STOCHASTIC COOLING FOR THE FAIR PROJECT

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Abstract

Stochastic cooling is used in the framework of the FAIR project at GSI for the first stage of phase space compression for both rare isotope and antiproton beams. The collector ring CR serves for the precooling of rare isotope and antiproton beams. The paper discusses mainly the stochastic accumulation in the RESR based on a new lattice design.

STORAGE RINGS IN THE FAIR PROJECT

The storage rings in the FAIR project are designed for the preparation of experiments with rare isotope (RI) or antiproton beams, which are produced by bombardment of short high intensity bunches from the SIS100 synchrotron [1] on appropriate production targets. As these beams have large longitudinal and transverse emittances, stochastic precooling is foreseen in the Collector Ring (CR) [2].

The antiproton beams are accumulated in the RESR storage ring [3]. High energy antiproton experiments make use of stochastic cooling in the HESR storage ring [4].

PRECOOLING IN THE COLLECTOR RING

The stochastic cooling systems in the CR have been described in [5] and [2].

The development of slotline electrodes for the CR is described in [6]. A prototype of the 1 GHz - 2 GHz power amplifier has been built and will be tested at GSI in the near future. The integration of the slotline structures into a complete pick-up tank is presently prepared.

STOCHASTIC ACCUMULATION IN THE RESR RING

Overview

Stochastic accumulation in the RESR makes use of the same principle which has successfully been used in the AA at CERN [7], [8] and in the Accumulator at FNAL [9]. In any case, the accumulation works in the longitudinal phase subspace. Figure 1 shows a sketch of the vacuum chamber at the pick-up which is used for accumulation.

The beam is injected at the injection orbit (i). It is then deposited by rf to a deposition orbit (d). Before the next shot arrives, the stochastic cooling system must be fast enough to shift these particles to the stack tail (t). The repetition interval between single injection shots is mainly given by the time it takes to perform the shift between (d) and (t). Then the same pick-up signal is used to shift the particles gradually into the core. The pick-up sensitivity of the stack tail cooling pick-up should decrease exponentially towards the core.

In order to achieve this goal, the vertical β function at the pick-up must be small and the dispersion large (see below). However, experience from the CERN AA shows that in addition a twofold staggered notch filter may be needed in order to get the system gain down in the core region.

New RESR Lattice

The new lattice of the RESR [10] has the following advantageous properties with respect to antiproton accumulation:

- The lattice enables a flexible choice of the transition gamma up to γ_t = 6.3.
- There are straight sections with large dispersion and small vertical betatron function for the accumulation pick-up.
- There is enough space in dispersion free sections to take up the stochastic cooling kicker tanks.

RESR Cooling Systems

Four cooling systems are envisaged for the RESR:

1. The stack tail cooling system (longitudinal, see above)
2. The core cooling system (longitudinal)
3. A horizontal betatron cooling system
4. A vertical betatron cooling system

Figure 2 shows the locations for pick-ups and kickers in the new RESR lattice. Figure 3 shows the Twiss functions of an optical setting with γ_t = 5.3.

In a first stage, the system will work in the 1 GHz - 2 GHz band. Due to the chosen γ value, an upgrade up to 4 GHz is feasible. The pick-ups and kickers will be of the Faltin [11] type. The core cooling system will use the same kicker as the stack tail system, just with an additional quadruplet of pick-up electrodes in the accumulation pick-up structure, and a low gain amplification.
Exponential Gain Profile and Vertical Chamber Height

For optimum accumulation we need an exponential gain profile [12] with the property

\[ g(x) = g_t \exp\left(\frac{x - x_t}{\delta x}\right) \]  

(1)

leading to an exponential particle distribution

\[ \Psi(x) = \Psi_t \exp\left(-\frac{x - x_t}{\delta x}\right) \]  

(2)

with \( \delta x > 0 \). Accumulation proceeds towards negative \( x \) (Figure 1). \( g_t \) and \( \Psi_t \) are the values of the gain and distribution functions at the stack tail orbit \( x_t \). If \( \Psi_c \) is the distribution at the core \( x_c \), then

\[ \delta x = (x_c - x_t) \ln\left(\frac{\Psi_t}{\Psi_c}\right) \]  

