

SIMULATION OF SELF-MODULATING PARTICLE BEAMS IN PLASMA WAKEFIELD ACCELERATORS*

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Abstract

Controlled self-modulation of long proton or electron beams is a new trend in plasma wakefield acceleration which sets a new goal for simulation codes. Long interaction lengths (tens of meters), long beams (up to hundred of plasma wave periods), motion of plasma ions, and violation of fluid approximation are factors that makes the problem too heavy for general purpose codes. Only specialized codes can attack this problem in real geometry. We describe recent upgrades of the code LCODE which enabled simulations of long dense proton beams and report results of numerical studies of proton beam-plasma interaction in the context of AWAKE project.

INTRODUCTION

Plasma wakefield acceleration driven by proton beams was recently proposed as a possible way to compact lepton accelerators of TeV energy level [1, 2]. This concept relies on strong electric fields possible in plasmas and huge energy contents of available proton beams, three orders of magnitude greater than those of laser and electron drivers. However, proton bunches in synchrotrons are typically tens of centimeters long and therefore cannot directly drive the wakefield. For efficient field excitation, the proton bunch must be transformed into one or many micro-bunches of a sub-millimeter length. The latter can be done by the plasma itself as a result of the self-modulation instability [3]. The experiment at CERN named AWAKE [4] is proposed to test micro-bunching of SPS proton beam and subsequent plasma wakefield acceleration of test electrons.

A numerical simulation of a self-modulating proton beam in the real geometry is a challenging problem. If measured in natural plasma units of the anomalous skin depth $k_p^{-1} = c/\omega_p$, the beams and interaction distances are very long. In AWAKE experiment ($k_p^{-1} = 0.2$ mm), the beam of the length up to $3000 k_p^{-1}$ must propagate $50000 k_p^{-1}$ in the plasma. The energy depletion length for this beam is about $10^6 k_p^{-1}$. For comparison, the record holder electron beam [5] was shorter than $10 k_p^{-1}$ and propagated up to $85000 k_p^{-1}$. At beam populations of interest, motion of plasma ions and breaking of the plasma wave come into play thus taking the problem out of applicability area of efficient fluid codes.

To some extent, the problem is facilitated by the long

time scale of beam evolution, which makes possible to use the quasi-static approximation for the plasma response [6]. For the beam energy $W_b = 400$ GeV, beam length $L_b \sim 20$ cm, and grid step $\Delta\xi = 0.01 k_p^{-1}$, the quasi-static approximation ensures roughly $\sqrt{W_b}/(511 \text{ keV}) \approx 10^3$ advantage in simulation time and requires roughly $L_b/\Delta\xi \sim 10^5$ fewer plasma macro-particles, as compared to general purpose particle-in-cell codes. With this performance, parametric scans are possible in the two-dimensional axisymmetric geometry.

CODE UPGRADES

To meet the new needs, the quasi-static code LCODE [7, 8] was upgraded to provide the required accuracy in calculation of the plasma response. The main source of numerical errors in the old plasma solver was a simple predictor-corrector algorithm. The accuracy was improved by adding one more iteration when calculating the plasma response at the next layer of the simulation window. Now we first move plasma particles from layer a to layer b by fields of the layer a (Fig. 1) and calculate currents in layer b , then calculate fields in layer b , then move plasma particles from layer a to layer b by average fields of layers a and b , then again calculate currents and fields in layer b , then again move plasma particles from layer a to layer b by the average fields. When the fields are calculated second time, the earlier found average radial field is used in equation (12) of [7] as \tilde{E}_r . Also, special efforts are made to suppress a small-scale (of the grid step size) plasma density noise.

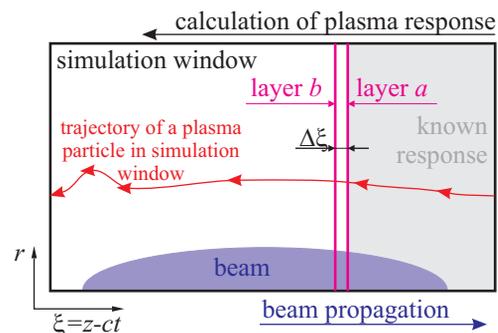


Figure 1: Calculation of plasma response in the quasi-static approximation.

