THE BUNCH LENGTHENING DUE TO ELECTRON CLOUD IN POSITRON STORAGE RING

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Abstract
The electron cloud generates not only the transverse wake field but also the longitudinal wake field. The mechanism of the bunch lengthening due to longitudinal wake produced by the electron cloud in the positron storage ring is analyzed. The longitudinal field, which depends on the density of the electron cloud in the storage ring, arises from the accumulation of the electrons near to the centre of the bunch during the bunch passage. Based on the longitudinal wake field, tracking method is used to simulate variation of the bunch longitudinal profile in different electron cloud density. According to simulation, the longitudinal action on the bunch from electron cloud is the same as the potential-well distortion to shift the bunch distribution and give a rise to the bunch deformation.

INTRODUCTION
The electron cloud accumulated in the vacuum chamber is usually associated with the transverse coupled bunch instability and bunch blow up. Experimental studies and numerical simulation have been developed for these phenomena [1]. In fact, electron cloud interacts with a positron bunch not only in transverse direction. Longitudinal instability was first discussed by G. Rumolo and F. Zimmermann as a single bunch effect, such as bunch lengthening [2]. The accurate simulation on the longitudinal interaction between electron cloud and the bunch need a fully self-consistent three dimensional plasma particle in cell model to solve Maxwell’s equation as code OSIRIS which will spend much longer time [3]. In this paper, a simplified 2D approach is use to estimate the longitudinal electron cloud wake excited by a bunch passing through the electron cloud. Based on the longitudinal wake field, tracking method is used to simulate variation of the bunch longitudinal profile due to electron cloud.

PHYSICAL MODEL FOR THE LONGITUDINAL WAKE
During the passage of a bunch through the electron cloud, the electrons are attracted by the beam electric field and accumulate around the positron beam. The positron bunches have to lose some mount of their kinetic energy to build the electron cloud during the interaction with the electrons. The energy variation inside the bunch can be seen as a longitudinal wake. The bunch particles have an additional energy spread due to the longitudinal wake from the electron cloud. The electromagnetic field of the electron cloud can drive from the Maxwell equations as following formula [4]

\[ \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \]
\[ \nabla \times \vec{H} = \vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \]
\[ \nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \]

where \( \rho \) and \( \vec{j} \) are the charge density and current density of the electron cloud, respectively. Here, \( \varepsilon_0 \), \( \mu_0 \) are the function of time during the bunch passing. For azimuthally symmetrical fields and the assumption that the electron cloud has a periodic structure, the equations can be expressed as

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r E_r \right) - \frac{\partial E_z}{\partial \tau} = \frac{\rho}{\varepsilon_0} \]
\[ \frac{\partial H_\phi}{\partial \tau} = j_r + \varepsilon_0 \frac{\partial E_z}{\partial \tau} \]
\[ -\frac{\partial E_z}{\partial \tau} = -\mu_0 \frac{\partial H_\phi}{\partial \tau} \]

where \( \tau = z/c \), is the time difference to synchrotron particles. According to the relation \( Z_0 = \frac{1}{\varepsilon_0 c} = 120\pi \Omega \), the integration in the radial direction and the boundary condition \( E_z(r = a, \tau) = 0 \), the longitudinal electric field of the electron cloud is expressed as

\[ E_z = Z_0 \int j_r dr \]

where \( Z_0 \) the impedance in free space and \( j_r \) is transverse current density of electron cloud.

As an example of BEPC II, assuming the bunch current is 9.8 mA and central density of the electron cloud is \( 0.1 \times 10^{13} \text{m}^{-3} \), the bunch longitudinal distribution and its electric field caused by electron cloud is shown in Figure 1.

From Figure 1, the longitudinal electric field is much smaller than the transverse filed which is on the order of \( 10^7 \text{V/m} \) [5]. During the passage of a positron bunch the transverse distribution of the electron cloud also has some significant change as displayed in Figure 2.
During the time of a positron bunch passing through the electron cloud, the electrons in the cloud are moving toward to the centre of the beam orbit and then moving back to the walls of vacuum chamber. So the sign of electric field is different. The head particles of the bunch lose energy but the tail particles get additional energy from the electron cloud.

**BUNCH LENGTHENING CAUSED BY THE LONGITUDINAL WAKE**

The bunch length and energy spread varied due to the longitudinal wake caused by the electron cloud. The longitudinal interaction between bunch and electron cloud depends on the bunch current and density of electron cloud. A normal method to simulate the process of bunch lengthening is to track the motions of many macro-particles presenting the bunch. The motion of macro-particles is described in the longitudinal phase by

\[
\begin{align*}
\mathbf{z}_i(n) &= \mathbf{z}_i(n-1) + \frac{\alpha \epsilon T_0}{E} \mathbf{E}_i(n) \\
\mathbf{E}_i(n) &= \mathbf{E}_i(n-1) - \frac{2T_0}{\tau_e} \mathbf{E}_i(n-1) + \\
&\quad 2\sigma_0 \sqrt{\frac{T_0}{\tau_e}} R_i(n) - U_0 + \\
&\quad V \cos(phi_i + \frac{2\pi h}{C} z_i(n)) + V_i[z_i(n)]
\end{align*}
\]

where \(\mathbf{E}_i(n)\) and \(\mathbf{z}_i(n)\) are the energy and position coordinates of the \(i^{th}\) particle after \(n\) revolutions in the storage ring. \(T_0\) is the revolution period; \(\tau_e\) the damping time; \(U_0\) the energy lost per turn; \(\phi_i\) the synchronous phase; \(h\) the harmonic number; \(C\) the ring circumference; \(E\) the bunch energy; \(\alpha\) the momentum compaction factor; \(\sigma_0\) the natural energy spread; \(R_i\) a random number obtained from a normal distribution with mean 0 and RMS 1. The wake potential \(V_i\) caused by the electron cloud depends on the longitudinal electric field of \(E_z\).

In the simulations \(10^6\) macro-particles are tracked over 6 longitudinal damping times and the bunch length are calculated by averaging particle positions in the last damping time. For BEPCII, positron bunch current 9.8mA, bunch natural length 13.53mm, the longitudinal electric field and the tracking result for bunch length in different electron cloud density are shown in Figure 3 and Figure 4.
According to the simulation results for bunch length in different electron cloud density, the electric field due to electron cloud can lead to the bunch lengthening. The bunch initial Gaussian distribution is shifted slightly in the forward direction for compensating the additional energy loss to the electron cloud. The longitudinal action between electron cloud and bunch can be seen as an electron cloud potential well which cause the possible bunch distortion.

CONCLUSION

The longitudinal electric field due to electron cloud is analyzed and simulated in the paper. The longitudinal effect of electron cloud serves as a potential well to interact the dynamics of bunch particles. By tracking methods to simulate the bunch length in different electron cloud density, results show that the bunch lengthening caused by the electron cloud density $1.0 \times 10^{14}$ m$^{-3}$ is just $2.0\%$ comparing to the natural bunch length. Its effect appears to be negligible for BEPCII.

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