MATCHED OPTICS OF MUON RLA AND NON-SCALING FFAG ARCS*

V.S. Morozov#, S.A. Bogacz, Y.R. Roblin, Jefferson Lab, Newport News, VA, USA
K.B. Beard, Muons, Inc., Batavia, IL, USA
Dejan Trbojevic, BNL, Upton, NY USA

Abstract

Recirculating Linear Accelerators (RLA) are an efficient way of accelerating short-lived muons to multi-GeV energies required for Neutrino Factories and TeV energies required for Muon Colliders. To reduce the number of required return arcs, we employ a Non-Scaling Fixed-Field Alternating-Gradient (NS-FFAG) arc lattice design. We present a complete linear optics design of a muon RLA with two-pass linear NS-FFAG droplet return arcs. The arcs are composed of symmetric cells with each cell designed using combined function magnets with dipole and quadrupole magnetic field components so that the cell is achromatic and has zero initial and final periodic orbit offsets for both passes' energies. Matching to the linac is accomplished by adjusting linac quadrupole strengths so that the linac optics on each pass is matched to the arc optics. We adjust the difference of the path lengths and therefore of the times of flight of the two momenta in each arc to ensure proper synchronization with the linac. We investigate the dynamic aperture and momentum acceptance of the arcs.

INTRODUCTION

A proposed [1] dog-bone-shaped muon RLA consists of a single linac with droplet return arcs. Reusing the same linac for multiple beam passes leads to significant cost savings. In a scheme with separate return arcs, different energy beams coming out of the linac are separated and directed into appropriate droplet-shaped arcs for recirculation. Thus, each pass through the linac requires a separate fixed-energy droplet arc, increasing the complexity of the RLA. In this paper, we present a novel return-arc optics design based on an NS-FFAG [2] lattice, which allows two consecutive passes with very different energies to be transported through the same string of magnets.

OPTICS DESIGN

Droplet Arc Geometry and Optics Requirements

We consider a scheme, in which a 0.9 GeV/c muon beam is injected in the middle of a 0.6 GeV/pass linac. The linac is then traversed by the beam four times. Therefore, one of the return arcs must accommodate 1.2 and 2.4 GeV/c muon momenta, while the other arc must accommodate 1.8 and 3.0 GeV/c momenta. Since the two arcs are designed using the same approach, here we focus our discussion on the 1.2/2.4 GeV/c arc whose design is somewhat more complicated than that of the 1.8/3.0 GeV/c arc due to the greater fractional momentum difference of the two passes.

Each droplet arc consists of a 60° outward bend, a 300° inward bend and another 60° outward bend so that the net bend is 180°. This arc geometry has the advantage that if the outward and inward bends are composed of similar cells, the geometry automatically closes without the need for any additional straight sections, making it simpler and more compact.

To transport different energy muons of both charges through the same arc structure, the arc must possess the following properties:

- For each transported momentum, both the offset and slope of the periodic orbit at the arc's entrance and exit must be zero to ensure that the beam goes through the center of the linac.
- The arc must be achromatic for each momentum to keep the linac dispersion free.
- The arc must be mirror symmetric, so that μ^+ and μ^- can pass through the same lattice in opposite directions. If such a symmetric arc is designed with a periodic solution for the optics, the periodic beta functions are equal at the arc's ends while the periodic alpha functions and the dispersion slope are zero at both ends.
- The times of flight of the two momenta must provide proper synchronization with the linac.
- The dynamic aperture and momentum acceptance must be adequate for a large-emittance muon beam.
- The orbit offsets as well as beta functions and dispersion for both energies should be small enough to keep the aperture size acceptable.

The linear NS-FFAG lattice presented in this paper meets all these requirements.

Linear NS-FFAG Arc Design

We earlier considered [3,4] a non-linear NS-FFAG design for the RLA return arcs. In this paper, we present a new design based on a linear NS-FFAG lattice, which has a number of advantages over the non-linear solution:

- much greater dynamic aperture and momentum acceptance,
- no need for a complicated compensation of non-linear effects,
- simpler adjustment of the path length and the time of flight at each energy,
- easier control of the beta-function and dispersion values, which simplifies matching to the linac,
- simpler combined-function magnet design with only dipole and quadrupole field components.

