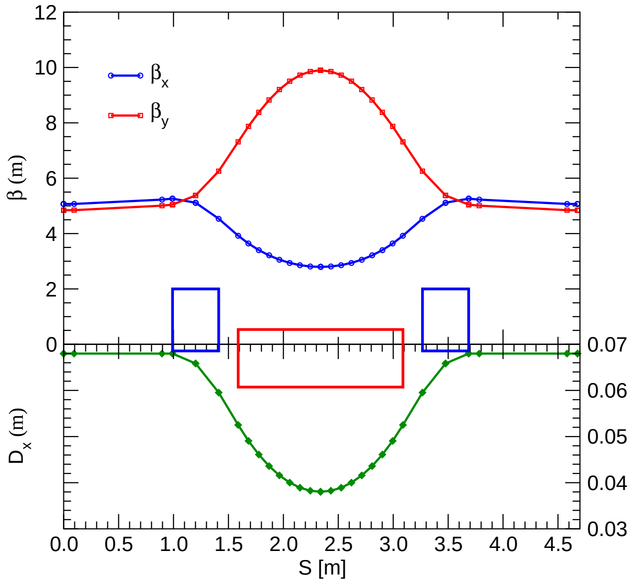

Figure 1: Beta (top) and dispersion (bottom) functions at the reference energy in the eRHIC lattice. Magnet positions are shown in the center.

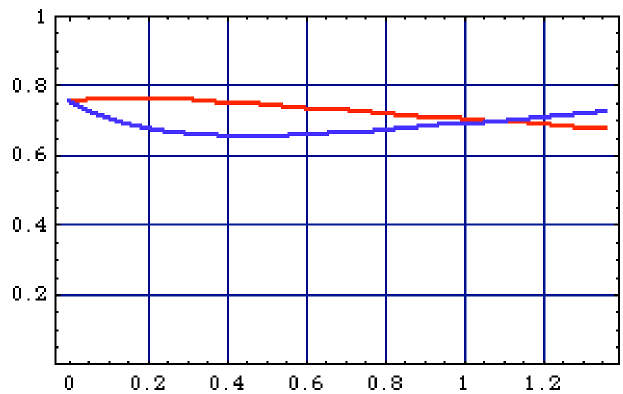

Figure 2: Fractional tunes as a function of energy in the 1.5 GeV nonlinear FFAG.

line scenario is to have a superconducting linac replace the booster [7], but an FFAG may be a much more cost-effective alternative.

A Nonlinear Non-scaling FFAG Design

The design of an FFAG for the AGS upgrade presented unique problems. Since protons or ions are being accelerated, the acceleration did not need to be extremely rapid: only small compared to the desired repetition period of the AGS (400 ms). For slower acceleration, a scaling FFAG is often desirable since its tunes are constant, and therefore resonances can be avoided. However, preliminary attempts to design a scaling FFAG for the booster replacement required fields that were higher than 0.3 T, which would preclude operation with H^- ions. Thus, a non-scaling design with a small tune variation was needed.

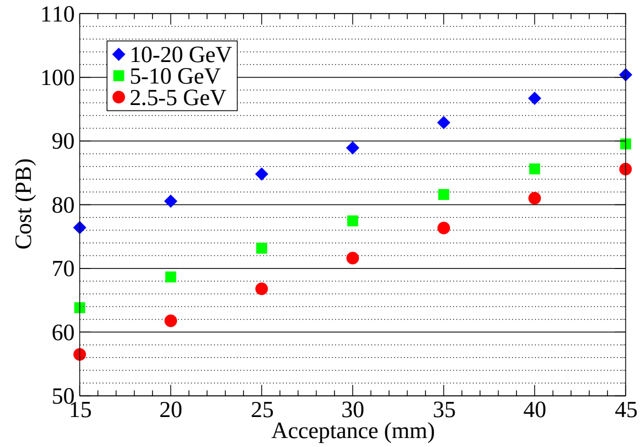

Figure 3: Dependence of muon FFAG cost on transverse lattice acceptance for various energies.

In [8], Ruggiero describes a method for achieving this: constructing a magnet such that on the closed orbit the gradient is proportional to the momentum. This not only requires a nonlinear transverse field profile, but a field profile that varies along the length of the magnet. While this method does not make the horizontal tune perfectly constant (and with end effects neither tune is perfectly constant), the deviations are small and decrease as the number of cells increases. Based on this, a 1.5 GeV FFAG was designed which could replace the booster [9]. It has 136 FDF triplet cells for a total circumference of just over 800 m. The fractional tunes as a function of energy are shown in Fig. 2, showing the high degree to which the tunes are constant (the integral parts of the tunes are 39 and 37). Furthermore, full-turn resonances should be relatively weak due to the fact that every cell is identical.

