Some Studies on Regenerative Beam Extraction in Synchrocyclotrons

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Abstract

In this paper it is shown that the extraction efficiency in a synchrocyclotron can be increased considerably by selecting proper values for the internal circulating beam parameters. A good energy homogeneity and a high microscopic duty cycle can be achieved simultaneously. A calculation for some relevant radial amplitudes and energy gains/turn gives the energy diagram in fig. 11. From this it can be found that, if the internal beam is composed of 0.4 - 0.6 cm incoherent radial amplitudes and if the useful acceleration voltages are 7.2 - 10.6 kV, the gross extraction efficiency is 40% for an energy spread of 300 keV. The duty cycle will be about 20%. These results are valid if the vertical amplitudes are small so that the median plane motion program is correct.

Introduction

In a regenerative extraction system\(^\text{1-5}\) the extraction efficiency, \(\eta\), depends critically on the quality of the internal circulating beam. In a synchrocyclotron \(\eta\) is usually very low. It has been found, however, that this extraction method can give a duty cycle of 25%\(^\text{a}\) and an energy spread of 0.2 MeV (FWHM) at 185 MeV/\(\text{cm}^2\) protons.

In order to study the influence of radial and phase oscillation amplitudes on the regenerative process and to find suitable dee-voltages, computer calculations have been performed in which the particles were followed during the whole extraction process. Below will be given explanations for the behaviour of the particles during the extraction process and for the compression of the energy spread that takes place. Finally it will be shown how a large gain in extraction efficiency, keeping the small energy spread, can be accomplished by an adequate choice of the internal circulating beam parameters.

The beam extraction of the Uppsala synchrocyclotron

In 1955 the beam was successfully extracted from the cyclotron by adopting a non-linear regenerative system. The extraction was considered to start at a radius \(r_0 = 100\) cm, and the beam had to pass a magnetic ridge of about 12 cm radial width before it could spiral out. The maximum useful radius was 102 cm where the field index \(n = 0.2\). After passing a magnetic channel at \(r = 108\) cm the phase-space distribution of the beam was changed by two "focusing channels" in the fringing field, as is shown in fig. 1. The extraction system is found to be very energy selective and only a small fraction of the internal circulating beam passes. Typically, the energy spread is 13 MeV FWHM at 180 MeV, as was measured by using the cyclotron magnet as a 180° analyzer. The energy spread in the external beam is decreased under these conditions. The measurements indicate that the extraction system accepts only relatively small amplitudes of the radial oscillation, as will be shown to some extent below. The external cyclotron beam is brought through an evacuated tube of about 20 meters length to the experimental area, where the beam can be focused by a quadrupole pair to a cross section of about 10 mm. The emittance is about 230 mm.mrad in both planes, and the intensity is normally \(3 \times 10^8\) protons/sec.

Median plane motion

Assume that the regenerator field and the fringing field start at the same radius \(r_0\). The radius of the equilibrium orbit for a particle moving in the field, and the amplitude and angular frequency of the betatron oscillation are denoted by \(r_0\), \(\rho_m\) and \(\omega_r\), respectively, where \(\omega_r\) is given by

\[
\omega_r = \omega(1-n)^{1/2}
\]

The maximum radial outswing \((r_0 + \rho_m)\) at azimuth \(\theta_m\) will precess around the magnetic centre of the cyclotron and the same holds for the orbit centre \(P\). The locus of the precession of \(P\) is essentially a circle with radius \(\rho_m(1-n)\). As soon as the regenerator field starts to influence the particle motion for some value \(\theta < \theta_s\), \(\theta_s\) being the regenerator azimuth, there is a certain increase in \(\rho_m\) per turn, as is shown in fig. 2. If the particle has not penetrated enough, \(P\) will "slip" past the regenerator azimuth and \(\rho_m\) starts to decrease. Due to the successive increase in \(r_m\) caused by the acceleration, the regenerator action is on the average stronger for \(\theta > \theta_s\) than for \(\theta < \theta_s\), and this implies that a slip through the regenerator always causes a net decrease in \(\rho_m\) which will somewhat affect...
the energy compression, as will be discussed later. The most critical passage at the regenerator is when $\delta \theta$, the total slip in $\theta$ per turn, is almost zero. This gives a slow precession during which much energy is gained, i.e., $r$ increases thus giving a strong regenerator action for $\theta > 0$. During the last stage of the extraction the motion of the particles is strongly locked in phase by the extraction system, i.e., $\Theta$ is fixed at some value $\Theta_f (\theta_f < \theta)$. The motion of $\Theta$ (or $p$) towards $\Theta_f$ can proceed in mainly two different ways. For particles experiencing a relatively weak regenerator action $\Theta \rightarrow 0$ for $\theta < \theta < \theta_f$, which means that $\Theta$ will decrease and return to $\Theta_f$, giving the motion of $p$ in the $p, \Theta$-plane the characteristic form of a "knee". For particles experiencing a relatively strong regenerator action, $\Theta \rightarrow p_m$ is always positive and $p$ approaches $\Theta_f$ from smaller values, which gives a more "straight" extraction. This can be seen in fig. 3, which is a result of a computer run. Particles A and B belong to the latter category and D and E to the former.

