DETERMINATION OF OPTICS TRANSFER BETWEEN THE KICKER AND BPMs FOR TRANSVERSE FEEDBACK SYSTEM

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Abstract

The knowledge of the transfer optics between the positions of the Kicker and the BPMs is required for the calculation of the correction signal in transverse feedback systems. Therefore, using nominal values of the transfer optics with uncertainties leads to feedback quality degradation, and thus beam disturbances. In this work, we propose a method for measuring the phase advances and amplitude scaling between the positions of the kicker and the BPMs. Directly after applying a kick on the beam by means of the kicker, we record the BPM signals. Consequently, we use the Second-Order Blind Identification (SOBI) algorithm to decompose the noised recorded signals into independent sources mixture. Finally, we determine the required optics parameters by identifying and analyzing the betatron oscillation sourced from the kick based on its mixing and temporal patterns. Results for the heavy ions synchrotron SIS 18 at the GSI are shown.

INTRODUCTION

In modern synchrotrons and collider facilities, there is always an increasing demand for higher beam intensities. However, stronger interaction between the accelerated beam and the accelerator objects can occur with increasing beam intensities. This excites coherent beam instabilities, when the natural damping becomes not enough to suppress the oscillations caused by the resonances of the interaction between the beam and the accelerator. This can occur in the horizontal, vertical, and longitudinal direction.

Furthermore, transversal beam oscillations can occur in synchrotrons directly after injection due to injection errors in position and angle, which come from inaccurate injection kicker reactions.

This paper focuses only on beam oscillations in the transversal direction, i.e., horizontal and vertical.

Beam transversal oscillations lead to emittance blow up caused by the decoherence of the oscillating beam. This decoherence comes from the tune spread of the beam particles. The emittance blow up deteriorates the beam quality since it reduces the luminosity [4]. Therefore, beam oscillations must be suppressed in order to maintain a high beam quality.

A powerful way to mitigate coherent instabilities is to use a feedback system. A Transversal Feedback System (TFS) measures beam oscillations by means of Pickups (PUs), and acts back on the beam by means of actuators called kickers [5, 4, 1, 2, 3]. The correction signal applied by the kicker must have a 90° phase advance from the betatron motion at the kicker position in order to be able to damp the beam oscillations. This can be achieved by passing the signal of one PU through an FIR filter with a proper phase response at the tune frequency [5].

Two or more PUs can be used to calculate the feedback correction signal without using an FIR filter [1, 2]. At least two PUs at different positions are required for defining the beam trace space since only beam displacements from the ideal trajectory can be measured by PUs, but not the angles of the beam. Multiple PUs can be used to minimize the noise contribution in the feedback correction signal, which enhances the feedback quality and reduces beam heating [2].

The proper coefficients of the FIR filter as well as the combination coefficients of multiple PU signals are calculated based on the accelerator optics parameters, i.e., the transfer matrices between the kicker and the PUs. Therefore, magnetic field imperfections and magnets misalignment lead to feedback quality deterioration, when using nominal optics values. Thus, optics transfer between the kicker and the PUs must be measured precisely to get real values and reach better feedback quality.

In this work, we address a method for measuring the phase advance and amplitude ratio between the beam oscillation at the kicker and the PUs positions. A rigid kick is applied on the beam, and the PU signals are recorded. The Second-Order Blind Identification (SOBI) algorithm [7, 6] is used Consequently to separate different beam oscillation sources from the noised PU signals. The required optics parameters determination follows then by identifying, analyzing, and fitting the betatron oscillation on horizontal and vertical direction.

SYSTEM MODEL

Let the TFS be composed of M PUs and a kicker located at different places around the accelerator ring. In order to make the required optics measurements for the TFS, we first of all perform a rigid initial kick on the beam by means of the kicker. We then start recording the turn-by-turn PU signals directly after the initial kick. The turn-by-turn PU signal is calculated by averaging the displacement over a whole bunch. This averaging reduces the thermal noise power generated by the PUs.

The recorded beam transversal motion contains components sourced from different phenomena, e.g., betatron oscillation, head-tail modes, and synchrotron oscillation.

