INTRODUCTION

The short lifetime of muons demands cooling which is several times shorter than the decay time [3]. Ionisation cooling is generated as the muon beam enters and passes through a low-Z absorber in the MICE beam line, losing energy through ionisation [4]. The beam momentum is reduced in the transverse and the longitudinal direction and the longitudinal momentum is restored by acceleration. Ionisation cooling has been proposed for reducing the phase space volume of an intense muon beam for a NF and a Muon Collider. The momentum distribution of the beam used in ST2 is symmetrical, with a mean momentum of $\bar{p} = 207$ MeV/c and momentum standard deviation of $s_p = 28$ MeV/c, as shown in figure 2.

\[ s = \frac{1}{n} \sum_{i=1}^{n} \left( x_i - \bar{x} \right)^3 \left( \frac{1}{n} \sum_{i=1}^{n} \left( x_i - \bar{x} \right)^2 \right)^{-\frac{3}{2}}, \]  

(2)

where $X$ is the material thickness, $X_0$ is the radiation length of the medium, $\beta$ is the velocity, $\beta_t$ is the betatron function, $E_\mu$ is the muon energy and $m_\mu$ is the muon mass. The negative part gives emittance reduction through energy loss of beam particles and the positive part emittance increase through multiple scattering. It is therefore important that the energy loss is dominant to achieve cooling.

SYMMETRICAL MOMENTUM DISTRIBUTION

The MICE cooling channel is based on Feasibility study 2 (ST2) [1]. It consists of liquid hydrogen absorbers and accelerating cavities in a magnetic field. The muon momentum distribution of the beam used in ST2 is symmetrical, with a mean momentum of $\bar{p} = 207$ MeV/c and momentum standard deviation of $s_p = 28$ MeV/c, as shown in figure 2.

Figure 1: The MICE beamline at step 1.

Figure 2: Momentum distribution of the ST2 muon beam [1].

The D1-scan is performed by holding the magnet strengths of D2 and the two quadrupole triplets Q4-Q6 and Q7-Q9 constant, while varying the strengths of the capture quadrupoles Q1-Q3, D1 and the decay solenoid proportionally to the reference pion momentum $p_{D1}$. The standard 6 mm - 200 MeV/c beam is used, the magnet currents can be found in [6]. The conversion from currents to fields are done by interpolating values from a magnet data sheet, it can be found in the MICE documentation [7]. D2 is set to select muons with momentum $p = 238$ MeV/c.

The skewness

\[ s = \frac{1}{n} \sum_{i=1}^{n} \left( x_i - \bar{x} \right)^3 \left( \frac{1}{n} \sum_{i=1}^{n} \left( x_i - \bar{x} \right)^2 \right)^{-\frac{3}{2}}, \]  

(2)
is used to indicate how symmetrical the momentum distribution is. A perfectly symmetric distribution will have $s = 0$. The distribution is negative skew if the left tail is more pronounced than the right, and positive skew if the opposite [8].

SIMULATIONS

D1-scan simulations have been performed for beams of both signs. The skewness is observed to decrease when the ratio between the pion momentum at D1 and muon momentum at D2 $p_{D1}/p_{D2}$ decrease and at some point become 0, such that the beam becomes symmetrical. The momentum distributions after the decay solenoid for $p_{D1} = 310$ and $p_{D1} = 408$ MeV/c are shown in figure 3 and the pion peaks are where one should expect them to be. To get a muon beam with a low pion contamination $p_{D2} < p_{D1}$ can be set to select the muons that decay backwards in the pion reference frame.

Table 1: Skewness for the positive and the negative beam calculated from the simulations. The positive/negative beam is on the left/right side of the slash.

<table>
<thead>
<tr>
<th>$p_{D1}$ (MeV/c)</th>
<th>s</th>
<th>$300/310$</th>
<th>$325/339$</th>
<th>$375/375$</th>
<th>$408/408$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>s</td>
<td>-0.82/-0.40</td>
<td>-0.13/0.17</td>
<td>0.51/0.45</td>
<td>0.65/0.68</td>
</tr>
</tbody>
</table>

The skewness values vs. $p_{D1}$ for both signs beams are shown in the table. According to these simulations one would expect the beams to be close to symmetrical when $p_{D1} \approx 325$ MeV/c.

Some pion contamination is expected in the beam, according to the simulations the pion contamination is low in the interval $p_{D1} \in [300, 425]$, shown in figure 4. The worst case is approximately 2.7 %, if one selects the most pessimistic value of the error bar from $p_{D1} = 300$.

The muon momentum distribution from MICE data and G4BL at TOF1 are compared in figure 5 and the agreement is fairly good.

SIMULATIONS AND MICE DATA

The statistics from the MICE data is low and it only exists for the negative beam at the moment, these results are therefore only preliminary. Higher statistics runs are expected to be taken and analysed in the near future. However, the statistics should be high enough to give estimates for the beam mean, skewness and standard deviation with reasonable error calculations.

Cuts are applied to eliminate the longest tails of the momentum distributions, a cut on the particle count was used, eliminating 2.5 % of the particles on each tail. Simulations have been normalised to the MICE data by integration. The cuts are especially important for the higher momentum distributions, a cut on the particle count was used, eliminating 2.5 % of the particles on each tail. Simulations have been normalised to the MICE data by integration. The cuts are especially important for the higher momentum distributions, a cut on the particle count was used, eliminating 2.5 % of the particles on each tail. The statistics from the MICE data is low and it only exists for the negative beam at the moment, these results are therefore only preliminary. Higher statistics runs are expected to be taken and analysed in the near future. However, the statistics should be high enough to give estimates for the beam mean, skewness and standard deviation with reasonable error calculations.

When analysing the MICE data the momentum distributions are approximated from the time-of-flight between TOF0 and TOF1, removing all particles in the TOF electron beam is on the left/right side of the slash.
Figure 6: The mean momentum at TOF1 as a function of $p_{D1}$. The ST2 muon beam has $\bar{p} = 207 \text{ MeV/c}$.

peak and assuming the rest of the particles to be muons.

The mean momentum at detector TOF1 lies in the interval $p \in [215, 235] \text{ MeV/c}$ when varying $p_{D1}$, as shown in figure 6.

The standard deviation of the momentum distribution is shown in figure 7 and the maximum distribution width is found at 325 MeV/c for the simulations, and falls when going to higher momenta. For the MICE data the maximum is found at the lowest $p_{D1}$.

Figure 7: The momentum standard deviation vs $p_{D1}$ at TOF1. The ST2 beam has $s_p = 28 \text{ MeV/c}$. The error bars correspond to 95% confidence intervals.

When calculating the skewness of the distribution the cuts are especially important. It is sensitive to outliers and therefore they are eliminated from distribution. The most symmetric distribution is found when $p_{D1} = 330 \text{ MeV/c}$ and for the MICE data the lowest data point has the lowest skewness.

**CONCLUSION**

If one argues that the MICE data can be extrapolated to agree with the simulations, then a symmetrical momentum distribution can be found when $D1$ is set to select pions with $p_{D1} \approx 325 \text{ MeV/c}$, the standard deviation of that beam is $s_p \approx 30 \text{ MeV/c}$. The mean momentum is $\bar{p} \approx 225 \text{ MeV/c}$, but the mean momentum will decrease further between TOF1 and the cooling section, further downstream, where some detector material and a diffuser will be placed.

**ACKNOWLEDGMENT**

We would like to thank Chris Rogers and Yordan Ivanov Karadzhov for all their help and the MICE collaboration for feedback and helpful discussions.

**REFERENCES**


