**Abstract**

The design of a dogbone Recirculated Linear Accelerator, RLA, with linear-field multi-pass arcs was earlier developed [1] for accelerating muons in a Neutrino Factory and a Muon Collider. It allows for efficient use of expensive RF while the multi-pass arc design based on linear combined-function magnets exhibits a number of advantages over separate-arc or pulsed-arc designs. Such an RLA may have applications going beyond muon acceleration. This paper describes a possible straightforward test of this concept by scaling a GeV scale muon design for electrons. Scaling muon momenta by the muon-to-electron mass ratio leads to a scheme, in which a 4.5 MeV electron beam is injected at the middle of a 3 MeV/pass linac with two double-pass return arcs and is accelerated to 18 MeV in 4.5 passes. All spatial dimensions including the orbit distortion are scaled by a factor of 7.5, which arises from scaling the 200 MHz muon RF to the frequency readily available at CEBAF: 1.5 GHz. The footprint of a complete RLA fits in an area of 25 by 7 m. The scheme utilizes only fixed magnetic fields including injection and extraction. The hardware requirements are not very demanding, making it straightforward to implement.

**MUON RLA WITH TWO-PASS ARCS**

A schematic layout of a dog-bone-shaped muon RLA, proposed for future Neutrino Factory [2] is illustrated in the top portion of Fig. 1. Reusing the same linac for multiple (4.5) beam passes provides for a more compact accelerator design and leads to significant cost savings. In the conventional scheme with separate return arcs [4], different energy beams coming out of the linac are separated and directed into appropriate arcs for recirculation. Therefore, each pass through the linac would require a separate fixed-energy arc, increasing the complexity of the RLA. We propose a novel return-arc optics design based on linear combined-function magnets with variable dipole and quadrupole field components, which allows two consecutive passes with very different energies to be transported through the same string of magnets [6].

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SCALED DOWN ELECTRON MODEL

Here, we propose a straightforward test of this concept by scaling the above GeV-scale muon RLA design for electrons. Scaling muon momenta by the muon-to-electron mass ratio (~ 200) yields a scheme, in which a 4.5 MeV electron beam is injected into the middle of a 3 MeV/pass linac with two double-pass return arcs and then is accelerated to 18 MeV in 4.5 passes.

The second scaling, would involve replacing the original low frequency (200 MHz) RF, required to accommodate inherently long muon bunches, with readily available high frequency CEBAF RF (1.5 GHz). All spatial dimensions including the orbit distortion would then scale down by the ratio of the two frequencies (factor of 7.5). As a consequence, the scaled down electron model would fit in a modest test cave of 25 by 7 meters.

For the remainder of this paper, we will describe the principle of multi-pass arc architecture, a possible magnet design and field requirements, as well as a complete conceptual RLA design.

MULTI-PASS ARC OPTICS

Design Concept

The droplet arc design consists of super cells, which are required to satisfy the following basic conditions at two discrete energies (6 and 12 MeV):

- Each super cell exhibits periodic solutions for the orbit and the Twiss functions.
- At the beginning and at the end of each super cell, the periodic orbit offset, dispersion and their slopes are all zero.

The first condition ensures that the super cells bending in the same direction are optically matched while the second one provides optical matching of the cells bending in the opposite directions. The second condition also implies that the beam is centered in the linac and that the linac is dispersion free.

Linear Optics

Optics solution satisfying the above conditions can be obtained using only same-direction bends, which significantly shortens the arc (by almost a factor of 3) compared to the conventional linear NS-FFAG lattice [3], which involves alternating the ‘outward-inward-outward’ bends in the underlying triplet structure. We make the bending angle of each combined function magnet variable with a constraint that the bending angles of all magnets in a super cell must add up to the required fixed total bend. Such a solution combines compactness of the design with all the advantages of a linear NS-FFAG [5], namely, large dynamic aperture and momentum acceptance essential for large-emittance muon beams, and no need for a complicated compensation of non-linear effects, simpler combined-function magnet design with only dipole and quadrupole field components. We use the maximum possible bend of 60° per super cell to accommodate the largest possible number of magnets in the super cell and therefore have the largest number of free parameters for optics tuning. The extra free parameters were used to control the maximum values of the orbit deviation, and beta functions and dispersion. Figure 2 shows solutions for the periodic orbit and dispersion of the outward-bending super cell at 6 and 12 MeV/c, respectively. An inward-bending super cell is identical to the outward-bending cell except that its bends are reversed. The super cell consists of 24 combined function magnets with dipole and quadrupole field components. The magnets are 6.5 cm long and are separated by 3 cm gaps. The total arc length is 16 m. In terms of magnetic field requirements, the maximum needed dipole field is about 650 Gauss while the maximum quadrupole gradient is about 850 Gauss/cm.

