STUDY ON SPACE CHARGE EFFECT IN THE SPIRAL INFLECTOR

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Abstract
This paper analyzes space charge effect of the beam in spiral inflector in order to increase the injection efficiency of the compact cyclotron. In the process, we used an ideal cylindrical beam which was approximately the same as the real beam for the calculation of the charged particle dynamics. The beam has azimuthal symmetry and infinite extent in the beam direction. According to the distribution difference of beam density, two modes were used to study the space charge effect, uniform distribution and gauss distribution. As a result, we developed a modified version of CASINO to calculate the space charge effect in the spiral inflector. For double check, a modification of TRACE3D to include the spiral inflector for the injection matching was implemented. The results comparison will be given in this paper too.

INTRODUCTION
In the field of high intensity accelerators the compact cyclotron has drawn an increasing attention due to its extensive and important application in recent years. Spiral inflector is a key part for this kind of machine, but the in-depth study on the space charge effect of the spiral inflector has rarely been carried out so far.

The matrix-based TRACE3D and TRANSOPTR can be easily applied to optics matching of the injection line, but compared to the software based on trajectory tracing, this kind of software is not sufficient to study in detail the beam optical quality of the spiral inflector that is based on numerical analysis including fringe field. However, the currently available software for spiral inflector design using trajectory tracing method could not meet the requirement to calculate the space charge force, including the software of CASINO of TRIUMF. As a result, the existing method is not enough to study the spiral quality in the condition of high beam injection.

Given this limit, we have tried some tentative work of improving on the CASINO to calculate the space charge effect, thus introducing software CASINO_SCE. In fact, Dragan had done some tentative work to solve the space charge effect problem of the spiral inflector in the RIKEN laboratory of Japanese. In his study, he put forward a simplified model of a “straight” cylindrical beam [1][2]. We developed a method to study the space charge effect with uniform distribution and gauss distribution respectively, and we noticed the difference of the result with Dragan’s.

To explain the difference of the two results and to verify our calculation as well, we apply the matrix of the spiral inflector to TRACE3D. In that case, TRACE3D can be used to match the injection line including spiral inflector, and this mended software was named TRACE3D_INF. Based on this work, we test the space charge effect of the CASINO_SCE excluding fringe field with software TRACE3D_INF and TRANSOPTR. By comparing the results, we can conclude that the result of the CASINO_INF is reasonable.

THEORETICAL ANALYSIS OF THE SPACE CHARGE FIELD
To describe particle motion in high current beams, we must have the electric and magnetic fields generated by the particles. So in the space charge effect calculation, we use an ideal cylindrical beam approximate to the real beam (Fig.1). The cylindrical beam can be considered as uniform or gauss distribution, at radius r, beam density \( \rho \), beam intensity \( I_0 \), and the maximum beam radius is \( a \). Ideally the cylindrical beam is axially linear, while strictly speaking, the beam should be spiral due to the function of the spiral inflector.

\[
\begin{align*}
\mathbf{E} &= \frac{\rho}{2\varepsilon_0}r \hat{e}_r, r \leq a \\
\mathbf{B} &= \frac{\mu_0 I_0}{2\pi a^2} \hat{e}_\phi, r \leq a
\end{align*}
\]

where \( I_0 \) is the maximum beam current, \( \varepsilon_0 \) is permittivity of vacuum, \( \mu_0 \) is permeability of vacuum, \( \hat{e}_r \) is the unit vector in radial direction and \( \hat{e}_\phi \) is unit vector in azimuthal direction.

Figure 1: Cylindrical beam propagating in the z direction.
**Gauss Distribution**

When the beam is in gauss distribution, according to gauss distribution formula, we can get:

\[
J_\theta \int_0^\infty \left( \frac{1}{\sqrt{2\pi} \sigma} e^{-r^2/2\sigma^2} \right) 2\pi r dr = I_0
\]

From the equation above, we can get the parameter \(J_\theta\):

\[
J_\theta = \frac{I_0}{\sqrt{2\pi} \sigma \left( 1 - e^{-r^2/2\sigma^2} \right)}
\]

So we can get the beam current at radius \(r\):

\[
I = J_\theta \sqrt{2\pi} \sigma \left( 1 - e^{-r^2/2\sigma^2} \right)
\]

Thus we can get the self-fields equations:

\[
E = \frac{I}{2\pi \epsilon_0 v_0} \hat{\epsilon}_r, r \leq a
\]

\[
B = \frac{\mu_0 I}{2\pi r} \hat{\epsilon}_\phi, r \leq a
\]

