SPECTRA OF COHERENT SMITH-PURCELL RADIATION OBSERVED FROM SHORT ELECTRON BUNCHES: NUMERICAL AND EXPERIMENTAL STUDIES

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Abstract

To stimulate and support the progress in areas of compact particle accelerators, within research fields including X-ray and THz (T-ray) sources of radiation, non-invasive, electron beam diagnostics that are capable of measuring a single femtosecond electron bunch are required. At the current stage such beam diagnostics for femtosecond-long electron bunches are still not available. The goal of the work presented is to understand the spectral characteristics of coherent Smith-Purcell radiation to enable its quick and reliable interpretation including the longitudinal profile reconstruction of electron bunches. The research presented comprises results from numerical modelling and experimental studies. We use a PiC numerical model to analyse the dependence of radiated spectra dependence on the electron bunch profile, and discuss our results in the context of new experimental data from the E203 experiment at SLAC.

INTRODUCTION

The recent advances in conventional and next-generation (plasma and dielectric wakefield) accelerators as well as beam requirements to build next generation of free electron lasers and compact sources of coherent radiation, have been driving the development of high performance beam diagnostic methods [1-5]. One of the promising methods is based on analysis of coherent Smith-Purcell radiation (cSPr), which occurs when a bunch of charged particles moves in a vicinity of a periodic structure (grating) [1,5-7]. It can be shown that in the far field region the wavelength of the emitted radiation detected is a function of the angle \( \theta \) between the grating and the electron beam and it is defined by the formula:

\[
\lambda = \frac{l}{m \beta - \cos \theta}
\]

where \( l \) is the period of the grating along \( z \) (fig.1), \( m \) is the integer, \( \beta \) is the electron beam relative velocity with respect to the speed of light \( c \), \( \theta \) is the angle in \( xz \) plane (fig.1) with \( \theta = \pi/2 \) corresponding to a normal direction toward the grating. The schematic illustration of this dependence as well as grating and the electron beam propagating above the grating are shown in figure 1.

To understand the characteristics of the cSPr surface current model in conjunction with electric-field integral equation method [5,6] was used. We also used 3D PiC code Vorpal [7,8] to simulate the phenomena and to verify the theory developed. As a beam diagnostic technique, the cSPr has the following assets: (1) non-invasive (no beam interception is required); (2) the broadest frequency spectrum measurement \textit{in situ} as compared with the other technique; (3) small footprint and relatively inexpensive. In conjunction with reconstruction techniques such as Kramers–Kronig (KK) algorithm, it allows measuring (RMS bunch length, etc.) and time profile reconstruction of femtosecond electron bunches [1].

In the theoretical frame of the surface-current model, we can calculate the angular distribution of the power radiated by one electron:

\[
\frac{dI}{d\Omega}_{sp} = \frac{2\pi q^2 Z n^2 \beta^4}{e^2 (1 - \beta \cos \theta)^2} R^2 \exp\left(\frac{-2 x_0}{\lambda c}\right)
\]

where \( x_0 \) is the distance of the detector from the grating, \( \lambda \) is the wavelength of the emitted radiation, \( R \) is the radius of the beam, and \( n \) is the refractive index of the medium.

Figure 1. Schematic of electron beam propagating above the grating and excitation of Smith-Purcell radiation. Expression 1 is illustrated by showing the dependence of radiation wavelength on the observation angle.
where \( \lambda_e \) is the an effective coupling parameter between electron beam and electromagnetic (EM) field (known as evanescent wavelength and given by the expression [4])

\[
\lambda_e = \left( \frac{\lambda}{2\pi} \right) \beta_0 \sqrt{1 + \beta_0^2 \gamma^2 \sin^2 \theta \sin^2 \phi}
\]

(3)

For a bunch of \( N_e \) electrons, the power radiated will be:

\[
\left( \frac{dt}{d\Omega d\omega} \right) = \left( \frac{dt}{d\Omega d\omega} \right)_{sp} \left[ N_e + N_e (N_e - 1) |F(\omega)|^2 \right]
\]

(4)

where \( F \) is the form factor of the bunch [1,5].

The aim of the studies is to enable measurement of the form factor and thus determination and reconstruction of the bunch profile.