OPTIMIZING DESIGNS

Since FFAGs consist of a single simple cell repeated around the ring, they are in principle very straightforward to design. One can even use automated techniques to design a minimum-cost lattice, according to some cost function. In [10], a cost model was used to do such designs for muon acceleration with linear non-scaling FFAGs. Several things (applicable to muon machines with high-frequency (200 MHz) RF), and in many cases beyond that) were learned from performing these cost optimizations:

- A doublet is a more cost-effective lattice cell than either a triplet or a FODO. While the triplet requires less RF voltage than the doublet, having three magnets per cell instead of two makes it more costly.
- For modest and even somewhat large sizes, a longer ring was less expensive than a shorter one, even ignoring RF costs. This is because the dispersion gives a significant contribution to the aperture in FFAGs, and the dispersion decreases as the number of cells increases in the ring. In many cases, the decrease in

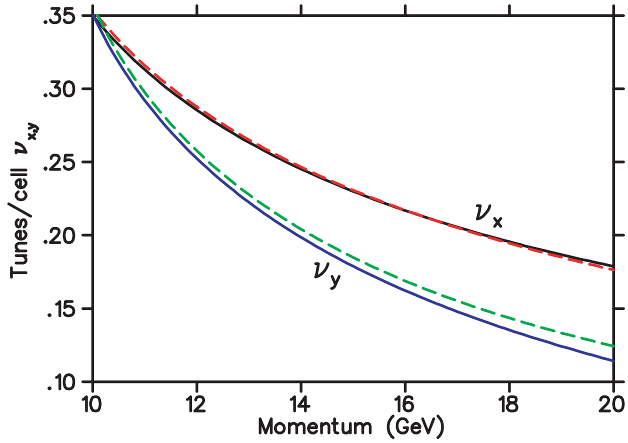

Figure 4: Tune as a function of energy in a triplet lattice, showing the ”exact” solutions computed with COSY [11] (dashed) and using the analytic approximation (solid).

aperture reduces the cost more quickly than the cost is increasing from the increased number of cells. Thus, the least expensive ring sometimes has unacceptable levels of decay; thus, the cost of the decay is incorporated into the optimization.

- Due to constraints of fitting the beam into the pipe and other tradeoffs, the cost optimum lattices have specific tune profiles which depend only on the type of lattice and the ratio of the initial and final energy. The tunes are split significantly over the entire energy range.
- The cost per GeV for a low energy FFAG is significantly higher than for a higher energy FFAG. Thus, for accelerating muons using high-frequency RF, FFAGs are unlikely to be useful in low-energy stages (below about 5 GeV).

The dependence of cost (and other merit factors) on various input parameters is easily obtained by these techniques (see Fig. 3 for an example).

ANALYTIC MODELS

Optimization techniques, especially if one wants reasonable accuracy over a large energy range, can be rather slow, requiring repeated tracking for finding the solution for the closed orbit and the linear map about it. For linear non-scaling FFAGs, one can make analytic approximations to the orbits in the magnets, and use that to find closed orbits and linear lattice functions [12]. This can then lead to much more rapid design and optimization of these lattices. An example of the approximation's accuracy is shown in Fig. 4. The accuracy is excellent except for very compact rings, since exact analytic formulas don't fully take into account the reference orbit curvature.

Simpler models using thin-lens approximations have also been used to compute lattice properties and design lattices for linear non-scaling FFAGs [13, 14]. While these methods are not as accurate, they have had some success in being used to produce lattice designs [14].

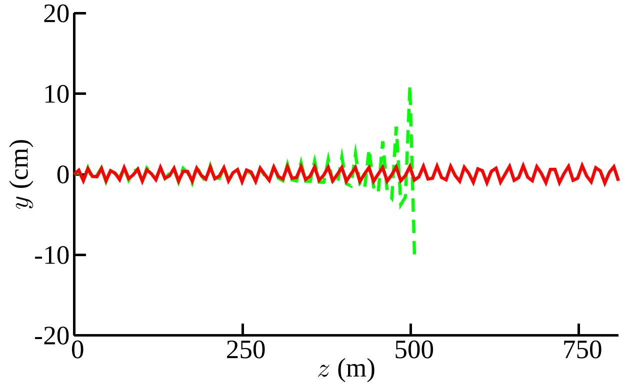

Figure 5: Tracking a large amplitude particle with uniform acceleration in a 5–10 GeV FFAG. The dashed line shows tracking with sextupole components on the ends of the magnet and no correction in the body, and the solid line shows the same thing with a body sextupole component to correct the particle loss.