Suppose we record the PU signals over N turns. Let

\[ x_i = [x_i(1), \cdots, x_i(N)] \in \mathbb{R}^{1 \times N} \quad (1) \]

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denote the signal recorded from the $i^{th}$ PU. This vector is the contribution of the beam transversal motion and the noise from the $i^{th}$ PU, i.e.,

$$x_i = \bar{x}_i + z_i,$$

where the vector $\bar{x}_i$ represents the beam transversal offsets over the $N$ turns at the position of the $i^{th}$ PU, and $z_i$ denotes the Gaussian noise vector. Let

$$X = [x_1^T, \cdots, x_N^T]^T \in \mathbb{R}^{M \times N}$$

denote the matrix of the recorded signals from all the $M$ PUs over $N$ turns.

Assume the beam transversal motion is composed of $K$ components driven by various sources. The recorded signals vector can be then written as

$$X = AS + Z,$$

where $Z$ denotes the noise terms assumed to be independent zero-mean Gaussian with variance $\sigma^2$, and $S \in \mathbb{R}^{K \times N}$ the source signals. $A \in \mathbb{R}^{M \times K}$ is the source mixing matrix, which represents the coupling between the sources on the betatron motion.

After kicking a bunch in the transversal direction, it will perform transversal oscillation with the coherent tune frequency. The initial phase of the recorded oscillation at some PU corresponds to the phase difference between the kicker position and the position of this PU.

Furthermore, decoherence of the transverse bunch oscillation will occur, meaning that the amplitude of the oscillation will decrease over the time. This is a direct result of the tune spread of the bunch particles due to the momentum spread and machine chromaticity. Recoherence of the transverse oscillation will follow after a while corresponding to the synchrotron period in a linear focusing lattice [8]. Therefore, the bunch transverse oscillation in vertical or horizontal direction can be stated as

$$x(n) = a(n) \sin(Qn + \phi_0),$$

where $Q$ denotes the coherent tune, and $\phi_0$ is the initial phase. $a(n)$ denotes the amplitude function of the transversal oscillation, and it can be written as [8, 9]

$$a(n) = a_0 \exp \left\{ -2 \left( \frac{\xi Q_0 \delta_p}{Q_s} \sin(\pi Q_s n) \right) \right\},$$

where $a_0$ denotes the initial amplitude depending on the kick strength, $\xi$ the chromaticity, $\delta_p$ the RMS momentum spread, and $Q_s$ the tune of the synchrotron oscillation. This amplitude has also an exponentially decreasing trend due to the Landau damping.

Fig. 1 shows recorded turn-by-turn beam oscillation at the heavy ions synchrotron SIS 18 at the GSI [8]. One can observe that this signal matches to the above described model.

Figure 1: Recorded turn-by-turn beam oscillation in the vertical plane at SIS 18.

**SECOND-ORDER BLIND IDENTIFICATION**

The SOBI is a source separation technique based on the coherence of the source signals, which is considered to be more robust in poor signal to noise ratios due to utilizing second-order statistics, i.e., spatial covariance matrices, of the measurements [7].

The covariance matrix of the source signals $C_{SS}(\tau) = E\{S(t)S(t + \tau)^T\}$ is diagonal assuming independent sources. From Eq. (4) one can see that

$$C_{XX}(\tau) = AC_{SS}(\tau)A^T + \delta(\tau)\sigma^2 I,$$

meaning that $A$ is a diagonalizer of the measurements covariance matrix for $\tau \neq 0$. This is a very important property to perform the source separation.