*Supported in part by US DOE STTR Grant DE-FG02-08ER86351
#morozov@jlab.org

The design concept of the linear NS-FFAG arc lattice is similar to that of the non-linear solution [3,4]. The droplet arc is composed of symmetric NS-FFAG super-cells. Each super cell possesses periodic solutions for the orbit and the Twiss functions at both momenta. The super cells are designed imposing the constraints that the periodic orbit offset, dispersion and their slopes must all be zero at the cell's entrance and exit for both momenta. Since the cells are symmetric and have periodic solutions for the beta functions, the beta functions are equal and have zero slopes (the alpha functions are zero) at the cell's entrance and exit. These properties ensure that two super cells bending in the same or opposite directions are matched to each other at both energies.

To study the optics of the NS-FFAG structure for large momentum offsets, we used the Polymorphic Tracking Code (PTC) module of MAD-X [5]. While perturbative method codes are not suitable for such a study, PTC allows symplectic integration through all elements with user control over the precision with full or expanded Hamiltonian.

An outward bending super cell consists of 12 outward-inward-outward bending triplets, a total of 36 combined function magnets with dipole and quadrupole field components. The magnets are 0.5 m long and are separated by 0.2 m gaps. Each magnet's bending angle is 2.5°. An inward bending super cell is identical to the outward bending cell except that its bends are reversed.

Because of the cell's mirror symmetry, there are 18 quadrupole gradient values available as free parameters in each super cell. When solving for the periodic orbit and the periodic Twiss functions, the initial values of the orbit offset, dispersion, their slopes and the alpha functions were set to zero. The initial values of the beta functions were chosen to provide easy matching to the linac and to keep the peak values of the beta functions inside the cell acceptable. The 18 quadrupole strength parameters were adjusted to attain zero slopes of the orbit offset, dispersion and beta functions at the center of the cell for the two momenta, a total of 8 constraints. The cell's symmetry then ensures the appropriate properties at the cell's exit. Solutions for the periodic orbit, dispersion, and beta functions at 1.2 and 2.4 GeV/c are shown in Figs. 1 and 2 respectively.

The additional free parameters were needed to limit the maximum values of the beta functions and dispersion and to control the path lengths at the two momenta. At these energies muons are not ultra-relativistic; therefore, one has to consider the difference of the speeds at 1.2 and 2.4 GeV/c when calculating the difference of the times of flight at the two momenta. In the solutions shown in Figs. 1 and 2, the path lengths for the whole droplet arc were adjusted to give a time of flight difference equal to exactly one period of 201.25 MHz RF. For the 1.8/3.0 GeV/c arc parameters, it was more straightforward to make the times of flight at the two momenta equal.

Changing the bending direction of the super cell does not affect the beta functions, but reverses the signs of the periodic orbit and dispersion. Since, at the cell's entrance and exit, the periodic orbit and dispersion are zero along with their slopes, any two super cells bending in the same or opposite directions are matched to each other at both energies. Moreover, since the net bend of each super cell is 30°, they can be combined together to form the 60° and 300° bends of the droplet arc.

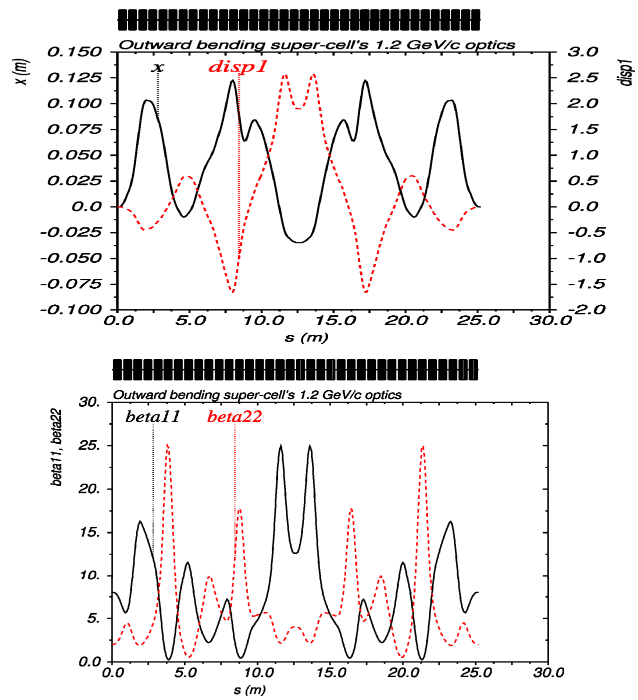

Figure 1: 1.2 GeV/c periodic orbit, dispersion (top) and beta functions (bottom) of the outward bending super cell.