In the following is developed a theory for the motion of the orbit centre $P$. We start at the moment the particle crosses the entrance to the linear field region and assume that this happens at $\theta = \theta_0$. One revolution later the particle will experience regenerator action and $\rho_m$ will increase by an amount of $\delta \rho_m$. The quantity $\delta \rho_m / \rho_m$ expressed as a function of the parameters $b = r - r_f$, $\theta_f$, and $\rho_m$, determines how strongly the extraction sets in, and, as the regenerator to the linear field region, the value of $\delta \rho_m / \rho_m$ should give a good idea of the subsequent motion of $P$. For particles with the same $\delta \rho_m / \rho_m$ the trajectories described by $P$ will be approximately uniform, if an essentially constant accelerating voltage (small phase oscillations) is assumed. In order to get an expression for $\delta \rho_m / \rho_m$ we define $x = r - r_f$ and $\theta_0 = \theta - \theta_f$. If $\nu$ is the betatron frequency corresponding to the equilibrium orbit $r_f$, the increase in $\rho_m$ due to the precession is

$$\delta \rho_m = \frac{2\pi (1-\nu)}{\nu}$$

The radius of the precession circle is $\rho (1-n) = \rho \nu^2$, and consequently the vector $F_1$ in fig. 2 is

$$F_1 = \delta \rho_m \cdot \rho_m \nu^2 = 2\pi \nu (1-\nu) \rho_m$$

After one revolution the outswing $x_1$ in the regenerator becomes

$$x_1 = 2\pi \nu (1-\nu) \rho_m \sin \theta_0 + \delta r_0$$

as can be found from fig. 2. The first term is thus caused by the precession and the second term is the increase in $r_0$ due to acceleration. The increment $\delta \rho_m$, caused by a regenerator of strength $T_{x_r} + W_{x_r}^2 \rho_m^2$ (fig. 2) in eq. (3) is roughly

$$\delta \rho_m = (T_{x_r} + W_{x_r}^2 \rho_m^2) \cdot \sin \theta_0$$

Inserting eq. (2) in eq. (3) gives

$$\delta \rho_m / \rho_m = T_{x_r} \sin \theta_0 + W_{x_r} \rho_m \sin^2 \theta_0 + \delta r_0$$

where

$$c_1 = 2\nu (1-\nu)$$

The first and second terms in (4) arise from the linear and quadratic terms, respectively, of the regenerator strength. The third term represents the contribution due to the acceleration. Since $\delta \rho_m$ is small compared to $T_{x_r}$ and $W_{x_r}$, it is seen that the third term is important only for small $\rho_m$. Graphs of $\delta \rho / \rho_m$ for $\rho_m = 0.2 \text{ cm}$ and $1.0 \text{ cm}$ are shown in fig. 4. The relation at the starting moment between $b, \rho_m$ and $\theta_f$ can be found from geometric considerations which yield