THE E203 EXPERIMENT AT FACET SLAC

The E203 experiment is dedicated to the development of single pulse diagnostic tool based on cSPr and capable of electron beam studies at the femtosecond timescale. The experiment description can be found in [1] and to reconstruct the bunch profile KK method is used. It is important to say that in the experiments conducted it was possible to measure the radiation in the wavelength range from \( \sim 20 \mu m \) (from 50\( \mu m \) grating) to \( \sim 2000 \mu m \) (from 1500\( \mu m \) grating) and to our best knowledge this is the widest frequency range covered by this type of diagnostics devices. This is important as for bunch profile reconstruction a reverse Fourier Transformation should be performed with integration over the full spectral range leading to necessary extrapolations to zero and infinity and interpolations between the measured points. As a result, a broadest measured frequency spectrum gives a more accurate time-profile. Also the radiation measured includes a non-negligible amount of background, which must be taken into account and subtracted before the pulse reconstruction. This is experimentally done by using a blank grating with the signal measured subtracted later from the signal observed from the grating. The aim of the paper is to discuss the property of the cSPr spectrum as compared with the numerical predictions observed using PiC code Vorpal. This should lead to development of better algorithm of measured spectrum extrapolations and “removal” of possible sources of radiation which can distort the signal from cSPr.

In figure 2 the preliminary result of electron bunch profile measured at FACET and reconstructed is shown. An analysis carried at high compression yields \( t_{rms}=2\times10^{-26} \) fs.

PARTICLE-IN-CELL SIMULATION

To study the properties of cSPr observed the numerical modelling was conducted using 3D PiC code Vorpal [8]. The code uses FDTD algorithm allowing solving time-dependent Maxwell’s equations using advanced processor parallelisation method which take advantage of multicore system. A typical cSPr simulation takes about two to three hours using parallelized 64 cores, depending on the duration and size of the grating chosen. In spite the use of multiprocessor technique it is still impossible to simulate fully the E203 experiment. In the model presented, we have set up a rectangular box of 20 mm over 10 mm within which the cSPr phenomena was investigated. To observe cSPr generation a Gaussian electron bunch of 300 microns (FWHM) with parameters close to one observed at FACET experiment was launched above (at 1mm distance) a grating of 500 microns period. The main difficulties were associated with modelling of EM field generation in a broad range of wavelength (from 2000 \( \mu m \) to 20 \( \mu m \)). To describe it, mesh steps of 5 \( \mu m \) were used. This limited the overall size of the simulated volume as well as duration of the process, always less than 1ns. For instance it took 4 hour modelling using 512 cores to simulate 1ns evolution of such system producing at the end 90 GB of history data. The numerical modelling has the advantage over experiment of providing us with the fields at any time and position, so we do not need to be in the far field zone to gain full knowledge of the radiated field.

Results of the Numerical Modelling

In figure 3a the contour plot of the electric field generated by electron bunch having initial Gaussian profile and propagating in the vicinity of the grating is shown. The grating is shown at the bottom while the concentric lines illustrate the EM field wave-fronts. Figure 3b shows the temporal dependence of the electric field measured at the point (1) located 2mm above the grating (the beam propagates 1mm above the grating). We note that the spectrum radiated by the electron bunch is practically invariant with respect to the position of the
observation point and the spectra measured at points (1), (2) and (3) were practically identical. The differences measured were attributed to relatively short time of observation (below 1ns).

In figure 3b two different phenomena can be seen: 1/ coherent transition radiation which manifests itself by the large spike at the start and 2/ coherent SP radiation with very specific feature, namely, frequency chirped oscillations (starting with high and finishing with low frequency). At the end one can see a small spike which is also associated with coherent transition radiation. The spectrum of the radiation measured in numerical model is shown in figure 4. To observe it the data from Vorpal was analysed using MATLAB FFT routine. It was expected that the spectrum generated by the Gaussian bunch should be close to Gaussian. The differences and “noise” came from the fact that the bunch had a truncated form (not pure Gaussian), the observation time was limited and electrons were modelled by a number of macro-particles resulting in higher “noise level” at high frequencies. The insert to the figure 4 is the experimental data observed at FACET, SLAC. The good agreement between numerical predictions and experimental data can be easily seen.

CONCLUSION

In this work we presented the first 3D numerical model of an electron bunch detector based on coherent Smith-Purcell radiation, capable of profiling femtosecond long electron bunches, using the PiC code Vorpal. The preliminary results of numerical modelling are presented and compared with experimental data. We discussed the challenges associated with modelling of such a device and show specific solutions. It was also shown that coherent transition radiation (due to the edge of the grating) may significantly affect the overall measured signal and the way to remove such contribution will be measuring signal from blank target and then subtracting it from the signal measured from the grating. In this work we illustrated that even at this preliminary stage a good agreement between numerical prediction and experimental data was observed. Further simulations are planned that will use different algorithms to allow high resolutions to be conducted faster and at a higher resolution than was possible in this work.

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REFERENCES


Figure 3. (a) Contour plot of the EM fields generated by 300fs electron bunch propagating above (1mm) of the 0.5mm period grating. (b) The electric field time dependence measured at the detector position as shown in figure 3a.

Figure 4. The spectrum of cSPr generated by near Gaussian electron bunch propagating above the 0.5mm period grating.