**NUMERICAL SIMULATION**

Based on the formulae above we modified the program CASINO [3] to calculate the space charge of the spiral inflector by tracing a single particle. In our calculation with CASINO SCE, we have taken into consideration the spiral inflector which is likely to be used in the 100MeV high intensity cyclotron being constructed at CIAE (CYCIAE-100). For the parameters of the spiral inflector, \(A=4.0\)cm, \(R_m=3.9695\)cm, \(k'=0.74\). The electric field distribution in the spiral inflector is an ANALYTIC field, the magnetic field comes from CYCIAE-100, and the injection energy is 40keV. In order to compare the results between uniform and gauss distribution, we choose the full width at half-maximum (FWHM) of gauss distribution as the maximum width of the gauss distribution, the integral area of uniform distribution is equal to the integral area of gauss distribution. In this case, the beam radius of the uniform distribution is 2mm, while the beam radius of the gauss distribution is 3mm, and \(\sigma=4.5\)mm. From these initial conditions, we can get the space charge effect on the particle for the two distributions as is shown in Fig.2, in which the abscissa represents RF time, and the ordinate represents the displacement in the ur and hr directions. Three different values at different beam intensity through the spiral inflector have been chosen in the calculation: \(I=0\)mA, 5mA and 10mA.

It can be seen from the figures that whether for gauss distribution or uniform distribution, the space charge effect is stronger in the hr direction than in the ur direction.

To make it clear, in order to get the difference of the space charge effect between gauss distribution and uniform distribution, we select the beam current \(I=0\)mA and 10mA to calculate the two case. Based on the results of Fig.2, we can get the comparison result shown in Fig.3, from which we can conclude that the space charge effect of uniform distribution is stronger than that of the gauss distribution.

**RESULT COMPARISON WITH DRAGAN’S**

Dragan had done some tentative work to solve the space charge effect of the spiral inflector in the RIKEN laboratory of Japanese. In his study, the injection ion is \(N_{i4}^{+}\) of which the charge-to-mass ratio is 0.375, and the injection energy is 50keV. The parameters for the selected spiral inflector are: \(A=2.2\)cm , \(R_m=1.6\)cm , \(k'=0.3\) .

The electric filed in the spiral inflector is computed from an electric map produced by RELAX3D, and the magnetic field is uniform field with a magnitude of 1.5T. The injector entrance emittances are \(\epsilon_u = 140\)\(\pi\)mm.mrad and \(\epsilon_h = 130\)\(\pi\)mm.mrad [2][4].

In his calculation, when the beam current varies from 10\(\mu\)A to 40\(\mu\)A, the maximum displacement in the ur is about \(\Delta u_r\) =0.3mm and the maximum displacement in the hr is about \(\Delta h_r\) =4.8mm.

According to Dragan’s inputting parameters, we do the same calculation using our program, and we find there is a difference between the results, as is shown in Table 1.

Table 1: the result comparison of space charge effect

<table>
<thead>
<tr>
<th></th>
<th>Dragan</th>
<th>Our calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta h_r) (mm)</td>
<td>~0.48</td>
<td>0.00695</td>
</tr>
<tr>
<td>(\Delta u_r) (mm)</td>
<td>~0.3</td>
<td>0.00155</td>
</tr>
</tbody>
</table>

**AMENDMENT OF TRACE3D & VERIFICATION OF SPACE CHARGE EFFECT RESULT**

In order to verify the computation of the space charge effect, the optical matching program TRACE3D and
The beam intensity changes 32π and \(A\) and the beam intensity changes from 1mA to 10mA. Figure 4 show the result of the space charge effect of CASINO_SCE, TRACE3D_INF and TRANSOPTR. In these figures, the abscissa is the beam current I with unit mA, and the ordinate is the offset between the cases with and without space charge effect at the exit, the unit is cm. From the figures we can see that the result of the CASINO_SCE, TRACE3D_INF and TRANSOPTR is accordant with each other, and the space charge effect on the h direction is bigger than the u direction. However, there are still some differences in the three results, because the calculation of CASINO_SCE uses numerical tracing, while TRACE3D_INF and TRANSOPTR use matrix resolution. But there are not any quantity differences among them. The difference also exists in numerical value between TRACE3D_INF and TRANSOPTR, because TRACE3D_INF uses pure analytic matrix, while TRANSOPTR uses Runge-Kutta routine integrate matrix.

![Figure 4: comparison of space charge effect for CASINO_SCE, TRACE3D and TRANSOPTR in h direction (left) and u direction (right)](image)

### CONCLUSION

Through the calculation, we can get the following conclusion:

The higher the beam intensity is, the stronger the space charge effect is. The space charge effect is stronger in the \(h\) direction than in the \(u\) direction.

In general, the space charge effect of uniform distribution is stronger than the effect of the gauss distribution.

### REFERENCES


