MODIFICATION OF A DOUBLE DRIFT BEAM BUNCHING SYSTEM TO GET THE EFFICIENCY OF A SIX HARMONIC BUNCHER

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Abstract
We have proposed a double drift bunching scheme where one can obtain a bunching effect equal to that of a six-harmonic buncher by using only two harmonics mixed in optimum proportions. In this case one has to apply voltages with frequencies $\omega$ and $2\omega$ (instead of only $\omega$) on the first buncher and $2\omega$ on the second buncher. The optimization of buncher parameters such as the voltage amplitudes of the various harmonics and the separation between the two bunchers has been carried out using linear programming technique.

1 INTRODUCTION
In a cyclotron using an external ion source only a small fraction of the injected continuous beam is accepted in the central region for further acceleration. By transforming the dc beam into a suitably bunched beam using a buncher prior to injection, the amount of accepted particles could be increased. A sawtooth voltage waveform is ideal for a buncher, however, it is very difficult to generate it at the required frequency and power level. Therefore, bunchers are fed with either a sinusoidal waveform or a nearly sawtooth-like waveform obtained by combining the fundamental wave with its various higher harmonics. Single-gap bunchers using a sinusoidal waveform are the most frequently used bunchers. They typically bunch about 50% of the original dc beam in a phase width of 30° of rf as required by the cyclotron. Harmonic bunchers [1] capture as much as 60-80%, depending upon the number of harmonics used. Double drift bunchers (DDB) [2,3] give a bunching efficiency better than what is obtainable with three harmonics on a single gap.

At the Variable Energy Cyclotron in Calcutta, a $K=500$ superconducting cyclotron is in an advanced stage of construction. The axial injection system will require an efficient buncher. While studying the design of the buncher to be used in the axial injection system, we have explored the possibility of increasing the bunching efficiency further by mixing the harmonics in the two gaps of a DDB. We have found that by applying voltages with frequencies $\omega$ and $2\omega$ (instead of only $\omega$) on the first buncher and $2\omega$ on the second buncher of a DDB, it is possible to obtain a bunching efficiency equal to that of a harmonic buncher containing six harmonics on a single gap. Maximum obtainable bunching efficiency has been calculated for various bunch widths and results have been compared with those of the other bunchers.

2 METHOD OF OPTIMIZATION
In a buncher, an axial RF electric field is produced which modulates the beam energy in time. This energy modulation gives rise to a density modulation after a drift space. The buncher parameters and the drift distance (time focus) are adjusted to get the maximum number of particles in a given bunch width.

\[
\begin{align*}
\omega & \quad 2\omega & \quad 3\omega & \ldots \\
\hline
\end{align*}
\]

Multi-harmonic buncher

\[
\begin{align*}
\omega & \quad 2\omega & \quad L - \ell \\
\hline
\end{align*}
\]

Conventional DDB

\[
\begin{align*}
\omega & \quad 2\omega & \quad 2\omega & \quad L - \ell \\
\hline
\end{align*}
\]

Improved DDB

Fig. 1. Schemes of various bunching systems

In a harmonic buncher the energy modulation is done by using a sawtooth like voltage waveform obtained by superposition of fundamental frequency $\omega$ with its various higher harmonics $2\omega, 3\omega, 4\omega$ etc. on a single gap. If $V = \sum V_n \sin(n\omega t)$ be the voltage applied on the buncher to produce bunching at a distance $L$ from the gap (Fig. 1.), then the phase of an arbitrary ion with respect to that of an unmodulated ion at the time focus is given by [4],

\[
\theta = \phi - \sum \mu_n \sin(n\phi)
\]
where,
\[ \mu_n = \omega L \left( \frac{q V_0}{m} \right)^{1/2} \left[ T(d) \frac{V_n}{V_0} \right] \] (2)

\( T(d) \) is the well known term due to the transit time effect for a gap of width \( d \) and \( q V_0 \) is the initial energy of the ion of mass \( m \) and charge \( q \). \( \phi \) is the phase of the rf voltage of frequency \( \omega \) when the ion crosses the gap.

A cyclotron has a fixed phase acceptance \( 2\theta_m \) within which ions are captured for further acceleration and depends largely on the central region geometry. The amplitude and shape of the voltage waveform applied to the buncher are adjusted in such a way that the largest possible number of ions arrive in this phase acceptance. Thus the problem reduces to finding out the maximum value of \( \phi (=\phi_m) \) for a given value of \( \theta_m \) in Eq.(1). Since Eq.(1) is linear in terms of \( \mu_n \), linear programming techniques can be used to minimize \( \theta_m \) for a given maximum value of \( \phi_m \). This is equivalent to maximizing \( \phi_m \) for a given value of \( \theta_m \). The linear constraints are that for each value of \( \phi \), up to \( \phi_m \), the value of \( \theta \) should be less than \( \theta_m \). In the computations it suffices to use the constraint only at a finite number of values of \( \phi \) so that number of constraints is finite and is workable. We have used the standard simplex method to minimize for a given number of constraints. Our experience with computations shows that taking \( \phi \) values at 1° interval yield satisfactory results. While using the simplex algorithm care should be taken to see that correct signs of various harmonics are used. It has been found that all the odd harmonics are positive while the even ones are negative in sign. This is expected, as a sawtooth wave also shows a similar pattern.