$$\cos \theta_0 = -\frac{b}{\rho_m \nu}$$

and this relation is shown in fig. 5 with $b$ as parameter. Lines of constant value of $\delta \rho / \rho_m$ in fig. 4 give a relation between $\rho_m$ and $\theta_f$ at the points of intersection with the curves. Plotting these points in fig. 5 and joining points of constant $\delta \rho / \rho_m$ one gets what may be regarded as equipotential levels for the relative strength of the regeneration. One of these levels forms the threshold between slip and knee motion. In fig. 5 the position of the orbit centre for a sample of numerically traced particles is marked when $x = 0$, i.e., when the particles reenter the linear field at the regenerator. Dots indicate that slip motion will follow and circles and triangles indicate knee and straight extraction, respectively. In order to trace some interesting particles from the slip position to the final regeneration state in the diagram, connecting lines have been drawn. Those lines "cover" the 30 - 40 turns needed for the last betatron cycle and they show the decrease in $\rho_m$ associated with a slip motion. The threshold level in this analysis is calculated by the computer and is shown in the diagram (upper "computed" curve). As a comparison the analytically calculated curve is also given, the position of which is chosen to be common with the computed curve for $\rho_m = 0.6$. Just below the threshold level there is a critical interval where the particles get a slow, attenuated precession. Particles occurring in this interval suffer a considerable decrease in amplitude and an excessive energy gain when slipping and even if all the
incoherent particles in the internal circulating beam initially have the same radial amplitude, it is impossible to avoid a range of amplitudes (almost down to zero), when the particles finally regenerate. This gives rise to an increased energy spread in the beam. The motion of \( P \), for some representative particles \( A, B, D \), and \( E \) has already been shown in fig. 3 and the same particles are marked in fig. 5. Note the strongly pronounced knee motion for particle \( E \), which starts just above the slip threshold, and turns at \( \theta = \theta_1 \). Consequently for small amplitudes particles closer to the threshold can be captured although \( \theta_1 \) has passed \( \theta \). Particle \( A \), first appearing in the critical interval, slips and is later on extracted straightly. The curve defining the lower limit of this interval is computed for particles having a slip of \( 2.8^\circ \) per turn when \( P \) passes \( \theta_1 \).

An accelerating voltage of 20 kV gives an average increase in \( b \) of about 0.35 cm per betatron cycle. A particle accelerated with a lower voltage has a slower "motion" through the slip region towards the extraction region (above the threshold) which can be found from fig. 5. This effect increases the probability of getting the particle into the critical interval, which gives rise to an uncontrolled energy spread during the extraction process. Hence, large phase oscillations are disadvantageous.

### Energy compression

In order to understand the factors causing the energy compression during the extraction we divide the whole regeneration process in two parts. The first includes the particle motion to the final regeneration state, and the second the final regeneration (knee or straight). During the first part the energy is compressed due to the fact that particles with larger amplitudes make more precession cycles, which can be found from the position of the threshold level in fig. 5. For example two amplitudes 0.2 and 1.0 cm, initially differing 0.8 cm in \( r_0 \), can differ from 0.35 to 0.65 in \( r_0 \) when the knee region is reached. During the second part of the regeneration state energy compression occurs for particles having the same amplitude (or lying within a narrow band of \( r_0 \)) while a certain de-compression takes place. If a large range of amplitudes are present, this can be found from eq. (4). If \( \rho_0 = \text{const} \), \( \rho_m \) increases with \( \theta_0 \), i.e., with \( r_0 \), and thus particles with large energy escape after fewer turns. If \( \theta_0 = \text{const} \), \( \rho_0 \sim \rho_m \) (approximately) and thus particles with larger \( r_m \) escape faster. But since \( \rho_m \sim \frac{1}{r_m} \), we conclude that the energy difference between particles with small and large \( \rho_0 \) has increased when escape occurs. This de-compression, however, is smaller than the compression during the first part of the regeneration and hence there is in general a net energy compression which is larger the smaller is the range \( r_m \) when the final regeneration starts.

