STUDIES OF DENSITY DISTRIBUTION AND EMITTANCE MEASUREMENT FOR HIGH CURRENT ELECTRONIC BEAM

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Abstract

Beam density distribution and emittance are the important parameters of an accelerator. The accurate emittance measurement has an important reference significance for the design of accelerating tube, and provides a design basis for the aperture size of accelerating tube. This paper introduces a beam measurement method which uses multiwire, can rotate in the horizontal plane and adjust in the Z coordinate. The results of simulation show that this method can accurately measure the beam density distribution and emittance, and the accuracy can meet the requirements of applied accelerator.

INTRODUCTION

In the accelerator research, it plays an extremely important role to obtain the electron beam profile information, for the evaluation of accelerator’s characteristics, accelerator debugging and the further improvement of accelerator [1]. Beam emittance is an important parameter of beam quality. Emittance shows the ability of lossless transmission from electron gun or ion source to the experimentation area in beam transmission [2]. For high current accelerator, beam emittance is especially important. Therefore, emittance’s measurement is essential for high current beam [3].

In the past, the measurement of electron gun beam emittance was the paper punch method in the practice of engineering. Two pieces of paper, and same thickness, place in different positions along the beam direction. When the beam penetrates the two pieces of paper, there will be two holes in the papers. According to the two apertures and the distances between two holes and electron gun exit to calculate the beam emittance. This method is rough, and the calculated error is relatively large. This paper introduces a beam measurement method which uses multiwire, can rotate in the horizontal plane and adjust in the Z coordinate. This method can accurately measure the beam density distribution and calculate emittance, and the accuracy can meet the requirements of applied accelerator.

MEASURING METHOD

The multiwire method is based on the improved single wire [4]. Measurement device as shown in Figure 1.

Figure 1: A simple diagram of measurement device.

By the formula (1) could obtain:

$$X_1 = \sum_{i=1}^{n} x_{1,i} \quad Y_1 = \sum_{i=1}^{n} y_{1,i}$$

$$X_2 = \sum_{i=1}^{n} x_{2,i} \quad Y_2 = \sum_{i=1}^{n} y_{2,i}$$

$$\vdots$$

$$X_a = \sum_{i=1}^{n} x_{a,i} \quad Y_a = \sum_{i=1}^{n} y_{a,i}$$

In Figure 1, there are three pairs of X-type measuring wire, A1 and A2, B1 and B2, C1 and C2. Each pair of X-type measuring wire is composed of two carbon filaments perpendicular to each other. The intersections of measuring carbon filaments are not below the beam, in order to avoid the carbon filaments below the intersections are obstructed and make a measurement error. At the same time, ignore the carbon filaments’ diameter, and all carbon filaments are in the same beam cross section plane.

This method uses multiwire, can rotate in the horizontal plane, in order to minimize the error. Using multiwire rotation could measure beam cross section in different angles. According to the measurement data of two vertical carbon filaments to reconstruct the beam density distribution. The reconstructed beam density distribution in the same beam cross section, average them as the beam density distribution. Because the carbon filaments measuring angles are different, the lengths are not the same on the frame. In order to avoid the special angle makes measuring carbon filament too long resulting in long track, which will result in the difficulty of measuring, so we take the rotated measuring device based on the same plane to measure different angles. Our measurement method uses three groups of six carbon filaments simultaneously measure, and can rotate 45°. The range of angle from 0° (reference point) to 175° in an increment of 15°. The angles more than 180° do not need to measure, because measuring carbon filament movement is at the same track, such as 15° and 195°.

BEAM PROFILE RECONSTRUCTION

Characteristic of beam is axisymmetric, beam density distribution could be represented by a matrix $Y$. $Y(x,y) (1 \leq x,y \leq n)$ is the density value of each point. $F_x=[x_1,x_2,...,x_n]$ and $F_y=[y_1,y_2,...,y_n]$ are obtained by sampling two carbon filaments’ measuring data [5].

$$x_1 = \sum_{i=1}^{n} x_{1,i}$$

$$x_2 = \sum_{i=1}^{n} x_{2,i}$$

$$\vdots$$

$$x_a = \sum_{i=1}^{n} x_{a,i}$$

$$y_1 = \sum_{i=1}^{n} y_{1,i}$$

$$y_2 = \sum_{i=1}^{n} y_{2,i}$$

$$\vdots$$

$$y_s = \sum_{i=1}^{n} y_{s,i}$$

By the formula (1) could obtain:
According to the characteristic of beam are axisymmetric and circular distribution. Suppose \( a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n \) are density values that through the beam’s center (matrix’s center) in X and Y directions.

\[
r = \left[ \sqrt{(x - (n+1)/2)^2 + (y - (n+1)/2)^2} \right] \tag{3}
\]

\( r \) is the integer part of the distance between an arbitrary point \((x, y)\) and the beam center point \(((n+1)/2, (n+1)/2)\).

\[
\text{If } r > (n+1)/2, Yx, y = 0.
\]

\[
\text{If } r \leq (n+1)/2, \text{ use the following formulas:
}
\]

\[
Y = \begin{bmatrix}
Y_1 \\
Y_2 \\
\vdots \\
Y_n
\end{bmatrix} = \begin{bmatrix}
\frac{k_{1,0}a_1 + k_{1,1}a_2 + \cdots + k_{1,n}a_n}{\sqrt{1 + \left( \frac{z(x_y)^2}{x_0^2} \right)}} \\
\frac{k_{2,0}a_1 + k_{2,1}a_2 + \cdots + k_{2,n}a_n}{\sqrt{1 + \left( \frac{z(x_y)^2}{x_0^2} \right)}} \\
\vdots \\
\frac{k_{n,0}a_1 + k_{n,1}a_2 + \cdots + k_{n,n}a_n}{\sqrt{1 + \left( \frac{z(x_y)^2}{x_0^2} \right)}}
\end{bmatrix} \tag{5}
\]