(3)

and

\[ \frac{\Psi_t}{\Psi_c} = \frac{g_c}{g_t} \]  

(4)

For the RESR, we want to inject \( 10^8 \) antiprotons per shot and accumulate up to at most \( 2 \cdot 10^{11} \) particles. Hence we must achieve a (voltage) gain drop of 66 dB over \( x_c - x_t \). This can only be achieved if the chamber height \( h \) is small compared to \( x_c - x_t \). An electrostatic model of the electrode sensitivity \( S(x) \) yields in the vertical midplane of a sum pick-up

\[ S(x) = \frac{2}{\pi} \arctan\left(\frac{\sinh(\pi w/2h)}{\cosh(\pi x/h)}\right) \]  

(5)

\( h \) is the vertical separation between pick-up plates, \( w \) is their horizontal width. For large \( |x|/h \), this scales as

\[ S(x) \propto \exp\left(-\frac{\pi |x|}{h}\right) \]  

(6)

In case of the RESR

\[ \frac{x_c - x_t}{h} \approx \ln\left(2 \times 10^3\right) \approx 2.42 \]  

(7)

In other words: If the gain profile is realized only by the sensitivity drop from the pick-up to the core (and not by additional notch filters), then the distance between the stack tail and the stack core should be about 2.4 times the chamber height. This leads to rather tight requirements for the chamber height. In the straight sections inside the arcs of
the RESR, the dispersion is about 13 m, and the vertical beta function is below 3 m along a distance of 3 m. These are almost ideal conditions for an accumulation pick-up. With the vertical emittance \( \epsilon_y \) of 10 mm mrad, one gets a beam height of \( \Delta y = 2\sqrt{\beta_y \epsilon_y} = 11 \) mm. Adding a safety margin of \( \pm 3 \) mm on each side, one arrives at a chamber height of 17 mm. With these parameters, one then would get a distance of at least 41 mm from the stack tail to the core. An analog requirement is that the stray field of the injection kicker must not disturb the beam at the stack tail. This requirement gives a limit for \( x_i - x_d \) (see Figure 1).

**Desired and Undesired Mixing**

Once the distance \( x_c - x_t \) is given, the product

\[
x_c - x_t = D (\delta p/p)_{ct}
\]

is also fixed. Here \( (\delta p/p)_{ct} \) is the relative momentum difference between tail and core, and \( D \) is the dispersion at the pick-up. For the RESR pick-up it follows that \( (\delta p/p)_{ct} = 3.2 \times 10^{-3} \).

This number is important as the product \( |\eta|(\delta p/p)_{ct} \) is important for the mixing number

\[
M = (mc|\eta|(\delta p/p)_{ct})^{-1}
\]

which should be of the order of unity. Here \( \eta = (\delta f/f) / (\delta p/p) \) is the frequency slip factor, and \( mc \) is the harmonic number in the center of the cooling band. On the other hand, the undesired mixing (bad mixing)

\[
B = \cos (\pi mc x \eta pk (\delta p/p)_{cd})
\]

must not be too small of even get negative. Here \( (\delta p/p)_{cd} \approx 4 \times 10^{-3} \) is the momentum width between the deposit and the core orbits. This number enters into the cooling rate equation for transverse cooling. In this equation \( x = (s_k - s_p)/C \) is the ratio of the path between pick-up and kicker along the closed orbit and the circumference of the closed orbit. \( \eta_{pk} \) is the local frequency slip factor between pick-up and kicker. It is assumed for simplicity that the cooling system is adjusted to the time of flight of a particle at the position \( x_c + x_d \)/2. One should require that the cooling decrement as a function of frequency should not have the wrong sign even at the upper limit of the cooling band, leading to an upper frequency limit:

\[
f_G = \frac{f_{rev}}{2xmc \eta_{pk} (\delta p/p)_{cd}}
\]

where \( f_{rev} \) is the revolution frequency.