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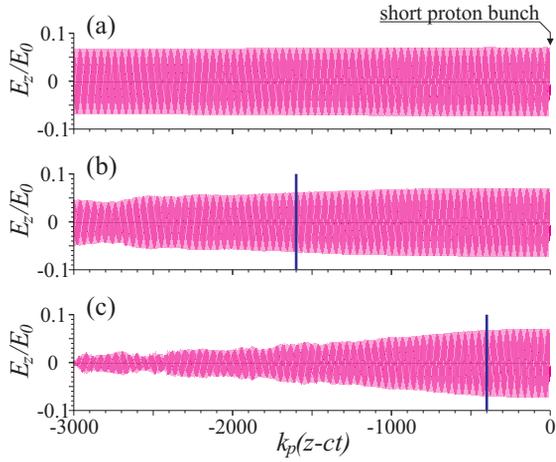


Figure 2: Simulations of the long lasting plasma with various resolutions: $0.01 k_p^{-1}$ (a), $0.025 k_p^{-1}$ (b), $0.05 k_p^{-1}$ (c).

TEST PROBLEMS

To verify applicability of the code to the problem of beam self-modulation, we have simulated two test problems for which the result is known. The first one is a long lasting plasma wave. A small amplitude linear plasma wave is excited by a short rigid proton beam of sizes $\sigma_r = \sigma_z = k_p^{-1}$ in the plasma with the immobile ion background. The wave amplitude is much smaller than the wavebreaking field E_0 and is constant for many periods. The code must reproduce the unchanging wave amplitude at the simulation window $3000 k_p^{-1}$ long [Fig. 2(a)]. The test is passed for rectangular grid of the size $\Delta r = \Delta \xi = 0.01 k_p^{-1}$.

This test also shows the importance of a small grid size for correct simulations of long beams. Commonly used second-order plasma solvers have uncontrollable relative errors of the order of the reverse grid size squared ($\propto \Delta \xi^{-2}$). At the distance $\sim L_x = k_p^{-1}/\Delta \xi^2$ behind the driver, the error of the wakefield period accumulates to make an amplitude-dependent phase shift of the order of unity, which causes nonphysical distortion of wave fronts and wave damping. Distances L_x for low-resolution runs are shown in Fig. 2(b,c) by vertical thick lines. The wave is seen to start damping after these lines.

Table 1: Parameters of test-2 and AWAKE (in parentheses if different) variants

Parameter & notation	Value
Plasma density, n_0	$7 \times 10^{14} \text{ cm}^{-3}$
Ion-to-electron mass ratio, M_i	∞ (157000)
Beam population, N_b	$1.15(3) \times 10^{11}$
Beam length, σ_z	12 cm
Beam radius, σ_r	0.02 cm
Beam energy, W_b	450 (400) GeV
Beam emittance, ϵ	8(9) $\mu\text{m mrad}$

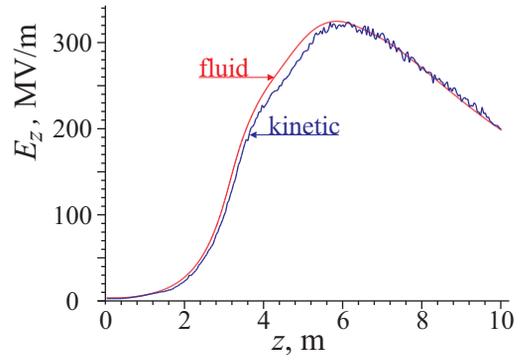


Figure 3: Second test problem simulated by different codes. Dependence of the maximum wakefield amplitude on propagation distance.

The second test is development of the seeded self-modulation instability in the regime accessible by fluid codes. Beam and plasma parameters for this test are listed in Table 1 and correspond to multiple-bunch operation of SPS synchrotron [9]. We assume only the second half of the beam interacts with the plasma created by a short co-propagating laser pulse, while the first half propagates in a neutral gas. The beam as seen by the plasma thus has a sharp leading edge that seeds the instability. Being the result of an unstable process, growth of the wakefield is very sensitive to accuracy of simulations. Fig. 3 shows that results of the new kinetic code are in excellent agreement with the high-resolution fluid LCODE [10], which is essentially different code though with the same name.

SIMULATIONS OF AWAKE EXPERIMENT

We have simulated self-modulation of the proton beam for AWAKE experiment with the newly developed code. The baseline beam parameters for the experiments are given in Table 1. Along with the maximum of the longitudinal field E_z , we also characterize the excited wave by the wakefield potential

$$\Phi(\xi) = k_p^{-1} \int_{\xi}^{\infty} E_z(0, \xi') d\xi', \quad \xi = z - ct \quad (1)$$

which is less noisy. For the sinusoidal linear wakefield, oscillation amplitudes of E_z and Φ are nearly the same. The stronger the wave nonlinearity, the greater the difference between the two. We see that the wakefield amplitude grows as high as 1 GV/m (40% of the wavebreaking field), while the seed wave is only ~ 5 MV/m (Fig. 4a).

