CSR-Driven
Longitudinal Single Bunch Instability Thresholds

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I. Theoretical Predictions
   I.1 Vlasov-Fokker-Planck Equation
   I.2 Instability Driven by Resistive Impedance
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   I.3 CSR-Driven Instability

II. Experimental Observations
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   II.1 MLS
   II.2 Similarity Between Resistive and CSR-wake
   II.3 Threshold Determination
   II.4 BESSY II

III. Summary
N > 10^9 electrons per bunch → smooth distribution in phase space → distribution function:

\[ f(q, p, \tau) \]

\[ q = z / \sigma_z \]

\[ p = -\Delta E / \sigma_E \]

\[ \tau = \omega_s t \]

\[ \frac{\partial f}{\partial \tau} + p \frac{\partial f}{\partial q} - [q + F_c(q, \tau, f)] \frac{\partial f}{\partial p} = \frac{2}{\omega_s t_1} \frac{\partial}{\partial p} \left( pf + \frac{\partial f}{\partial p} \right) \]  

(M. Venturini)

RF focusing  Collective Force  Damping  Quantum Excitation

Numerical solution based on  
Other numerical solutions:  
S. Novokhatlski, EPAC 2000 and SLAC-PUB-11251, May 2005
II.1 Vlasov-Fokker-Planck-Equation (VFP) „Wave Function“ Approach

original VFP-equation:

\[
\frac{\partial f}{\partial \tau} + p \frac{\partial f}{\partial q} - \left[ q + F_c(q, \tau, f) \right] \frac{\partial f}{\partial p} = \frac{2}{\omega_s t_l} \frac{\partial}{\partial p} \left( pf + \frac{\partial f}{\partial p} \right)
\]

Ansatz – “wave function” approach: Distribution function, \( f \), expressed as product of amplitude function, \( g \):

\[
f = g \cdot g
\]

\[
\frac{\partial g}{\partial \tau} + p \frac{\partial g}{\partial q} - \left[ q + F_c(q, \tau, g^2) \right] \frac{\partial g}{\partial p} =
\]

\[
\frac{2}{\omega_s t_l} \left( \frac{g}{2} + p \frac{\partial g}{\partial p} + \frac{1}{g} \left( \frac{\partial g}{\partial p} \right)^2 + \frac{\partial^2 g}{\partial p^2} \right)
\]

\( f \geq 0 \) and solutions numerically more stable

Details on the simulations in my ICAP ‘12 contribution
I.1 Parameters Used in the Simulations

Simulations for 6 – 10 damping times, step size $\Delta=\sigma/20 \ldots \sigma/10$, time step $\sim 2\pi/1024$. Over last 64 periods the line density is stored 64 times per period for analysis: FFT gives CSR-spectrum, and integrated spectral power is proportional to instantaneous CSR signal. FFT of this signal corresponds to observed signal.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>BESSY II</th>
<th>MLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy, $E_0/\text{MeV}$</td>
<td>1700</td>
<td>629</td>
</tr>
<tr>
<td>Bending radius, $\rho/\text{m}$</td>
<td>4.35</td>
<td>1.528</td>
</tr>
<tr>
<td>Momentum compaction, $\alpha$</td>
<td>$7.3 \times 10^{-4}$</td>
<td>$1.3 \times 10^{-4}$</td>
</tr>
<tr>
<td>Cavity voltage, $V_{\text{rf}}/\text{kV}$</td>
<td>1400</td>
<td>330</td>
</tr>
<tr>
<td>Accelerating frequency, $\omega_{\text{rf}}/\text{MHz}$</td>
<td>$2\pi 500$</td>
<td>$2\pi 500$</td>
</tr>
<tr>
<td>Revolution time, $T_0/\text{ns}$</td>
<td>800</td>
<td>160</td>
</tr>
<tr>
<td>Natural energy spread, $\sigma_E$</td>
<td>$7.0 \times 10^{-4}$</td>
<td>$4.36 \times 10^{-4}$</td>
</tr>
<tr>
<td>Zero current bunch length, $\sigma_0/\text{ps}$</td>
<td>10.53</td>
<td>1.549</td>
</tr>
<tr>
<td>Longitudinal damping time, $\tau_l/\text{ms}$</td>
<td>8.0</td>
<td>11.1</td>
</tr>
<tr>
<td>Synchrotron frequency, $\omega_\text{s}/\text{kHz}$</td>
<td>$2\pi 7.7$</td>
<td>$2\pi 5.82$</td>
</tr>
<tr>
<td>Height of the dipole chamber, $2h/\text{cm}$</td>
<td>3.5</td>
<td>4.2</td>
</tr>
</tbody>
</table>
Weak instability theory by K. Oide, Part. Accel. 51, 43 (1995) - black solid line
Numerical results for the Diamond Light Source (DLS) and BESSY II

DLS: \( V_{\text{rf}} = 2, 4, 8, \) and 16 MV
BESSY II: \( V_{\text{rf}} = 0.7, \ldots 14000 \) MV.