TRACKING RESULTS

Linear non-scaling FFAGs are a relatively new invention, and until recently, very little tracking has been done on them. One must carefully consider the nonlinear effects in these machines to find the dynamic aperture. The magnets are relatively short compared to their apertures, and thus end effects in the magnets become very important. So whatever tracking code is used must properly include these effects. COSY Infinity [11] has excellent built-in handling of magnet ends, but its use of truncated power series can at times (but not always) be problematic for the large energy acceptances required for FFAGs [12, 15]. ICOOL [16] has extensive facilities for handling end fields, as does ZGOUBI [17], and both of these codes have been used to do tracking for FFAGs [18, 19].

In particular, a linear non-scaling FFAG was examined using ICOOL, including sextupole contributions expected on the ends of the magnets [18]. When the sextupole ends were added, it was found that there was significant particle loss at a particular energy, apparently due to a 1/3 resonance. This loss could be corrected, as shown in Fig. 5, by adding a sextupole component to the body of the magnet. The integrated sextupole required to eliminate the loss was determined by tracking to be only 0.68 times the integrated sextupole strengths of the ends.

Tracking using ICOOL has also been done on the PRISM lattice [20]. It was demonstrated that there is a large difference between the horizontal dynamic aperture when there is no vertical amplitude and a small vertical amplitude. Furthermore, if the scaling PRISM design was replaced with a non-scaling design, there was a substantial increase in the dynamic aperture [21], as shown in Fig. 6.

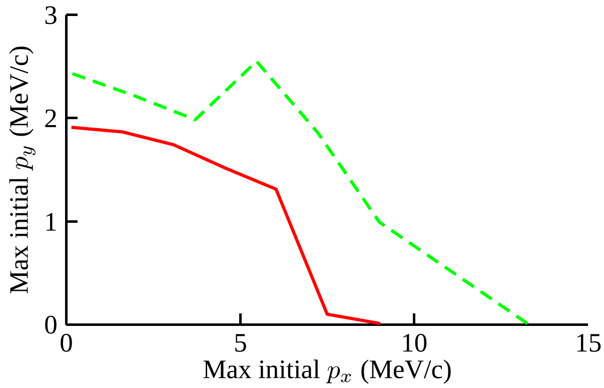

Figure 6: Dynamic aperture for PRISM using a scaling lattice at 68 MeV/c (solid) and linear non-scaling lattice at 80 MeV/c (dashed).

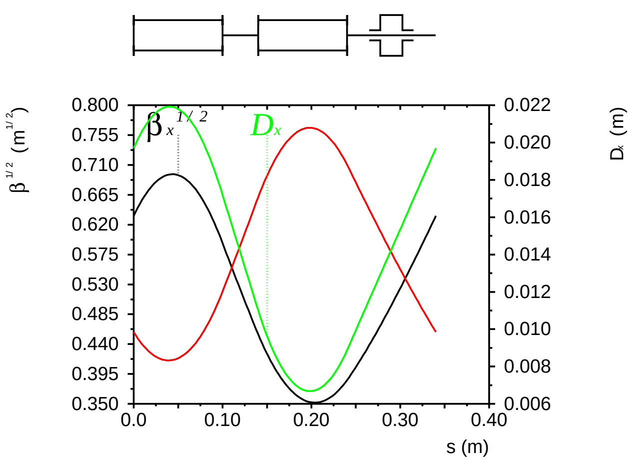

Figure 7: Lattice functions at the central energy and layout for an electron non-scaling FFAG demonstration ring.

ELECTRON MODEL

While several scaling FFAGs have been built [23, 24, 25, 26], a non-scaling FFAG has never been built. There is great interest in building a model of a non-scaling FFAG that accelerates electrons, to demonstrate both our understanding of the transverse dynamics in such a lattice as well as the unique longitudinal acceleration mode that will probably be used for muon acceleration with high-frequency RF [22].

Several authors have produced parameter sets for electron models [27, 28, 29, 14, 30, 31]. Two [28, 31] have been more extensively analyzed, including hardware considerations. Figure 7 shows a cell from one of the lattices; that lattice consists of 45 cells, and accelerates electrons from 10–20 MeV.

CONCLUSION

Extensive progress has been made recently in the design of non-scaling FFAGs. Understanding of how to optimally design linear non-scaling FFAG lattices for various applications has increased, and we are beginning to perform more detailed nonlinear analyses of these lattices. The idea of non-scaling lattices is even being extended beyond the linear non-scaling lattices. Finally, we are considering the idea of building a low-cost model of a linear non-scaling FFAG to demonstrate our understanding of these machines.

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