**Abstract**

 neutrino factories and muon colliders require rapid acceleration of short-lived muons to multi-GeV and TeV energies. a recirculating linear accelerator (RLA) that uses superconducting RF structures can provide exceptionally fast and economical acceleration to the extent that the focusing range of the RLA quadrupoles allows each muon to pass several times through each high-gradient cavity. a new concept of rapidly changing the strength of the RLA focusing quadrupoles as the muons gain energy is being developed to increase the number of passes that each muon will make in the RF cavities, leading to greater cost effectiveness. we discuss the optics and technical requirements for RLA designs, using RF cavities capable of simultaneous acceleration of both $\mu^+$ and $\mu^-$ species, with pulsed Linac quadrupoles to allow the maximum number of passes. the design will include the optics for the multi-pass linac and droplet-shaped return arcs [1].

**MULTI-PASS LINAC OPTICS**

The superconducting accelerating structure is by far the most expensive component of the accelerator complex. maximizing the number of passes in the RLA can significantly lower the cost [2] of the overall acceleration scheme. The RLA linac consists of uniformly spaced RF cavities phased for a speed-of-light particle. The injection energy into the RLA and the energy gain per linac pass were optimized so that a tolerable level of RF phase slippage along the linac could be maintained. Furthermore, to minimize phase slippage at the lowest energy pass, an injection at the middle of the linac was chosen.

The key element of the transverse beam dynamics in a multi-pass ‘Dogbone’ RLA is an appropriate choice of multi-pass linac optics. The focusing profile along the linac (quadrupole gradients) needs to be set, so that one can transport (i.e. provide adequate transverse focusing for a given aperture) multiple pass beams within a very large energy range.

Since the beam is traversing the linac in both directions throughout the course of acceleration, one would like to maintain a 90° phase advance per cell for the lowest energy pass (the initial half-pass) by scaling the quad gradients with increasing energy along the linac. In order to mitigate the beta beating due to reduced focusing for the subsequent passes, the other half of the linac would have the inverted scaling of the quadrupole gradients. The resulting mirror symmetric focusing profile of the linac is illustrated in Figure 2.
Now we consider a ‘Pulsed’ linac Optics for the same RLA layout. Here we assume a time varying quad strength in the RLA linac described in the previous section. A feasible quad pulse would assume a 500 Hz cycle ramp with the top pole field of 1 Tesla. That would translate to a maximum quad gradient of $G_{\text{max}} = 2$ kGauss/cm (5 cm bore radius) ramped over $\tau = 1$ ms from the initial gradient of $G_0 = 0.1$ kGauss/cm. We have used a fairly conservative rise time based on similar applications for ramping the new corrector magnets for the Fermilab Booster that have 1 kHz capability [3].

For simplicity, we consider a linear ramp according to the following formula:

$$G(t) = G_0 + \frac{G_{\text{max}} - G_0}{\tau} t$$  \hspace{1cm} (1)

A single bunch travelling with a speed of light along the linac with quads ramped according to Eq.(1), ‘sees’ the following quad gradient passing through the $i$-th cell along the linac ($i = 1, \ldots, 20$)

$$G_i = G_0 + \frac{G_{\text{max}} - G_0}{\tau} \frac{\ell_{\text{cell}}}{c} i$$  \hspace{1cm} (2)

where $\ell_{\text{cell}}$ is the cell length and $i$ defines the bunch position along the linac.

For multiple passes through the linac (the index $n$ defines the pass number) the above formula can be generalized as follows:

$$G_i^n = G_0 + \frac{G_{\text{max}} - G_0}{\tau c} \left[(n-1) \left(\frac{\ell_{\text{linac}}}{2} + \frac{\ell_{\text{arc}}}{2}\right) + i\ell_{\text{cell}}\right]$$  \hspace{1cm} (3)

where $\ell_{\text{linac}}$ is the full linac length and $\ell_{\text{arc}}$ is the length of the lowest energy droplet arc. Here we also assume that the energy gain per linac is much larger than the injection energy. Figure 3 illustrates the multi pass optics for the pulsed linacs. As one can see below, there is sufficient phase advance to support up to 12 passes.

Pass 8 (31-35 GeV)

Pass 12 (47-51 GeV)

Figure 3: The 8-th pass and the last one (12-th) of the pulsed linac optics. By pulsing the focusing quads as described in Eq.(3), the additional 4 passes increase the output energy from 35 to 51 GeV. Red is horizontal and green is vertical.
‘DROPLET’ ARCS

In a ‘Dogbone’ RLA one needs to separate different energy beams coming out of a linac and to direct them into appropriate ‘droplet’ arcs for recirculation [1]. For multiple practical reasons, horizontal rather than vertical beam separation was chosen. Rather than suppressing horizontal dispersion created by the Spreader, it is smoothly matched to the horizontal dispersion of the outward 60° arc. Then by the appropriate pattern of removed dipoles in three transition cells, one ‘flips’ the dispersion for the inward bending 300° arc, etc. The entire ‘droplet’ Arc optics architecture is based on 90° betatron phase advance cells with uniform periodicity of Twiss functions. The resulting ‘droplet’ Arc optics based on FODO focusing [2] is illustrated along with its ‘footprint’ in Figure 4.

![Arc 1](image1)

![Arc 4](image2)

Figure 4: ‘Droplet’ Arc optics and its ‘footprint’ – uniform periodicity of beta functions and dispersion. The design offers both compactness and modularity; the top plot illustrates the lowest energy arc. The higher arcs based on the same bending field are configured by adding periodic cells in the outward and inward bending sections, extending the circumference and increasing the quadrupole strength according to the momentum. The bottom plot illustrates Arc 4 Optics.

CONCLUSIONS

A Recirculating Linear Accelerator (RLA) can provide exceptionally fast and economical acceleration to the extent that the focusing range of the RLA quadrupoles allows each muon to pass several times through each high-gradient cavity. A new concept of rapidly changing the strength of the RLA focusing quadrupoles as the muons gain energy has been developed. It significantly increases the number of passes that each muon will make in the RF cavities, leading to greater cost effectiveness. A complete linear lattice for the RLA has been designed (multi pass linac with pulsed quads and 12 droplet arcs) Technical feasibility, ultimate limitations, and cost effectiveness of such schemes is presently under studies [4], [5].

REFERENCES

[4] G. Wang et al., this Conference
[5] K. Beard et al., this Conference