Oide's dimensionless parameter:

\[
'k \cdot R' = \frac{I_{\text{threshold}}[A] \cdot R[\Omega] \cdot T_0[s]}{dV_{\text{rf}} / dt[V/s] \cdot \sigma_0^2[s]} 
\]

\( I_{\text{threshold}}(\sigma_0 = \text{const}) \propto \sqrt{dV_{\text{rf}} / dt} \)

weak instability – damping time matters
I.3 Results for BBR-Wake

\[ S_{BBR} = \frac{2N_{e}}{\gamma cZ_{0}v_{s}^{2} \sigma_{e}} \cdot \frac{2\pi F_{res} R_{s}}{Q} \]

K.L.F. Bane, et al., "Comparison of Simulation Codes for Microwave Instability in Bunched Beams", IPAC'10, Kyoto, Japan and references there in
I.3 Results for BBR-Wake – BESSY II

\[ 2\pi \cdot 27 \text{ GHz} \cdot \sigma_0 \approx 1.8 \]
I.4 Shielded CSR-Impedance

Broad band resonator with low Q:

\[ F_{\text{res}} = c\sqrt{\pi / 24}\rho^{1/2}h^{-3/2} \]

BESSY II: \( F_{\text{res}} \sim 100\) GHz

MLS: \( F_{\text{res}} \sim 44\) GHz

\[ S_{csr} = \frac{N r_e}{2\pi v_s \gamma \sigma \varepsilon} \cdot \rho^{1/3} (c \sigma_0)^{-4/3} \]

\[ F_{res} = c\sqrt{\pi/24} \rho^{1/2} h^{-3/2} \]

\[ S_{csr} \sim 0.5 + 0.12 \cdot X \quad (\text{Bane, et al., IPAC'10}) \]
1.4 Shielded CSR-Wake – BESSY II

\[
S_{csr} = \frac{N r_e}{2\pi v_s \gamma \sigma_\varepsilon} \cdot \rho^{1/3} (c\sigma_0)^{-4/3} \quad F_{res} = c\sqrt{\pi / 24} \rho^{1/2} h^{-3/2}
\]
\[ S_{csr} = \frac{N_{re}}{2\pi n_s \gamma \sigma_\varepsilon} \cdot \rho^{1/3} (c\sigma_0)^{-4/3} \]
\[ F_{res} = c\sqrt{\pi / 24} \rho^{1/2} h^{-3/2} \]
II.1 CSR-Threshold Currents for the MLS

Solution of Vlasov-Fokker-Planck equation

$2\pi \cdot F_{res} \cdot \sigma_0 \approx 1.84$

$2\pi \cdot 44\text{GHz} \cdot 3.3\text{ps} \approx 0.92$

II.1 CSR-Threshold Currents for the MLS

II.2 Comparison of CSR- and Resistive Wake

MLS: \( V_{rf} = 330 \text{kV}, \ \alpha = 1.3 \times 10^{-4}, \ \sigma_0 = 1.55 \text{ps} \)
II.2 CSR-Threshold Currents for the MLS

II.2 CSR-Threshold Currents for the MLS

Figure 1: Temporal fluctuation of the THz power as a function of the single bunch current plotted in the frequency domain. The measurement was performed using an InSb hot electron bolometer at a the storage ring parameters $\alpha_0 = 1.3 \times 10^{-4}$ and $V = 330$ kV.
Simulated temporal CSR spectra – multi particle tracking with CSR-wake, 1 Mio. particles, $\alpha_0=1.3\cdot10^{-4}$ and Vrf=330 kV
II.3 Threshold Determination

MLS – Theoretical Result

Simulated temporal CSR spectra – multi particle tracking with CSR-wake, 1 Mio. particles, $\alpha_0=1.3 \cdot 10^{-4}$ and $V_{rf}=330$ kV
Simulated temporal CSR spectra – multi particle tracking with CSR-wake, 1 Mio. particles, $\alpha_0=1.3\cdot10^{-4}$ and $V_{rf}=330$ kV
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II.3 CSR - Theoretical Threshold Determination

- CSR - Theoretical Threshold Determination

![Graph showing bunch length and energy spread as functions of normalized bunch length and energy spread, with particle tracking and VFP calculation annotations.]

- Particle tracking $N=I_0 \cdot I/e$
- Particle tracking $N=1e6$
- VFP calculation

![Graph showing spectral peak in arbitrary units as a function of $I_{sb}/\mu A$.]

- Particle tracking $N=I_0 \cdot I/e$
- Particle tracking $N=1e6$
- VFP calculation

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Many modes visible in the Fourier transformed CSR – Equilibrium fluctuations due to finite number of particle – Schottky noise effect, longitudinal beam diagnostics
II.4 CSR-Threshold Currents for BESSY II

In fair agreement with predictions – bunch lengthening explains shift

II.4 CSR-Threshold Current Measurement

BESSY II, $F_{\text{syn}} = 1$ kHz, $\sigma_0 \sim 1.5$ ps

$F_{\text{inst}} / F_{\text{syn}} \sim 3.1$  

instability mode number
II.4 First Unstable Modes BESSY II

Slope agrees with resonance $F_{\text{res}} \sim 100\ \text{GHz}$
• Predictions using the shielded CSR-wake are in surprisingly good agreement with measurements at BESSY II and the MLS.
• The observed resonance-like features show the importance of the vertical gap of the dipole vacuum chamber.
• Simulations demonstrate the weak nature of the CSR driven instability - also in the region of very short bunches where shielding is less important.
• Below the instability threshold multi-particle-tracking in better agreement with observations than “noise free” VFP-solutions.
• Equilibrium fluctuations due to finite number of particle and very sensitive THZ-detectors useful for longitudinal bunch diagnostics.
• Experimental determination and scaling of threshold currents requires attention – region of weak instability, low mode numbers.
• Results for very high RF-gradients (higher harmonic, double RF-system) have shown not quite the expected increase of instability thresholds.

• The support of Karsten Holldack, Jens Kuszynski, Fjodor Falkenstern and Dennis Engel is acknowledged.