Abstract
Injection phase space painting enables the mitigation of space charge and stability issues, and will be indispensable for the Project-X at Fermilab [1], delivering high-intensity proton beams to HEP experiments. Numerical simulations of multi-turn phase space painting have been performed for the FNAL Recycler Ring, including a self-consistent space charge model.

INTRODUCTION
The Project-X [2] accelerator facility will provide a 1 MW proton beam source upgradable to multi-megawatt. The accelerator chain includes a new 8 GeV H superconducting Linac with an 20 mA average beam current bunched in a 325 MHz beam structure with a 1.25 ms beam pulse and 5 Hz rep. rate, injecting into the existing Recycler storage Ring (RR) and the Main Injector (MI), responsible for the stripping/accumulation and further acceleration of 1.6×10^{14} protons/1.4sec up to 120 GeV, respectively.

In this paper we report initial study efforts on charge-exchange multi-turn injection from the Linac into RR in the presence of space charge, considering both transverse and longitudinal phase space painting.

LAYOUT, MAIN PARAMETERS AND COMPUTATIONAL TOOLS
Injection into the Recycler occurs in the newly formed symmetric straight section. Phase space painting is accomplished, through time varying horizontal closed orbit manipulations and vertical angle mismatch from the transport line, with fast painting dipoles. The injection chicane and horizontal painting dipoles, shown in Fig. 1, are wholly contained within the quad doublets defining the straight section.

Merging of the H⁻ ions with the proton closed orbit takes place within the second, low field, chicane dipole. The thin carbon stripping foil is located in the fringe field of the third chicane magnet and designed to strip approximately 98% of the incoming H⁻. Any H⁻ missing the foil or any H⁺ not stripped by the foil are converted into H⁰ in the increasing fringe field of chicane dipole three. The neutral ions in the ground state (n=1) or excited state, n=2 from the foil are converted to protons by the thick foil in front of chicane dipole four, which directs them toward the injection waste beam dump.

Figure 1: Injection chicane layout.

We used STRUCT [3] and ORBIT [4] codes for our simulations. Both codes were designed for accelerator rings and use macro particles for tracking. The lattice may consist of linear and non-linear elements. STRUCT was already used to study painting for the proton driver project [5]. This code represents a single particle model with analytical inclusion of space charge effects during tracking, which though was not applicable for painting simulations. The ORBIT code is more general, aiming to self-consistent inclusion of space charge effects into the model. It is a primary computational design tool for SNS.

ORBIT allows the extensions, implementing new “physics modules” to be run under the driver shell, by adding new classes and methods in the original C++ source code. We developed a block for the injection of a chopped beam, with arbitrary notch in the train of micro-bunches from linac. We used ORBIT and cross-checked the results with STRUCT calculations [6] and other previous results [7].

TRANSVERSE PAINTING
A uniform transverse phase distribution (KV-like) has lesser tune shift due to space charge and therefore is advantageous, in comparison, with the Gaussian-like ones. The resultant phase space distribution, after injection, is dependent on the form and sense of the time dependent horizontal closed orbit manipulation and vertical injection angle changes. Although a preliminary investigation [6] considering quadratic and exponential functional forms, without the inclusion of space charge, yielded a slightly smaller final emittance for the quadratic form, a more detailed study is planned. In this paper we report the results of the quadratic functional painting...
waveforms and the influence of space charge. The temporal form of the horizontal closed orbit and vertical steering dipoles is shown in equation 1 with the total injection period \( N_{\text{paint}} \) covering 277 turns.

\[
B = B_0 \left\{ 0.78 + 0.22 \left[ 1 - \sqrt{\frac{2N}{N_{\text{paint}}} - \left( \frac{N}{N_{\text{paint}}} \right)^2} \right] \right\}
\]

for \( N < N_{\text{paint}} \)

\[
\begin{align*}
B &= B_0 \left[ 0.78 - \frac{N - N_{\text{paint}}}{8.9743} \right] \\
\end{align*}
\]

for \( N_{\text{paint}} < N < 7 \times N_{\text{paint}} \)

\[
Y' = Y'_0 \sqrt{2 \frac{N_{\text{paint}} - N}{N_{\text{paint}}} - \left( \frac{N_{\text{paint}} - N}{N_{\text{paint}}} \right)^2}
\]

with \( Y'_0 = 0.22 \text{ mrad} \)

The current investigation considers an anticorrelated painting scenario. Future investigations will consider both correlated and anticorrelated phase space painting in the presence of space charge.