To get an idea of how strong is the wakefield at various positions along the beam, we show in Fig. 4b the wakefield amplitude as a function of ξ and propagation distance z . Decay of the wakefield at $|\xi| \gtrsim 25$ cm is due to ion motion. Decay of the wakefield at $z \gtrsim 6$ m is due to beam destruction by the instability, which always takes place in uniform plasmas [11].

Fig. 4c shows the location of the accelerating and focusing fields along the bunch as a function of propagation dis-

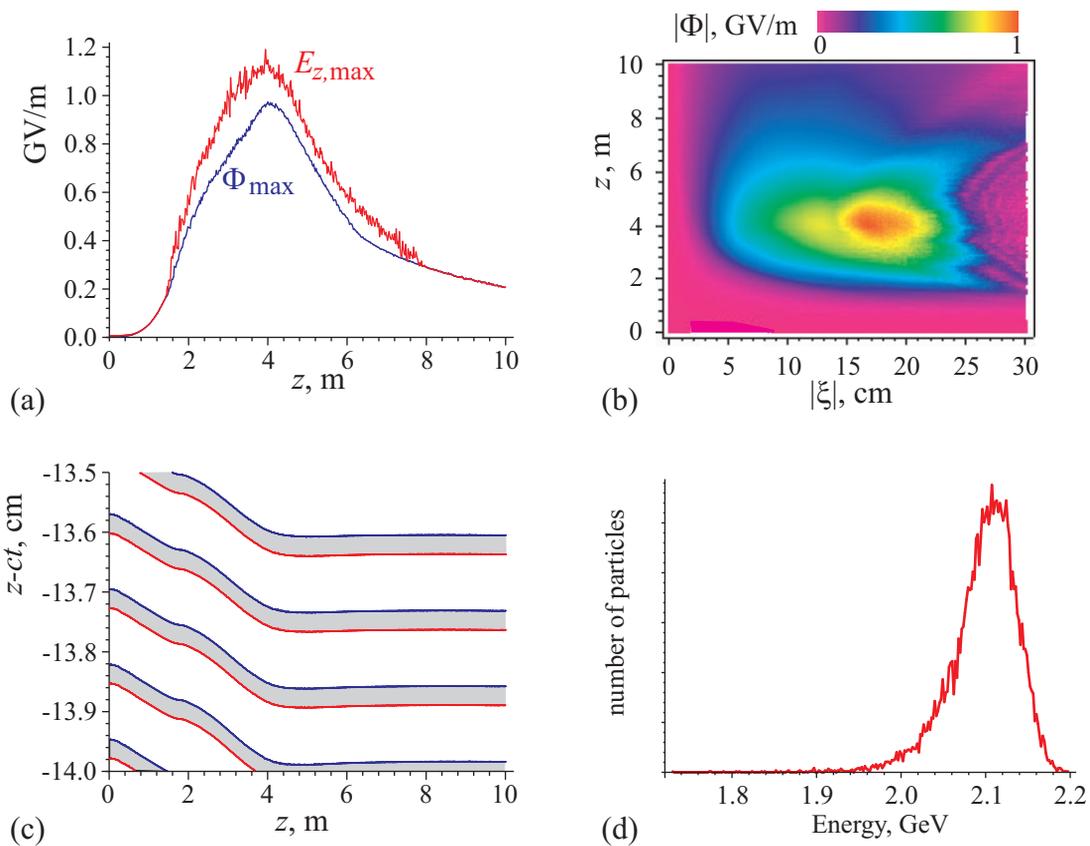


Figure 4: Simulations of AWAKE experiment: maximum values of the on-axis electric field $E_{z,\max}$ and wakefield potential Φ_{\max} versus propagation distance (a), map of the wakefield amplitude $|\Phi(\xi, z)|$ (b), positions along the bunch where the wakefields are both accelerating and focusing for witness electrons (shown in grey) versus propagation distance (c), energy spectrum of accelerated electrons (d).

tance along the plasma. As the instability grows, the interplay between bunch radius and wakefield amplitude leads to an effective wakefield phase velocity slower than that of the drive bunch [12]. Once the instability saturates, these two velocities become equal, and the wave can accelerate electrons. If properly injected at $z \approx 4$ m, test electrons are accelerated to approximately 2 GeV with a narrow energy spread (Fig. 4d).

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