Abstract

PAMELA (Particle Accelerator for MEdicaL Applications) is a design for a non-scaling Fixed Field Alternating Gradient (ns-FFAG) accelerator facility for Charged Particle Therapy, using protons and light ions such as carbon to treat certain types of cancer. A lattice has been designed which constrains the variation of betatron tunes through acceleration and thus avoids integer resonance crossing and beam blow-up. This paper outlines the design and performance of this proposed PAMELA lattice.

INTRODUCTION

The aim of the PAMELA project is to design a Charged Particle Therapy (CPT) facility, using the features of ns-FFAG technology to improve performance over existing facilities [1, 2]. Unlike synchrotrons and cyclotrons, the fixed magnetic field of the ns-FFAG allows kHz repetition rates, whilst still allowing variable energy extraction, making it an ideal candidate for a CPT accelerator.

Clinical considerations require that a proton/carbon CPT facility provides protons from 70 to 250 MeV and Carbon (C⁶⁺) from 68 to 400 MeV/u. To achieve this, two accelerating rings are required. The first will accelerate protons to 250 MeV and carbon ions to 68 MeV/u and the second will accelerate carbon to 400 MeV/u. The design of the first, smaller ring is outlined in this paper.

In addition to providing variable energy beams with a high repetition rate, the lattice design for PAMELA is required to [3]:

- Be compact enough to fit in a hospital environment.
- Provide sufficient long straight sections (> 1m) for RF, injection and extraction.
- Constrain both horizontal and vertical total betatron tunes to within an integer to avoid resonance crossing and beam blow-up.

DESIGN

Various studies have shown that if a densely-packed linear lattice, such as that used for EMMA (Electron Model for Many Applications), is used for a proton/carbon ns-FFAG for CPT, the crossing of betatron resonances throughout acceleration would lead to severe beam blow-up and unrealistic alignment tolerances[4, 5].

In order to avoid resonance crossing it is necessary to constrain the total betatron tune to within an integer throughout acceleration. In other types of accelerators two methods are used which can achieve this. The first, employed in synchrotrons, is chromatic correction, adding higher order multipoles to achieve a constant tune. The second, used in a scaling FFAG, employs a magnetic guide field which follows the scaling law $B = B_0(r/r_0)^k$, where $r$ is the radial co-ordinate, $k$ is the field index defined as $k = (r/B_y)(dB_y/dr)$ and $y$ is the vertical direction. This results in a constant tune throughout acceleration. The PAMELA lattice design, as in Fig. 1 employs a combination of these two methods.

Starting with the concept of a radial-sector F-D-F triplet scaling FFAG, a number of changes and simplifications are made [6]. Firstly, the magnetic field becomes non-scaling, by expanding the scaling field profile and retaining only the dipole and first few multipoles. This significantly changes the magnet design, removing the need for iron-cored magnets with complicated pole shaping used in a scaling FFAG. To ease cost, construction and alignment issues, the magnets are made rectangular rather than sector-shaped and are aligned along a straight line in each cell rather than along an arc, further violating the scaling law.

![Figure 1: Layout of PAMELA lattice with injection orbit and maximum energy orbit. Scale is in metres.](image)

A number of parameters can be used to further describe the PAMELA lattice design, including the field index, $k$ and geometrical factors including the lattice packing factor, magnet length and average radius.

Field Index

The field index ($k$) influences both the magnetic focusing strength and the orbit excursion, that is, the difference
in radial position of the maximum and minimum energy orbits, as shown by the two blue lines in Figure 1. A large field index results in a small orbit excursion, which is advantageous as it reduces the bore of the magnets, the beam pipe aperture and the RF aperture. However, the field index is limited when using the first stable region of Hill’s equation, where the phase advance per cell is less than 180 degrees.

In this design the second stable region of Hill’s equation is used, with a horizontal phase advance per cell greater than 180 degrees. This allows for a larger field index to be used, resulting in a smaller orbit excursion. The available working points as a function of field index and D/F magnet strength ratio are shown in Fig. 2.

![Figure 2: Available working points for the PAMELA lattice at reference energy (118 MeV protons). The 2nd stable region with $\nu_x > 0.5$ is shown in green.](image)

By carefully choosing the field index to be as large as possible, we then select a D/F ratio which results in cell tunes of around $\nu_x = 0.75$, $\nu_y = 0.25$.