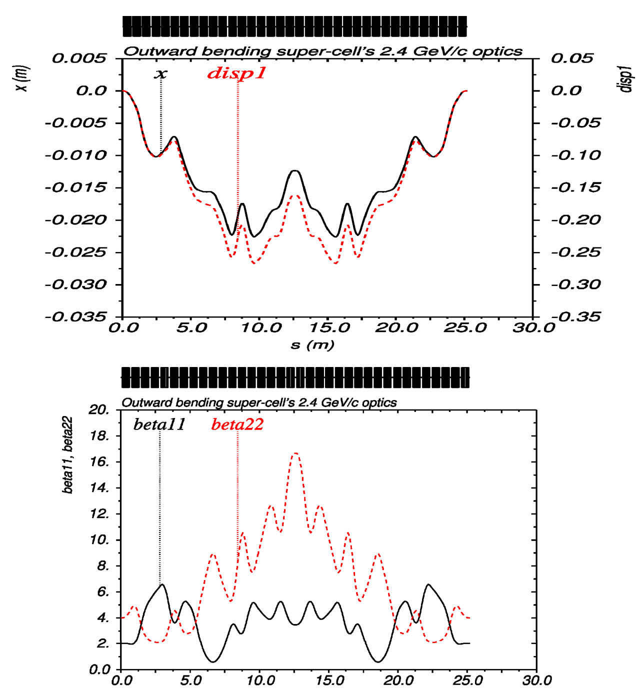

Figure 2: 2.4 GeV/c periodic orbit, dispersion (top) and beta functions (bottom) of the outward bending super cell.

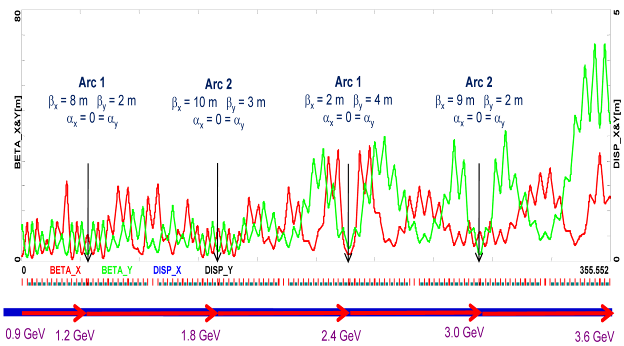

Figure 3: Linac optics matched to both NS-FFAG arcs for all passes simultaneously.

We also developed linac optics that is matched to both arcs at all energies. The matching is done for all passes simultaneously by adjusting the strengths of the linac's fixed-field quadrupoles. The linac's initial quadrupole configuration used as a starting point in matching was such that the quadrupole strengths increased linearly from the linac's center toward its ends. The matched linac optics for all passes is shown in Fig. 3.

Dynamic Aperture and Momentum Acceptance

We investigated the dynamic aperture of the linear NS-FFAG arcs by locating a stable phase-space trajectory with the maximum amplitude. Figure 4 shows such horizontal and vertical phase-space trajectories at 2.4 GeV/c and compares them with those for the non-linear NS-FFAG design [4]. The linear solution clearly has a significant advantage over the non-linear one.