For simulations we assumed that horizontal and vertical geometric rms emittances of injected beam were equal to \( \varepsilon_{x,y} = 0.044 \pi \text{ mm·mrad} \) and after painting the emittances increased up to \( \varepsilon_{x,y} = 4 \pi \text{ mm·mrad} \). The total number of protons after the multi-turn injection was taken to be \( 1.6 \times 10^{14} \), represented by 100,000 macroparticles. After that the number of particles remained unchanged.

Figs. 2, 3 demonstrate the initial stage of painting during initial 5 turns in the Recycler, with the painting kicker magnets operating according to formulae (1).

In Fig. 3 one can see the density splashes in the \((y,y')\) areas where bunches overlap due to betatron motion.

The phase space distributions after 500 turns for the both zero space-charge (left) and full space-charge (right) are shown in Fig. 4. Horizontal and vertical distributions with and without space charge are shown in Fig. 5. Casual observation shows a less uniform distribution in the vertical plane for the case including space charge and a slight increase in the tails of the distribution.

For the chosen wave-form (1) the presence of space charge deteriorates the flatness of phase distributions, as shown in Fig. 5, in the comparison with zero space charge in Figs. 4.

**LONGITUDINAL PAINTING**

Since the transverse space charge tune shift is proportional to the longitudinal bunching factor defined as the ratio of peak longitudinal current to average current, it becomes important to produce as uniform longitudinal distribution as possible. The non-integer ratio...
of RF frequencies, 325 MHz Linac beam bunch structure and the 52.8 MHz RF system in the Recycler creates a phase “slippage” or mismatch between the linac beam and the Recycler RF on a turn by turn basis. An RF shift factor is equal to 0.15 (325 MHz divided by 52.8 MHz is equal 6.15). Therefore the RR separatrix can comprise only 6 micro-bunches (1 : 0.15 = 6.6). In the meantime, the very left and the right micro-bunches will be in the proximity of the separatrix nodes (±π) and will be lost. Therefore the injection “window”, accommodating only four micro-bunches from the original train, was chosen in the interval [-2.05,+2.05] rad. The fast chopper in the 2.5 MeV MEBT removes all micro-bunches out of that window. See Fig. 6.

A comprehensive one-dimensional modeling of multi-turn injection into MI was performed in [7] with the code ESME [8], omitting the transversal dynamics in (x,x’,y,y’). The initial results of our simulations in six-dimensional phase space (x,x’,y,y’,Δφ,ΔE) with ORBIT are plotted in Fig. 7. We took for the first 53 MHz RF harmonic the amplitude voltage of 800 kV. We obtained the distribution of charge density in Fig. 8 which was similar to that from [7].

We developed a block for longitudinal painting, which works with arbitrary notches in incoming micro-bunch buckets. The appropriate choice of the amplitude of the second harmonic of RF field will help to flatten the RF-bucket contours, as was demonstrated in 1D simulations [7]. Non-linear lattice issue will be also addressed.

CONCLUSION

The goal of our studies was to study the injection painting with inclusion of 3D space charge, using the ORBIT tracking code.

In a current scenario the painting lasts for 110 turns, twice faster, than we considered in this paper. The optimal wave-forms for painting kickers, which ensure the flatter phase distributions, should be found. So far we used a simplified model for painting kicker strength (implemented as the “ideal bump” in ORBIT). We will include a more realistic field map for the chicane magnets. Additional stripping simulations will be combined.

REFERENCES