LOSS OF LANDAU DAMPING FOR INDUCTIVE IMPEDANCE IN A DOUBLE RF SYSTEM

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Abstract

In this paper the thresholds of the loss of Landau damping due to the presence of inductive impedance in a single and double harmonic RF systems are determined, both from calculations and particle simulations. A high harmonic RF system, operating in bunch lengthening mode is used in many accelerators with space charge or inductive impedance to reduce the peak line density or stabilize the beam. An analytical approach, based on emerging of the discrete Van Kampen modes, shows that improved stability in a double RF system can be achieved only below some critical value of longitudinal emittance. Above this threshold, a phase shift of more than 15 degrees between the two RF components is proven necessary to stabilize the bunch. These results, confirmed also by particle simulations, now are able to explain observations during the \( \rho \bar{p} \) operation of the SPS. The thresholds in bunch shortening mode as well as in a single RF case are compared with this regime.

MOTIVATION

A double harmonic RF system is used in many accelerators either to change the bunch shape or to increase the synchrotron frequency spread inside the bunch, providing more effective Landau damping of beam instabilities. In most of the cases and especially in low energy machines, where space charge effects are important, the operating mode of the 2 RF systems is selected so that the voltage derivative is zero at the centre of the bunch (bunch lengthening mode, BLM). This flattens the bunches and minimizes the peak line density.

This operating mode (BLM) was also used in the SPS during the time of operation as a \( \rho \bar{p} \) collider, where a 100 MHz RF system was installed in addition to the existing 200 MHz RF system [1]. At that time transverse space charge de-tuning effects together with microwave instability were the main bunch intensity limitations. By operating in BLM it was possible to significantly increase the intensity.

The bunches were injected into the SPS with a nominal longitudinal emittance of \( \varepsilon_{l1} = 0.65 \text{ eVs} \). However, the synchrotron frequency spread introduced by the second harmonic RF component was barely sufficient for stability and any injection errors were un-damped with the bunch oscillating along the injection plateau (26 GeV/c). Furthermore, for larger emittances instability was occurring in the tails of the bunch, which the feedback loops were not able to damp. To counteract this instability a phase shift between the 2 RF systems was introduced [2].

Previous studies of a double RF system already pointed out that in BLM Landau damping can be lost for particles in the region, where the synchrotron frequency distribution has its maximum outside the bunch centre [3]. Recently, an analytical approach made it possible to find this threshold through the onset of a discrete Van Kampen mode (coherent mode without Landau damping) by solving numerically the linearized Vlasov equation [4].

The latter method is used in this paper, together with particle simulations, to explain the observations during the \( \rho \bar{p} \) operation for inductive impedance. Only the dipole modes \( (m = 1) \) are addressed since they are expected to have the lowest threshold and no coupling between different azimuthal modes is considered. The analysis is also expanded to the cases of bunch shortening mode (BSM) and single RF to give a better understanding of the different operating modes.

ANALYTICAL CALCULATIONS

In the semi-analytical approach [4] the steady state problem is solved iteratively, where the phase-space density as a function of action \( F(J) \) and the wake function are defined as an input. Then as a next step, stability is determined by solving the linearized Vlasov equation for a small perturbation \( f(J, \phi, t) \) in \( F(J) \), taking into account the potential well distortion. Using the Oide-Yokoya expansion [5] one ends up with a standard eigen-value problem of linear algebra, which is solved numerically to get the spectrum of the Van Kampen modes.

This spectrum consists of continuous and discrete parts. The continuous spectrum is described by singular eigenfunctions coinciding with the incoherent synchrotron frequencies inside the bunch \( \omega_s(J) \), corresponding to some Landau damping. On the other hand, the discrete modes emerge above a certain intensity \( N_{th} \) and are described by regular functions. By definition these modes lie outside \( \omega_s(J) \) implying that Landau damping is lost.

SIMULATIONS

Numerical simulations presented here were performed using a Matlab code with \( 5 \times 10^5 \) macro-particles. The initial matched distribution was created again iteratively and placed into the RF bucket with a small phase error of \( \phi_0 = 3^\circ \), enough to excite the rigid dipole motion of the bunch. Tracking the particles for \( \sim 300 \) synchrotron periods \( T_s \), was adequate to study the effect of Landau damp-
The criterion used here to estimate the value of the threshold is based on the relative change of the dipole oscillation amplitude $\phi_{\text{max}}$ (envelope of the oscillations in Fig. 1), averaged after 100 $T_s$ (transients). The ratio $\phi_{\text{max}}/\phi_0$ is plotted in Fig. 2 for different emittances for the BLM. The threshold was selected to be 80% (horizontal line) and although being a rather random choice it affects only the absolute values and not the physical interpretation.