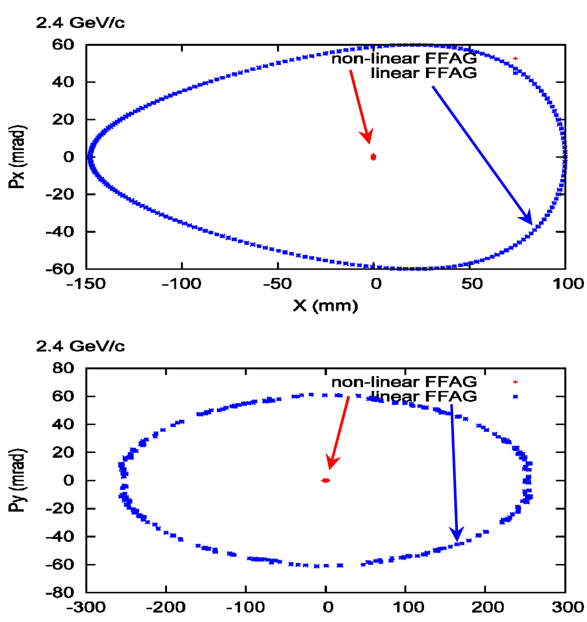

Figure 4: Horizontal (top) and vertical (bottom) maximum-amplitude stable phase-space trajectories for the linear and non-linear NS-FFAG designs.

Figure 5 shows the results of tracking a 2.4 GeV/c bunch for one pass through the droplet arc. The bunch shape is well-preserved for an rms relative momentum spread of 0.01. A significantly greater momentum spread, however, causes a significant bunch distortion and would require chromaticity compensation.

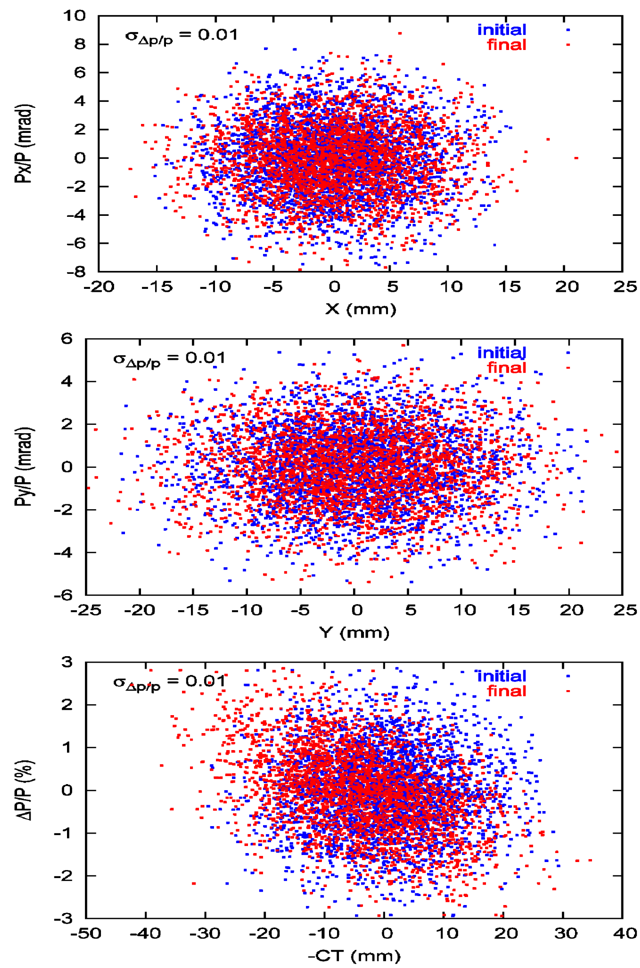

Figure 5: Horizontal (top), vertical (middle), and longitudinal (bottom) phase space distributions of a 2.4 GeV/c bunch before and after passing through the arc.

REFERENCES

[1] S.A. Bogacz et al., "Pulsed-focusing recirculating linacs for muon acceleration", in Proc. PAC 09, Vancouver, BC, Canada.

[2] D. Trbojevic, E. Courant, and M. Blaskiewicz, Phys. Rev. ST Accel. Beams 8, 050101 (2005).

[3] G.M. Wang et al., "Multipass arc lattice design for recirculating linac muon accelerators", in Proc. PAC'09, Vancouver, BC, Canada.

[4] V.S. Morozov et al., "Muon acceleration with RLA and non-scaling FFAG arcs", in Proc. IPAC'10, Kyoto, Japan.

[5] MAD-X, http://mad.web.cern.ch/mad/