In the long straight straight sections, the dispersion vanishes. These sections are used for the kickers. The section consists of a central part with a total length of 18m, delimited by a quadrupole doublet on each side. Between each doublet and the adjacent dipole there are additional dispersion free straight sections (7 m) with a vertical waist (\( \beta_y \) at most 7.8 m). The vertical phase advance here amounts almost exactly to 90 degrees. Of these straight sections, three are occupied by injection or extraction septa, which are located close to the dipoles. The horizontal pick-up is located close to the next dipole (Figure 2), reserving space for an optional electron cooler. Because the vertical cooling pick-up should be at moderate beta functions. it is placed at the beginning of the northern arc, where the dispersion is still below 0.7 m.

Table 1 shows some parameters of the new stochastic cooling paths. \( s_k - s_p \) is the length of the central closed orbit between pick-up and kicker. Shortcut is the length of
the straight connection across the ring between the end of
the pick-up and the beginning of the kicker. This number
is needed for the evaluation of the time which is available
for electronic processing (amplifiers, filters, etc.). The free
signal processing time $T_{\text{free}}$ is calculated by assuming a
signal transmission velocity of 0.95 $c$ across the ring. $\gamma_t$
and $\eta$ are the local parameters between pick-up and kicker.$f_G$ is the upper operating frequency limit (see eq. 11).
The longitudinal kicker could be placed at the opposite
side of the long straight section, just before the injection
septum magnet. This choice leaves a comfortable time
interval of 91 ns for signal processing, but still allows for
increasing the operating bandwidth in a possible future sys-
tem upgrade.

**Approximate Optimum Frequency Slip Factor**

The transverse cooling rate can be written

$$
\frac{1}{\tau_{\perp}} \approx \frac{2W}{N} \left[ 2Bg_{\perp} - (M + U) |g_{\perp}|^2 \right]
$$

(12)

If one works at the optimum gain

$$
|g_{\perp}|_{\text{opt}} = \frac{B}{(M + U)}
$$

(13)

the optimum cooling rate is

$$
\left( \frac{1}{\tau_{\perp}} \right)_{\text{opt}} = \frac{2WB^2}{N(M + U)}
$$

(14)

Under these conditions one can deduce an approximate op-
timum value for the frequency slip factor, if one assumes in
addition that

1. the diffusion due to Schottky noise dominates the dif-
fusion due to thermal noise, i.e. $M \gg U$,

2. if we vary $\eta$ then we vary $\eta_{pk}$ proportionally, i.e if we
  change the optical setting then the ratio of these values
  remains approximately constant.

Then the optimum cooling rate can be written in the form

$$
\left( \frac{1}{\tau_{\perp}} \right)_{\text{opt}} = \alpha \eta \cos^2 b \eta
$$

(15)

where $\alpha$ is independent of $\eta$ and $b = \pi m_c x (\delta p/p)_{\text{tot}}$. Here
$(\delta p/p)_{\text{tot}}$ is the total range of momenta to be cooled. This
expression can be treated as a function of $\eta$ It has a max-
imum if $2b\eta \tan b\eta = 1$ or if

$$
|\eta|_{\text{opt}} = \frac{0.208}{m_c x (\delta p/p)_{\text{tot}}}
$$

(16)

This expression can serve as a guide to estimate the opti-
mum $\eta$ value. It should be noted that it is independent of
$\Delta p/p$. For example we get for the RESR ($m_c = 1236$,
$(\delta p/p)_{\text{tot}} \approx 4 \times 10^{-3}$, and $x \approx 0.5$) an optimum value of
$|\eta|_{\text{opt}} \approx 0.084$, whereas the actual value with $\gamma_t = 5.3$ is
$\eta = 0.022$.

If $U$ has the same order of magnitude as $M$, then the
 optimum $\eta$ becomes smaller than the analytic estimate, it is
zero in the case $U \gg M$, because then the desired mixing
is worthless.

On the other hand, because during the process of cooling
the momentum width becomes smaller, larger values of $\eta$
become desirable. If one cannot or does not wish to ramp
the quadrupoles during cooling, one would have to chose
whether fast initial cooling or high equilibrium phase space
density are more important issues.

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