A double drift bunching system consists of two bunchers that are separated in space and are driven independently and phase locked together. The second buncher is driven at twice the frequency of the first. Such a bunching system has been discussed by Goldstein and Laisne [2] and its detailed numerical calculations have been performed by Milner [3]. This system gives a bunching efficiency better than what is obtainable by using three harmonics on a single buncher. Let us assume that the two bunchers are separated by a drift distance \( l \) and the distance of time focus from the first buncher is \( L \) (Fig. 1). In this case the final phase \( \theta \) of the ion at the time focus with respect to the unmodulated ion is given by [6]

\[ \theta = \phi - \sum_n \mu_{1n} \sin(n\phi) - (1-b) \sum_p \mu_{2p} \]
\[ \times \sin \left[ p \left( \phi - b \sum_n \mu_{1n} \sin(n\phi) \right) \right] \] (3)

with \( b=l/L \). Here \( \mu_{1n} \) is the term similar to that defined in Eq.(2) for the first buncher and \( \mu_{2p} \) is that for the second buncher and \( n \) and \( p \) are the corresponding harmonic numbers. For a conventional DDB system one has \( n=1 \) and \( p=2 \). In the proposed DDB we take \( n=1,2 \) and \( p=2 \). We see that Eq.(4) is not linear in \( \mu_{1n} \)'s and hence linear programming method can not be used here directly. We have used the simplex algorithm in this problem iteratively by using initial guess values of \( \mu_{1n} \)'s. After each iteration the initial guesses are replaced by the calculated ones.

The bunching efficiency defined as the ratio of the number of particles within the bunch width to the total number of particles in an rf cycle, is given by

\[ \epsilon = \frac{\phi_m}{\pi} \] (4)

3 RESULTS AND DISCUSSION

Fig.1 shows the new scheme of bunching compared to the conventional DDB (\( \omega,2\omega \)). In the improved DDB (\( \omega,2\omega,2\omega \)) we apply the frequencies \( \omega \) and \( 2\omega \) in the first gap and \( 2\omega \) in the second gap. So one requires to generate only two frequencies and apply them in proper proportions to the two separated gaps.

The improvement in bunching in the proposed DDB is clear from the plot of the final phase \( \theta \) of the particles as a function of the initial phase \( \phi \) as shown in Fig.2. Increase in the value of \( \phi_m \) (which results in an increase in the bunching efficiency) is clearly visible in the figure. We observe that in the improved DDB we get a larger number of oscillations in the final phase than that obtained in the case of conventional DDB. We also note that the amplitudes of the oscillations are the same for all the peaks. When the parameters are not properly optimized these amplitudes come out to be unequal.

![Fig. 2. Variation of final phase θ as a function of initial phase φ optimized for 2θm=12.3°.](image)

Fig.3 compares the calculated bunching efficiency for the improved DDB with those of the conventional DDB and various multi-harmonic bunchers. It is seen that for all bunch widths the efficiency in the improved DDB is comparable to that of a multi-harmonic buncher with six harmonics. We also see that a conventional DDB gives an efficiency greater than that of three harmonic buncher and is comparable to a
four harmonic buncher for larger bunch-widths. The DDB with three harmonics \((\omega, 2\omega - 3\omega)\) [7] gives a bunching efficiency slightly better than a four harmonic buncher for large bunch widths. For smaller bunch widths it is equivalent to a three harmonic buncher.

Fig.3: Comparison of the bunching efficiencies of various bunching systems as a function of bunch widths.

We have also studied a DDB using the frequencies \(\omega\) and \(2\omega\) in both the gaps. This gives a bunching efficiency better than that of the proposed DDB (see Fig. 3). But in this case the optimum voltages required on the second buncher come out to be several times higher than those required in the proposed DDB. Fig. 4 gives the optimized values of the parameters \(\mu_{11}, \mu_{12}, \mu_{21}, \mu_{22}\) and the parameter \(b\) for various bunch widths. As expected all the parameters go on increasing with the bunch width.

A prior knowledge of the buncher parameters is useful in setting up and tuning a bunching system without much experimentation. The method of optimization based on linear programming has helped us in obtaining the required buncher parameters of the proposed DDB. This method can be applied to any type of buncher used in the accelerator laboratories. In our optimization procedure we have not considered the effect of space charge. So the results are applicable to the cases where the beam currents are low.

Fig.4: Optimized values of the buncher parameters as a function of bunch width. The value of the parameter \(b\) in the case of DDB(\(\omega, 2\omega - 3\omega\)) has been taken to be 0.8 for all bunch widths.

4 REFERENCES