### Numerical results

In the program set up in order to study the particle motion, accelerated particles are followed from the instant the maximum radial oscillation reaches \( r = r_0 \). Due to the incoherent radial amplitudes, this can occur at any azimuth \( \theta_0 \). The acceleration and the motion in electric phase space are included. The electric phase at start, the synchronous phase and the phase oscillation amplitude, as well as the dee voltage, are parameters of the program. Thus one has complete freedom in choosing the initial conditions. The magnetic fields are approximated by analytical functions. The fringing field in the median plane is described by a hyperbola and the regenerator field by a parabola given by LeCouturier's parameters \( T = 0.2 \) and \( V = 0.08 \). The azimuthal distribution of the regenerator field is rectangular ("hard edge") and has a length of \( 12^\circ \) (21 cm). In fig. 6, \( \rho_0 \), as a function of the number of turns, \( N \), is shown for 6 particles with representative initial amplitudes, all particles starting with \( P \) at the same azimuth. A maximum energy gain per turn of 40 keV and an electric phase at start of 50° yield the equilibrium orbit expansion \( dr \), which is slightly curved due to regeneration. Particle \( B \) starts the final regeneration when \( N = 10 \). Because \( B \) starts at the same azimuth as \( A \), they are all influenced by the regenerator when \( N = 10 \), at which time \( B - F \) are well below the threshold. Therefore they make another precession cycle, after which they reenter at \( N = 40 \) and experience a stronger regeneration action than previously because of the increase in \( r_0 \) due to the acceleration. Now \( B \) regenerates while \( F \) just below the threshold, slips over the regenerator for a new betatron cycle. Therefore the precession is slow at the passage, and due to this, particle \( F \) finally regenerates with a comparatively greater energy increase than the other particles, which can be seen from the table in fig. 6. Since the initial energy difference of 2.33 MeV is reduced to 1.60 MeV for the particles \( A - F \). In fig. 7, the corresponding \( \rho_0 \) plots are shown. Particle \( A \), having an extraordinary knee motion, spends about 40 turns in the final regeneration state. Particle \( B \) regenerates straight in around 15 turns. The other particles exhibit knees of different magnitudes. For all of them \( \theta_0 \) approaches \( \theta_1 \) asymptotically.

Due to the incoherent radial oscillations and due to different acceleration histories, particles which initially had the same amplitude can start the final regeneration with a range of \( r_0 \)-values. If \( r_0 \) is the value of \( r_0 \) at this moment, the number of turns \( N_0 \) needed for a particle to escape depends on \( r_0 \) as explained earlier. A small \( r_0 \) was found to correspond to a large \( N_0 \), depending on the knee motion. Hence it is expected that the energy spread due to different entrance values of \( r_0 \) can be made small by a proper choice of the accelerating voltage. Large phase oscillations, yielding a large range of...
The quantitative behaviour in the final regeneration state is best seen from the $N(r_0)$ diagram in fig. 8. The diagram has been constructed using the data collected for a number of incoherent particles traced through the extraction system; $\delta r$ has been chosen to be 0.01 cm. The parameter $\rho_m$ is the value of $\rho$ at $r_0 = r_1$. A particle with $\rho_0$ corresponding to $\rho_m$ in the $\rho_1$ plane is small regenerates with the centre of curvature making a knee motion in the $\rho_\theta$-plane. Particles starting the regeneration in this region need a relatively small $\delta r_0$ to homogenize the final energies. The arbitrary choice of $\delta r_0$ does not appreciably change the $N(r_0)$ plots, because the particle trajectories are not very sensitive to the acceleration voltage, when the regeneration is once initiated. On the other hand, the final energies are affected greatly by this choice. Particles occurring on the portion of the curves, where $dr/dN$ is large, regenerate straight. Particles entering with these large $r_1$ need between 15-20 turns to regenerate. In order to get a homogenous energy, 56 kV dee voltage will be needed in this region. This is too high a voltage for a synchrocyclotron. We conclude, therefore, that the useful energy homogenizing occurs mainly because of the knee motion. Particles making knee motions need relatively small voltages to compensate for the energy spread due to the different entry values of $r_0$. The $N(r_0)$ plots are not shown above $N_0 = 36$, at which number the $r_0$ values are approximately 0.01 cm larger than those of the asymptotes which are found from the threshold equipotential line in fig. 5.

The median plane motion program does not account for the good energy selection of the extraction system in the Uppsala machine. A channel 1 cm wide will transmit the particles $E$, $C$, $E$, $F$ as is shown in the emittance diagram in fig. 9, and from the table in fig. 6 the total energy spread is found to be 1.6 MeV, which is more than the corresponding value of the measured 0.2 MeV FWHM. In order to explain the good energy selection the vertical motion must certainly be included. Also recalling the fact that the external proton energy is close to the highest possible determined by the $B-r$ product, we conclude that only particles with small radial amplitudes are extracted.

