cold to warm equal to one, and a thermal contraction of the coil set at $10 \times 10^{-5}$. Simulations were carried out in parallel on both models. An experimental program to clarify the modelization of prestress loss is in progress.

3 SENSITIVITIES ON TOLERANCES

To control field quality during series production, corrective actions should be investigated, eventually based on appropriate shimming strategies. We thus estimated the effect of additional shims on various regions of the coil-collar interface shown in Fig. 2. Sensitivities to field-shape harmonics were computed for shim thickness of 50 $\mu$m. Indeed, the response is linear for shim thickness larger than 100 $\mu$m.

Table 1: Sensitivities on 50 $\mu$m shift in collar regions labelled in Fig. 2, left-right symmetric (upper part) and left-right antisymmetric (lower part)

<table>
<thead>
<tr>
<th>Region</th>
<th>$b_3$</th>
<th>$b_5$</th>
<th>$b_7$</th>
<th>$c_1$</th>
<th>$c_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1+2</td>
<td>+0.8</td>
<td>-0.16</td>
<td>+0.05</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>3+4</td>
<td>+0.6</td>
<td>-0.02</td>
<td>-0.01</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>5+6</td>
<td>+0.4</td>
<td>-0.09</td>
<td>0.00</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>7+8</td>
<td>-0.5</td>
<td>+0.05</td>
<td>0.00</td>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Region</th>
<th>$b_2$</th>
<th>$b_4$</th>
<th>$b_6$</th>
<th>$c_1$</th>
<th>$c_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>+2.1</td>
<td>-0.13</td>
<td>+0.04</td>
<td>±6</td>
<td>0</td>
</tr>
<tr>
<td>3-4</td>
<td>+1.5</td>
<td>+0.10</td>
<td>-0.02</td>
<td>0</td>
<td>±5</td>
</tr>
<tr>
<td>5-6</td>
<td>+0.2</td>
<td>+0.12</td>
<td>-0.07</td>
<td>±1</td>
<td>0</td>
</tr>
<tr>
<td>7-8</td>
<td>-3.0</td>
<td>-0.16</td>
<td>+0.00</td>
<td>±6</td>
<td>±4</td>
</tr>
</tbody>
</table>

Figure 2: Interface regions between collars and coils

The estimate was made using the four available models to describe the thermo-mechanical properties of the dipole cross section, i.e. linear and nonlinear models with fitted coil parameters, and linear and nonlinear models with nominal coil parameters. In the four cases, the results differ by less than 30 %, and agree with estimates given in [6]. In Table 2 we show the results relative to the fitted nonlinear finite element model. Sensitivities on the allowed harmonics $b_3$, $b_5$ and $b_7$ are obtained with left-right symmetric shims. Instead, sensitivity on the non-allowed harmonics $b_2$, $b_4$ and $b_6$ is obtained with left-right antysymmetric shims, i.e. the shim is larger than nominal in one side and shorter in the other side. The last two columns of Table 1 give the variation of prestress $\sigma_i$ induced in the inner layer and $\sigma_o$ induced in the outer layer. Note that field harmonics are practically insensitive to the shape of the outer collar (regions 9, 10 and 11 of Fig 2), and therefore data are not shown for the sake of brevity.

The results of Table 1 show that shimming techniques can be used in appropriate linear combination to correct multipoles. Indeed, this technique has a limited effect if
one wants to avoid large stress variations, to preserve the optimum stress window of the coil layers.

4 SHIM SIZE AND FIELD QUALITY

The first three final dipole prototypes with austenitic steel collars, i.e. MBP2N2, MBP2O1 and MBP2A1, were collared with shim thicknesses different from the nominal one [5]. The three prototypes feature relevant differences in the low-order allowed harmonics in the collared coil; such differences are still preserved in cold measurements.

An extremely simplified modelization of these discrepancies can be made by assuming that field quality variations are only due to different shim thickness. We analyse the difference in the measured $b_3$ and $b_5$ between the prototypes MBP2O1, MBP2A2, and MBP2N2, considered as a reference magnet. Data are shown in Table 2, where the averages between the apertures have been considered. On the other hand, we can compute using the sensitivity table the effect due to different shim thicknesses (Table 2, columns 3 and 5). This simplified modelization already explains most of the shift observed in $b_3$, and a significant fraction of the shift in $b_5$. A dipole that features sections with different shim thicknesses in order to have a direct experimental measure of the sensitivities is foreseen.

Table 2: Differences in $b_3$ and $b_5$ between three prototypes: experimental data and numerical estimates

<table>
<thead>
<tr>
<th></th>
<th>$b_3$ meas.</th>
<th>$b_3$ comp.</th>
<th>$b_5$ meas.</th>
<th>$b_5$ comp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>O1-N2</td>
<td>-5.1</td>
<td>-3.4</td>
<td>+0.4</td>
<td>+0.4</td>
</tr>
<tr>
<td>A2-N2</td>
<td>-4.8</td>
<td>-5.1</td>
<td>+1.4</td>
<td>+0.8</td>
</tr>
</tbody>
</table>

5 MONTECARLO ANALYSIS

We performed a MonteCarlo analysis of the effect of tolerances on collars and coils on multipoles and prestress [7]. We evaluated 100 different mechanical structures within tolerances (collars and coils randomly varied) using the fitted nonlinear finite-element model. We generate 100 different geometries, considering error realisations with a Gaussian distribution truncated at $3\sigma$, where $3\sigma$ is set as the nominal tolerances. For each geometry we evaluate pre-stresses and multipoles.

In Fig. 3, the obtained r.m.s of the harmonics are drawn in a semilogarithmic scale as a function of the harmonic number $n$. In the same plot there are also the r.m.s. harmonics measured along the axis of the 15 m long dipole prototype MBP2N2 and the so-called target harmonics, i.e. the maximum random multipoles tolerable for long-term beam stability during LHC operation. We implicitly assume that the spread of multipoles along the axis of one dipole is a good estimate of the random fluctuations of the average harmonics during production: this hypothesis has to be verified.

There is a good agreement between the measured values and the estimates through the MonteCarlo. Both are much smaller that the target values, $a_4$ being the closest to the upper bound. This is an encouraging fact, since other effects, such as the random fluctuations of the persistent currents and of the iron magnetisation, are not yet included.

6 CONCLUSIONS

We have analysed the problem of modeling the thermo-mechanical behaviour of the LHC main dipole in view of evaluating sensitivities of field-shape errors on mechanical tolerances. Two finite element models have been developed: the ambiguity stems from the difficulty in getting the right pre-stress loss with known material properties (the same difficulty was found for the SSC dipoles [2]).

Sensitivities do not strongly depend on the used model, and we evaluate them for the collar-coil interfaces. Such sensitivities are used to estimate the impact of different shim sizes on allowed low-order harmonics, showing that they can explain most of the differences measured in the collared coils of the first three LHC dipole prototypes. A MonteCarlo method is used to estimate the random errors due to tolerances. The obtained estimates are in agreement with the measured multipole variations along the axis.

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REFERENCES