Both curves, being in a very good agreement, show that $N_{\text{th}}$ increases with emittance $\varepsilon_l$ until some value of $\sim 0.5$ eVs. After this point further increase in $\varepsilon_l$ leads to threshold reduction. In fact, an inspection of the incoherent synchrotron frequency distribution in Fig. 4 (red curve) shows that the flat region where $\omega_s(J) = 0$ (vertical line) corresponds to the critical bunch size $\varepsilon_{\text{cr}} = 0.65$ eVs.

This result can actually explain the un-damped oscillations at the injection plateau during the $p\bar{p}$ operation, since for the nominal (0.65 eVs) or larger emittances the threshold for the loss of Landau damping is very low. Although

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**RESULTS**

As mentioned above, both calculations and simulations were applied for the case of SPS during the $p\bar{p}$ operation. The 2 RF systems were set up in the BLM with voltage amplitudes $V_{100} = 0.6$ MV and $V_{200} = 0.3$ MV, while for the phase-space density the distribution $F(J) = (J_{\text{lim}} - J)^2$ was used, close to the one fitted from measurements. The intensity thresholds of the loss of Landau damping were defined for different longitudinal emittances. These thresholds $N_{\text{th}}$, found in calculations from the onset of the discrete Van Kampen mode and in simulations from the crossing of the horizontal line in Fig. 2 with the curves of different emittance, are presented in Fig. 3 (red color).

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This result can actually explain the un-damped oscillations at the injection plateau during the $p\bar{p}$ operation, since for the nominal (0.65 eVs) or larger emittances the threshold for the loss of Landau damping is very low. Although
the spread of the $\omega_s (J)$ inside the bunch is still big, the lack of stability in this case is determined by the non-monotonic behavior of the $\omega_s (J)$ in the tails of the bunch.

The effect of the phase shift ($\Delta \phi$) between the 2 RF systems around $\varepsilon_{cr}$ was also studied. The calculated $N_{th}$ are presented in Fig. 5, where one can see that although for small shifts the threshold goes down, after around 15° a dramatic increase takes place, explaining again the cure of the instability which was found empirically during the $\rho\pi$ operation. However, in this case the flatness of the bunches is lost since the potential well is not anymore symmetric.

For completeness, the same studies were also applied for the cases of a single RF and a double RF in the BSM. The results are presented again in Fig. 3, where we can see that for both of them $N_{th}$ keeps increasing with the emittance, as was expected from the monotonic behavior of their $\omega_s (J)$ distributions (Fig. 4). For bunches with $\varepsilon_t < 0.2$ eVs, BLM is the preferable mode, while after this value the threshold of the BSM is rapidly increasing, making this mode a better choice for stability. However, it is clear from Fig. 6 that the BSM is unacceptable above 0.6 eVs due to the lack of longitudinal acceptance, which would lead to significant particle losses. For $\varepsilon_t > 0.6$ eVs single RF seems to be the best option.

Today the SPS is used as the LHC injector, where the beam is captured and accelerated with the 200 MHz RF system. In addition the 4th harmonic RF system (800 MHz) is used in the BSM to stabilize the beam [7]. Since the nominal values of the injected emittances are around 0.35 eVs the bunch size is bigger than $\varepsilon_{cr}$ for the BLM (~0.15 eVs) and thus in this mode, no Landau damping is present. For the 4th harmonic RF system, studies [8] have shown that even in BSM, for high voltage ratio between the two RF components ($\frac{V_{800}}{V_{200}} = 0.25$) there is a region outside the bunch centre, where $\omega_s (J) = 0$, which can lead to strong beam instabilities. Landau damping can be recuperated by shifting the phase between the two RF systems or by reducing the voltage ratio, both leading to a monotonic $\omega_s (J)$ distribution. In fact, a voltage ratio of $\frac{V_{800}}{V_{200}} = 0.1$ is used during cycle in daily operation.

Figure 5: Loss of Landau damping thresholds versus the phase shift between the 2 RF systems found from calculations.

Figure 6: Bucket area versus emittance for a double RF (BLM - red, BSM - blue) and a single RF (black) in the cases corresponding to the $N_{th}$ in Fig. 3. The black straight line is the limit where $\varepsilon_t$ is equal to the bucket area.

CONCLUSIONS

Many accelerators in our days are operating with a double RF system in BLM in order to decrease the peak line density and to increase the synchrotron frequency spread inside the bunch, making Landau damping more effective. However, it was proven here for inductive impedance, both from simulations and calculations, that there is a critical value of the emittance, above which the Landau damping threshold decreases rapidly to zero. A phase shift between the two RF components of more than 15° can help to increase the threshold but the flatness of the bunches is lost. These results are able to explain observations during the $\rho\pi$ operation of the SPS.

Furthermore, this analysis agrees very well with the present situation in the SPS with a 4th harmonic RF, were no use of BLM is possible, while operating in BSM with high voltage ratio between the two RF systems a phase shift is again needed to stabilize the bunch. The current study clearly shows that local maxima of the synchrotron frequency distribution inside the bunch, but outside the bunch centre, can lead to loss of Landau damping.

REFERENCES