Choice of the internal circulating beam parameters

The choice of the proper ranges of radial amplitudes and dee voltages yielding a large duty cycle and a good energy homogeneity is based on the following considerations. Three radial amplitudes 0.2, 0.4 and 0.6 cm are examined. Smaller amplitudes than 0.2 cm are not likely to occur and a larger range of amplitudes will not be sufficiently compressed. A sample of particles with the chosen amplitudes and with uniformly distributed values of $r_0$ are shown in fig. 10. The $r_0$ values below the $r_1$ line are initially at $\theta = \theta_0$ one betatron cycle prior to final regeneration. Three accelerating voltages are examined and for all possible combinations of amplitude and voltage the critical intervals are marked (dashed regions). Here $r = r_0$ denotes the threshold and $r = r_1$ the value which gives a slip of 2°/turn, when $\theta = \theta_0$. This means a damping of the amplitude by 5°/-cycle to be a tolerable limit. The particles occurring inside the critical interval regenerate with final energies which are impossible to control. The critical intervals increase for larger radial amplitudes, and smaller voltages. Each particle is represented by a three character index of which the first denotes the initial radial amplitude, the second gives the distribution of equilibrium orbits, and the third index is related to the voltage. After one betatron cycle the particles appear above $r_0$ with $r = r_1$. The $r_1$ values are calculated by adding the equilibrium orbit expansion during the last betatron cycle to the $r_0$ values below the critical interval. The voltage has to be kept sufficiently low in order to avoid a large increase in $r_1$, i.e., to prevent the particles from reaching regions in the $N_1(r_1)$ diagram where $dr/dN$ is large (straight extraction). On the other hand, the voltage must be high enough for as many particles as possible to avoid the critical interval. Furthermore, a large range of voltages is required to yield a good duty cycle. This, however, is in contradiction with the need for a constant voltage as $\delta r_0$ should be equal to $dr/dN$ to compensate for the different values of $r_0$. The dee voltages 14.4, 10.8 and 7.2 kV have been chosen with the mentioned facts in mind. They correspond to $\delta r_0$ equal to 0.01, 0.0075 and 0.005 cm, respectively. It is now possible to calculate the final energies as the remaining turns $N_1$ are known from fig. 8. Since the values of $\rho_m$ in the figure are close to the amplitudes being investigated, the curves are quite representative. The calculated final energies are shown in fig. 11. As expected there is an energy spread due to the radial amplitudes. The large voltage values appear to be less useful than the small values. For a certain amplitude there is a correlation between final energy and voltage. This can be seen as a duty cycle limitation, which can be partly overcome if, for example, the two amplitudes 0.4 and 0.6 cm are chosen and if the largest voltage value is excluded. This yields a gross extraction efficiency of 40 % for an energy spread of 300 keV. A duty cycle of 21 % is given by the useful voltage range 7.2 - 10.8 kV. In this investigation the loss to the magnetic channel has been neglected. This loss should, however, not be more than 50 %.

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Fig. 1. Lay out of the extraction system for the Uppsala Synchrocyclotron.

Fig. 2. Motion of centre of curvatures (P) due to precession (F1) and regenerator action (F2) during one revolution for the case $\theta_m < \theta_p$. P' is the new position of the orbit centre.
Fig. 3. $\rho_m(\theta_m)$ plots in the final regeneration state.

Fig. 4. Curves showing relative increase in amplitude due to regenerator action as a function of $\theta_d$ with $\rho_m$ as parameter.

Fig. 5. $\theta_d(\rho_m)$-plots with $b$ as parameter when regenerator action starts ($x_r = 0$). Solid curves indicate the relations between $b$, $\theta_d$ and $\rho_m$. Final regeneration occurs above the critical interval.
Fig. 6. \( p_m(N) \) plots. \( E_i = \) initial energy. \( E_f = \) final energy.

Fig. 8. \( N_1(r_{01}) \) with \( p_{ml} \) as parameter.

Fig. 9. a) Radial emittance for particles A - F from Fig. 7.

Fig. 7. \( p_m(\theta_m) \) plots in the final regeneration state for the particles in Fig. 6.
DISCUSSION

HAGEOORN: Are you concerned about improvements in the central region of the cyclotron to improve the beam quality?

SVANHEDEN: Yes, certainly. Wednesday I will present a paper about our plans for conversion of our cyclotron. We plan to use a calutron ion source, to program all the acceleration process with puller slits, and to combine sector focusing and very small rf band width. If you modulate the rf so as to keep the phase constant during the whole acceleration process, you may end up with small radial as well as small phase oscillations. That should provide for high extraction efficiency from such machines.

Fig. 10. Level diagram for equilibrium orbits for three different amplitudes and accelerating voltages.

Fig. 11. The final energy distribution for the particles arriving at the magnetic channel.