Figure 2: 6 MeV (top) and 12 MeV (bottom) periodic orbits and dispersions of the outward bending super cell.

Figure 3: Layout of the 6 and 12 MeV reference orbits.
Figure 3 illustrates geometric layouts of the 6 and 12 MeV closed periodic orbits. Note that because of the varying bending angles, the arc is not perfectly circular. The largest orbit separation occurs only in a small number of magnets and is caused by the necessity to spread/recombine the different momenta orbits at the beginning of the arc.

**ARC TO MULTI-PASS LINAC MATCHING**

As a proof-of-principle, one can design a multi-pass linac with energy gain of 3 MeV per pass, which is matched by-design to previously described two-pass arcs for both passes simultaneously. As described in [4] the above multi-pass linac optics can be accomplished by appropriately ‘tailored’ focusing profile along the linac -the strengths of individual linac’s fixed-field quadrupoles. Here, they are treated as free parameters used to control the beta functions at linac’s ends for all consecutive passes. The proof-of-principle solution [6] was designed by modifying the so-called bi-sected linac profile [4], where the quadrupole strengths increase linearly (in a mirror-symmetric fission) from the linac’s center toward the ends.

**MAGNET DESIGN AND FIELD QUALITY**

*Design Choice*

Each of 7 super-periods required to complete the droplet arc is configured with 24 individual combined function magnets: 6.5 cm in length and with 5 cm of the horizontal aperture. These magnets will be mounted on a rectangular vacuum chamber; 2.3 meter long and shaped into a 60° arc. As for the magnet design, a combined function Panofsky quadrupole with integral dipole windings, similar to Jlab’s FEL design [7], provides a very attractive solution for an independent electrical control of both the magnetic field and its gradient.

Relatively weak magnets, satisfying our strength requirements, can be built ‘flattened’ with no compromise to their field gradient uniformity and have a window frame-like yoke that, at full quadrupole current, is not near saturation [7]. Superposition of a dipole onto the Panofsky quad adds current to one of the four coils and subtracts it from its opposing coil. This is accomplished by adding variable current coils to the vacant corners of the original Panofsky design as illustrated in Fig. 4.

**Field Quality Requirements – Error Sensitivity**

The two-pass arc optics, illustrated in Fig. 2, was checked for error sensitivity. We launched a mini Monte Carlo simulation by creating 25 virtual arc lattices with statistically distributed magnet mis-alignment (200 μm, rms, displacement error) as well as magnet mis-powering (with 10⁻⁴ relative field error, rms) for both the dipole and quadrupole components. Using pairs of horizontal and vertical correctors placed after each magnet the resulting orbit deviation was steered back to the design orbit within 20 μm level, which corresponds to the accuracy of the orbit measurement (BPM accuracy).

**CONCLUSIONS**

We propose a straightforward test of a GeV scale muon RLA by scaling the muon design for electrons (via the muon-to-electron mass ratio). Presented ‘electron model’ features: a 4.5 MeV electron beam injected at the middle of a 3 MeV/pass linac combined with two double-pass return arcs. The beam is accelerated to 18 MeV in 4.5 passes. All spatial dimensions of the full scale RLA are shortened by a factor of 7.5, as a consequence of using a readily available 1.5 GHz CEBAF cavity, rather than the original 200 MHz RF. The footprint of a complete RLA ‘demo’ fits in an area of 25 by 7 m. The scheme utilizes only fixed magnetic fields including injection and extraction. Engineering design and fabrication of linear-field combined-function magnets does not seem to present a challenge [7].

**REFERENCES**