\( K \) is the specific value has been calculated. Using similar method to construct matrix \( T \) to represent the beam section density distribution, so as to meet the axisymmetric, circular distribution, \( a_k = b_k \) and use \( q_k \) to express \( b_k \).

\[
T_{x,y} = \begin{bmatrix}
k_{1,0}a_1 + k_{1,1}a_2 + \cdots + k_{1,n}a_n & \cdots \\
\vdots & \ddots & \vdots \\
k_{n,0}a_1 + k_{n,1}a_2 + \cdots + k_{n,n}a_n & \
\end{bmatrix} \tag{6}
\]

Matrix \( H \) satisfies the following formulas:

\[
H = T \times \begin{bmatrix}
(a_1, a_2, \ldots, a_n) \\
(a_1, a_2, \ldots, a_n)
\end{bmatrix} = T \times (1, 1, \ldots, 1) \tag{7}
\]

\[
= Y \times (1, 1, \ldots, 1) \tag{7}
\]

\[
= (x_1, x_2, \ldots, x_n) \tag{7}
\]

Matrix \( H \) as follows:

\[
H = \begin{bmatrix}
0 & k_{1,1} & \cdots & k_{1,n} \\
\vdots & \ddots & \ddots & \vdots \\
k_{n,1} & \cdots & k_{n,n}
\end{bmatrix} \tag{8}
\]

By the formula (7) can obtain. \( (a_1, a_2, \ldots, a_n) = H^{-1} \times (x_1, x_2, \ldots, x_n) \)

Similarly, also can get \( (b_1, b_2, \ldots, b_n) = H^{-1} \times (y_1, y_2, \ldots, y_n) \)

After calculating matrix \( a \) and matrix \( b \) values, we could calculate the beam density distribution by the formula (5).

**CALCULATION OF BEAM EMITTANCE**

For the calculation of beam emittance, we need to measure and calculate beam density distribution on three different positions \((z_1, z_2, z_3)\) along with the transmission direction of beam. After getting the beam density distribution, can draw the contour of the beam density distribution through computer, and determine the beam envelope boundary. From the three beam section density distributions we can determine three beam envelope boundaries. The longest distances of each boundary to the respective beam centers were recorded as \( x_{p1}, x_{p2}, x_{p3} \).

![Figure 2: Drift space standard beam and A, B phase space.](image)

In the Figure 2, the left side expresses beam exit from accelerator, the right side says A and B phase spaces, the long axis of the oblique ellipse does not come from the long axis of the ellipse. \( x_p \) is known as the elliptic half width, with the change of \( Z[6] \).

\[
x_p = x_0 \sqrt{1 + \left( \frac{z(x_y)^2}{x_0^2} \right)} \tag{11}
\]

\( x_0 \) and \( x_0' \) are the maximum displacement and divergence in the beam waist position, and phase space is ellipse in the beam waist position.

If the distances between three measuring sections \((z_1, z_2, z_3)\) and the beam waist position are as follow: \( z, z+a, z+a+b \). By the formula (11) can obtain.

\[
x_{p1} = x_0 \sqrt{1 + \left( \frac{(z+a)^2(z_y)^2}{x_0^4} \right)} \tag{12}
\]

\[
x_{p2} = x_0 \sqrt{1 + \left( \frac{(z + a + b)^2(z_y)^2}{x_0^4} \right)} \tag{12}
\]

\( x_0, x_0' \) are the half shafts of phase space ellipse in the beam waist position. Through calculating the beam emittance can be obtained, \( \epsilon = x_0x_0' \).

**SIMULATION ANALYSIS**

The accelerator electron beam condition is simulated by E-gun [7]. Simulation on the dynamitron 5MV accelerator gun beam extraction and track in the first tube. As shown in the Figure 3.
In order to make electron beam of uniform field flows into the drift tube, so the electrode voltage values are set to the same from the fifth electrode. Electron beam trajectory simulation on two accelerating tubes, each section of the accelerating tube with 14 electrodes, the electron beam trajectory as follow figures.

As could be seen from the figures, the beam with the increment of the propagation distance, will spread and become larger. We can obtain three beam status information from Figure 5, Figure 6, Figure 7 output files.

In the output files, we can get the distance from beam envelope boundary to the center, record as $R$. From the three output files, find out three corresponding $R_{\text{max}}$. $R_{\text{max}}$ instead of $x_{p1}$ in Figure 5, $R_{\text{max}}$ instead of $x_{p2}$ in Figure 6, $R_{\text{max}}$ instead of $x_{p3}$ in Figure 7, $a=b=100\text{mm}$. Through the formula (12), we can calculate $x_{0}$, $x_{0}'$, and the beam emittance be obtained. $\epsilon = x_{0}x_{0}'$.

**DISCUSSION**

Simulation result shows that this measurement method is feasible, we can build a measurement system. The system can not only measure the density distribution of beam cross-section, the accuracy of measurement is more precise than the double wire measurement system, but also through the measured data to calculate the beam emittance. The measurement system structure is simple, precision can meet the Application Accelerator requirements. The system is applied widely, can be used to measure Industrial Application Accelerator gun exit beam, such as the 5MV accelerator gun.

**REFERENCES**