**Geometrical Factors**

The lattice packing factor, $\alpha$, is the length of the F-D-F triplet compared to the machine circumference. In order to access the second stable region of Hill’s equation for an F-D-F triplet focusing lattice whilst maintaining suitable dynamics, it is preferable to have a small ($\alpha \approx 0.5$) packing factor [7]. However, too small a value for $\alpha$ results in short magnet lengths with very long straight sections. This may result in unachievable magnetic fields or, to compensate, a very large radius of the accelerator. For these reasons $\alpha$ is chosen to be suitably large to achieve a compact accelerator whilst preserving the stable dynamics.

Once the packing factor and average radius are chosen, the peak magnetic field can be reduced slightly by extending the magnets into the short drift space, ensuring the magnetic centre remains fixed.

The lattice design parameters for PAMELA are outlined in Table 1.

### Table 1: Lattice Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Inj.</th>
<th>Ref.</th>
<th>Extr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton Kin. Energy [MeV]</td>
<td>30.95</td>
<td>118.38</td>
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<tr>
<td>$C^6+$ Kin. Energy [MeV/u]</td>
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<td>$B_\rho$ [Tm]</td>
<td>0.81071</td>
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<tr>
<td>$r_0$ [m]</td>
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<tr>
<td>Magnet length [m]</td>
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<td></td>
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<tr>
<td>Packing factor $\alpha$</td>
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<tr>
<td>Field index, $k$</td>
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</tr>
<tr>
<td>Orbit excursion [m]</td>
<td>0.176</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**PERFORMANCE**

The performance of the PAMELA lattice was studied using the ray-tracing code ZGOUBI [8] for tracking studies and S-Code [9] for the variation of basic lattice parameters. The beta functions in one cell, as calculated with S-Code, are shown in Fig. 3.

![Figure 3: Beta functions in one cell of the PAMELA lattice.](image)

The variation of betatron cell tunes throughout acceleration is shown in Fig. 4. The variation of the total horizontal tune is 0.116 and the total vertical tune is 0.655, both of which are well within an integer, as indicated by the dotted lines. These results illustrate that the design method for constraining betatron tunes is successful.

A typical tracking example using ZGOUBI including acceleration is shown in Fig. 5, clearly showing the small natural orbit excursion which is 17.6cm in total.

**Sensitivity to Alignment Errors**

This lattice was designed to overcome the high sensitivity of densely-packed ns-FFAG lattices to alignment errors. To ascertain the sensitivity of the current design, alignment errors were introduced and a single particle was tracked to extraction with four RF cavities at a voltage of 30 kV/Cavity and a synchrotron phase of 50°. Full acceleration is achieved in around 2500 turns in this scheme.

Random patterns of alignment errors were introduced,
using 20 different error sizes and 10 different lattices with each error size. The errors are gaussian distributed with a cut-off at $3\sigma$, with a different random value applied to each of the magnets. The orbit distortion is defined as the maximum deviation of the tracked particle from the closed orbit of the error-free lattice.

$$A = \frac{\langle \text{Orbit distortion [mm]} \rangle}{\langle 1\sigma \text{ Alignment Error [mm]} \rangle} \quad (1)$$

The distortion is quantified by an ‘amplification factor’ $A$, as in Equation 1. In the horizontal direction, the amplification factor is found to be $7.9 \pm 3.5$, implying an alignment error of $50 \mu$m would induce an orbit distortion of around $0.4$mm. This is a factor of around 50 smaller than in the linear ns-FFAG case, and presents an achievable alignment tolerance.

**Dynamic Aperture and Phase Space**

The typical size of the beam used in charged particle therapy is $10\pi$.mm.mrad. This is the un-normalised beam emittance at the patient. Phase space plots for the full dynamic aperture are shown in Fig. 6. The horizontal dynamic aperture is around $400 \pi$.mm.mrad and the vertical is around $200 \pi$.mm.mrad, which is more than adequate for the current application.

**CONCLUSIONS**

An ns-FFAG lattice has been designed which successfully constrains betatron tune variation, reduces orbit excursion and simplifies magnet design, construction and alignment issues. Both the horizontal and vertical betatron tunes are successfully constrained to within an integer, avoiding resonance crossing. Furthermore, a preliminary study of sensitivity to alignment errors indicates that the design is feasible in terms of alignment tolerances. The design also provides adequate dynamic aperture for the application of charged particle therapy.

**REFERENCES**