The SOBI algorithm is performed to determine $A$ and $S$ in the following steps [7, 6]:

- Perform eigenvalue decomposition on $C_{XX}(0)$
- Put the $K$ largest eigenvalues into a diagonal matrix $\Lambda_S$ and the corresponding eigenvectors into a matrix $U_S$. These eigenvalues represent the source signals and the smallest ones represent the white noise.
- Define $V = \Lambda_S^{-\frac{1}{2}}U_S^T$, and $\Psi = VX$,
- Calculate the covariance matrices $C_{\Psi \Psi}(\tau_k)$ for some values $\tau_k$, and $\tilde{C}_{\Psi \Psi}(\tau_k) = \frac{C_{\Psi \Psi}(\tau_k) + C_{\Psi \Psi}(-\tau_k)}{2}$.
- Find a matrix $W$ which jointly diagonalizes all of the $\tilde{C}_{\Psi \Psi}(\tau_k)$ for the selected time lags, i.e.,

$$\tilde{C}_{\Psi \Psi}(\tau_k) = WDW^T.$$  

- The mixing matrix is thus given by $A = V^{-1}W$, and the corresponding source signals are $S = W^TVX$,
Using the output of the SOBI algorithm, the phase and amplitude transfer between the kicker position and the PUs can be determined in the vertical and horizontal direction.

**OPTICS TRANSFER DETERMINATION**

After the source signals and the corresponding mixing matrices have been determined, the components of the betatron oscillation in some direction, e.g., horizontal direction, can be identified by its tune frequency, which must be closed to the nominal value. This betatron oscillation must have two components with orthogonal phases since the PU signals include this oscillation with different phases.

The betatron oscillation should follow the model described in Eq. (6) and Eq. (7) in beam offset and angle. Therefore, the beam oscillation following the kick will occur according to the decoherence/recoherence model with a maximum amplitude corresponding to the kick, and zero initial phase since the oscillation starts from zero offset, assuming a stable beam. Thus, the phase transfer between the kicker position and the PUs is equal to the initial phases of the betatron oscillation recorded at each PU. The amplitude transfer is then the ratio of the maximum amplitude and the kick for each PU.

Within the components of the betatron oscillation, part of the noise is superimposed. Therefore, a nonlinear fitting approach is used to determine the initial phase and amplitude parameters in the two betatron components. Let \(\hat{\alpha}_1\) and \(\hat{\alpha}_2\) denote the estimated initial amplitude of these components, and \(\hat{\phi}_1\) and \(\hat{\phi}_2\) denote the estimated phases. The amplitude transfer from the kicker position and PU \(i\) can be written as

\[
T_{\beta_i} = \frac{\sqrt{\hat{\alpha}_1^2 (A_{ib_1})^2 + \hat{\alpha}_2^2 (A_{ib_2})^2}}{\beta_K D_K},
\]

and the phase transfer

\[
\Delta \Phi_i = \arctan \left( \frac{\hat{\alpha}_1 (A_{ib_1})}{\hat{\alpha}_2 (A_{ib_2})} \right) + \hat{\phi}_1,
\]

where \(A_{ib_j}\) denotes the element corresponding to the PU \(i\) and betatron component \(j \in \{1, 2\}\) within the mixing matrix, and \(D_K\) represents the kick strength. \(\beta_K\) is the value of the beta function at the kicker position, which is assumed to be known precisely from a previous measurement for instance.

Since the two betatron components are orthogonal, the following must hold

\[
\hat{\phi}_2 \approx \hat{\phi}_1 + \frac{\pi}{2}.
\]

This can be exploited to enhance the phase estimation from the nonlinear fitting. In this case, both phases can be estimated from each component and averaged over the two estimates.

**RESULTS**

In this section, we show simulation results for the kicker and PUs of the SIS 18 at the GSI.

In the SIS 18, there are 12 PUs for both transversal directions located periodically along the ring. There is also one kicker for each transversal direction. We have generated a betatron oscillation in horizontal direction with \(Q_x = 4.29\). A small coupling part from the vertical and the longitudinal oscillations, as well as Gaussian measurement noise with standard deviation of 25% of highest signal amplitude were added. Using the above described method we were able to determine the phase advances with about 2° error, and the amplitude scaling with a less than 5% error.

**CONCLUSION**

In this paper, we have presented a method which is able to measure the phase and amplitude transfer between the kicker position and the PUs precisely for a better performance of the TFS. This method however assumes knowing \(\beta_K\) precisely. The TFS can be alternatively designed to be robust against optics uncertainties like in [3] in order to achieve better performance even without carrying out the measurements.

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**REFERENCES**


