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- Transverse Resonance in a Linac
- Experiment: Set-up & Results
- Parametric Resonance (inter-plane coupling long. ↔ transv.)
- Experimental Results

campaign embedded into HIPPI: High Intensity Pulsed Proton Injectors

HIPPI was part of the EU-supported CARE activity
Transverse Resonance in a Linac

- perturbing device (magnet, cavity, ...) acts on particle just once
- single devices cannot cause resonant perturbation
- high beam current:
  - space charge (sc) acts on particle
  - sc force acts permanently
  - sc force varies with envelope size
  - periodic change of envelope $\rightarrow$ periodic sc force on particle

$$\sigma_{\text{part}} < \sigma_{\text{env}} = 360^\circ$$
Model for Resonance

- matched envelope
- envelope has radial symmetry
- periodically breathing envelope, phase advance $\sigma_{\text{env}}$
- particle experiences:
  - constant external focusing $\sigma_o$
  - electric field of breathing envelope with radius $R(s)$

Envelope charge density depends on radius $r$:

$$\rho(r) = \rho_o(s) \cdot \left[ 1 - \frac{r^2}{R(s)^2} + O(r) \right]$$

density component ($r^2$), $r \gg$ neglected

Creating a field:

$$E_r = \frac{18 \cdot I}{\pi \epsilon_o \cdot R(s)^2 \beta c} \left[ r - \frac{r^3}{2R(s)^2} \right], \quad r \leq R(s)$$

octupolar field component ($r^3$)
Single Particle Motion

single particle equation (lattice + sc):

\[ r'' + \left[ \sigma_0^2 - \Delta \sigma^2 \right] r = a \cdot r^3 \cdot e^{i \sigma_{env} s} \]

\[ r'' + \sigma_0^2 r = a \cdot r^3 \cdot e^{i \sigma_{env} s} \] perturbed oscillator
depressed phase advance

resonance condition:

\[ \sigma_{env} = 4\sigma \]

envelope oscillates 4 times faster than single particle

\[ \sigma_{env} = 360^\circ \rightarrow \sigma = 90^\circ \]

4th order resonance occurs at \( \sigma = 90^\circ \), i.e. \( \sigma_o \geq 90^\circ \)
\[ f_{\text{oscillation (envelope)}} = 4 \times f_{\text{oscillation (particle)}} \]

→ resonant excitation of single particles

4 wings: characteristic feature of 4th order resonance

initial phase space distribution

final phase space distribution
Direct measurement requires:

- measurement of the phase space distribution (no quad scan)

- 100% beam transmission (resonant "wing" particles lost first)

- matched beam envelope:
  - periodic perturbation by space charge
  - mitigate mismatch emittance growth

DTL matching with space charge is most difficult part of experiment
• never observed directly
• simulations by D. Jeon (SNS/ORNL) suggested measuring resonance at GSI UNILAC
• simulations predicted dominance over envelope instability
→ PRST-AB 12, 054204 (2009)

Experiment at GSI UNILAC:

• install beam emittance measurement unit behind first DTL tank
• exploit experience from previous experiments to optimize UNILAC settings (matching !)
• measure phase space distributions and extract rms emittances
ions: protons to uranium
acceleration: 1.4 – 3.6 MeV/u
108 MHz
synchr. rf-phase -30° → σ_{l,o} = 43°
F-D-D-F focusing
15 full lattice periods
length ≈ 12 m
max. transv. phase advance σ_o:
- protons : 180°
- \(^{40}\text{Ar}^{10+} : 180°
- \(^{238}\text{U}^{27+} : 62°\)
Matching:

Reconstruction of Beam rms Parameters at DTL Entrance

1. selfconsistent backtracking finding \((\alpha, \beta, \varepsilon)_\parallel\) that fit to measured bunch length
2. verification: settings reproduce 100% transmission, no low-energy tails

L. Groening, *Experimental Observation of Space Charge Driven Resonances in a Linac*
Matching to Periodic Solution with Space Charge

- beam parameters at beginning of matching section from emittance measurement
- periodic solution at DTL entrance calculated numerically
- section to be set to match this solution
- 7 knobs: 5 quadrupoles + 2 re-bunchers
- rms envelope equations to obtain beam Twiss params. at DTL entrance
- seven variables to minimize one value, i.e. the sum of mismatches hor., ver., and long.
- solved numerically
• strong growth approaching $\sigma_o \approx 100^\circ$

• tune depression: $\sigma_o \approx 100^\circ \rightarrow \sigma \approx 90^\circ = 360^\circ / 4$

• good agreement with three simulation codes

• strong hint for space charge driven 4th order resonance
Proof for 4th Order Resonance in the UNILAC

4 wings were observed

L. Groening, Experimental Observation of Space Charge Driven Resonances in a Linac
simulated envelopes
→ no instability at $\sigma > 90^\circ$

DTL too short and/or mismatch too small for envelope instability growth

simulated rms emit. growth
KV beam, no 4th order term → no growth

5000 particles → residual num. noise creates small artificial 4th order term
• first direct measurement of space charge driven resonance

• resonance dominates envelope instability as predicted by D. Jeon in PRST-AB 12, 054204 (2009)

• evidence for enveloped-matched operation of the UNILAC DTL

• details in PRL 102, 234801 (2009)
Parametric Resonances from Inter-Plane Coupling

• Hofmann charts: well excepted linac design tool
• simulations: just $\sigma_\parallel \approx \sigma_\perp$ harmful to machine performance
• no experimental verification
• experiment done at GSI UNILAC, first DTL tank
Evidence for Parametric Resonance at the UNILAC

- Tune ratio approaches 1.0 → increased transverse growth measured
- Result in good agreement with simulations
Evidence for Parametric Resonance at the UNILAC

- transv. growth comes along with longitudinal emittance reduction
- strong evidence for long.→ transv. emittance transfer

Hofmann's Charts confirmed, details in PRL 103, 224801 (2009)
- first direct measurement of 4th order space charge driven resonance

- UNILAC DTL: 4th order resonance dominates envelope instability (exp. confirmation)

- first experimental confirmation of Parametric Resonance (Hofmann Charts)