STUDY OF GEOMETRY DEPENDENT MULTIPACTING OF A SUPERCONDUCTING QWR*

ZHOU Kui#, LU Xiang-Yang, YANG Liu, YAO Zhong-Yuan, QUAN Sheng-Wen, LUO Xing
State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing, China

Abstract
A superconducting quarter wave resonator (QWR) of frequency=162.5 MHz and β=0.085 has been designed at Peking University. This paper focus on the multipacting (MP) study for the QWR with CST Particle Studio. The simulation results for the initial designed model reveal that there is no sign of MP with its normal operating accelerating gradients in the range of 6-8 MV/m. The accelerating gradient range that may incur MP is from about 1.4 MV/m to 3.2 MV/m, and the places where MP may be encountered are mainly located at the top part of the QWR. So the effect of different top geometries on MP has also been studied in depth. Our results show that inward convex round roof is better than other round roofs, and plane roofs have an obvious advantage over round roofs on the suppression of MP in general. While considering the optimization of its electromagnetic (EM) design, our initial designed model is also acceptable.

INTRODUCTION
Multipacting (MP) is a resonant discharge process in which an electron avalanche builds up via secondary emission driven by radio-frequency (RF) field [1]. When MP effect occurs, these multiplied electrons can cause several severe problems, such as deteriorating the vacuum, absorbing incident power, preventing the increase of accelerating gradient, leading to quenching the cavity, even damaging RF devices. MP effect is an inevitable issue when a superconducting RF cavity is designed, especially for low β superconducting cavities, such as quarter wave resonator (QWR), half wave resonator (HWR), or spoke resonator.

A superconducting QWR of frequency=162.5 MHz and β =0.085 to accelerate high current proton beam has been designed [2]. Its electromagnetic (EM) design and optimization have already been finished [3]. The current paper focuses on the MP study of the QWR. First, the initial designed QWR model will be checked. The accelerating gradient range and the location, where MP may occur, are to be found out. Then, we change the shapes of the QWR where MP may occur, and explore the effect of different geometries on MP. The following sections will present more details.

MODEL SETUP
The QWR model is based on the optimized results of its EM design [3]. Its normal operating accelerating gradient range is from 6 MV/m to 8 MV/m. The model consists of two parts, the inner component being a vacuum chamber and the outer component being a 2.8 mm cavity wall made of niobium after 300 °C bake (Fig 1. a). The vacuum part is used for calculating the EM field and the trajectories of the electrons, while the cavity wall is the area generating initial electrons and the boundary of the electron motion.

The module of CST Particle Studio can calculate the EM field distribution in eigenmode solver, as well as import external EM field files from CST Microwave Studio [4]. We choose the latter method, which is more powerful and efficient. Firstly, the “vacuum” model is imported into CST Microwave Studio for field calculation. Then its field files are imported into CST Particle Studio for MP simulation. There are five regions considered to be the potential areas where MP may occur (Fig 1. d).

MP SIMULATION
The secondary emission model used in CST Particle Studio is based on the Furman probabilistic model [5]. The particle sources provide the primary electrons uniformly distributed over the 5 regions. Their energies are set to be uniformly distributed from 0 to 4 eV and their initial emission angles are set to be randomly distributed from 0° to 180°. The number of primary electrons per region ranges from 4000 to 5000. For each region, since all the primary electrons are launched simultaneously during the same RF period, we need to check different initial phases and find the most noteworthy phase of MP.

Two conditions need to be fulfilled to give rise to MP. One is the secondary emission yield greater than 1, which is mainly determined by proper material, treatment of the surface, appropriate incident energy and incident angle of the primary electron. The other one is the relatively stable trajectory, which is mainly affected by the initial phase of primary electrons, appropriate EM field distribution and appropriate EM field intensity. However, for a given cavity and fabrication material, the factors mainly influencing MP remain only two: the initial phase of primary electrons and the EM field intensity.

For Region 1, we set the $E_{ac}=2$ MV/m and scan...
different initial phases from 0° to 360° and find out the most noteworthy initial phase, about 120°. Then the initial phase is fixed and we change the value of \(E_{acc}\) from 1 MV/m to 10 MV/m, checking if there exists MP under different EM field intensities. MP can be found in the accelerating gradient range from 1 MV/m to 3 MV/m, while there is no MP in the accelerating gradient range from 4 MV/m to 10 MV/m. A more detailed gradient scan shows that the accelerating gradient range where MP may occur is from about 1.4 MV/m to 3 MV/m and at \(E_{acc}\approx 1.8\) MV/m, MP is manifested dramatically.

For Region 2 and 3, the same methods are adopted and the results are similar except that the noteworthy initial phases change. The accelerating gradient range where MP may occur is from about 1.4 MV/m to 3.2 MV/m. For Region 4, MP between the inner conductor and the outer wall can be spotted only when accelerating gradient drops to 0.1 MV/m. For Region 5, the results show that it is very difficult to form stable trajectories at the bottom of the QWR. In a word, the most sensitive place that may incur MP is located at the top part of the QWR.

**FURTHER STUDY**

In order to have a better understanding of the effect that geometries of the top part have on MP, we compare another three QWRs with different round roofs and four QWRs with different plane roofs to our initial designed model (Fig. 2). Their geometrical parameters are listed in Table 1. The differences on the top part will cause some changes in the resonant frequency, which is compensated by adjusting their cavity heights.

![Figure 2: The cross sections of different roofs of QWRs.](image)

(a is the initial designed roof, b is the inward convex round roof, c is the outward convex round roof, d is the symmetrical round roof, e is the inward plane roof 1, f is the inward plane roof 2, g is the outward plane roof 1 and h is the outward plane roof 2. R1 is the curvature radius of the outer blend edge of round roof, and R2 is the curvature radius of the inner blend edge of round roof. R is the curvature radius of plane roof, W is the width of plane roof)

<table>
<thead>
<tr>
<th>Model</th>
<th>R1 or R (mm)</th>
<th>R2 or W (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial designed model</td>
<td>35.5</td>
<td>11.5</td>
</tr>
<tr>
<td>Inward convex round roof</td>
<td>39.5</td>
<td>7.5</td>
</tr>
<tr>
<td>Outward convex round roof</td>
<td>13.5</td>
<td>33.5</td>
</tr>
</tbody>
</table>

**Table 1: Geometrical parameters of different roofs**

EM field accuracy is a vital issue to obtain good simulation results. Tetrahedral mesh is adopted to calculate their EM fields. Compared with hexahedral mesh, tetrahedral mesh division is based on finite element analysis, which can get very good precision with much fewer mesh cells. About 10 thousand tetrahedral mesh cells are set for EM field calculation. EM field files are separately imported into CST PARTICLE STUDIO for MP simulation. In each case, we get a MP curve, which shows the total number of electrons versus time and export its plot data for further data analysis. By exponential curve fitting, we can get their growth rate at each accelerating gradient.

![Figure 3: Growth rate vs \(E_{acc}\) curves of different top geometry models.](image)

The left map is the result of round roofs and the right map is the result of plane roofs. The vertical axis is their growth rate, which means the average multiplication factor of each collision. That is to say, if one electron impacts on the cavity wall under certain accelerating gradient, the growth rate refers to the number of electrons emitted from the surface of the wall in average. So there is MP, if the growth rate is above 1.

For the round roofs, relative to the symmetrical round roof and outward convex round roof, the inward convex round roof occupies lower and narrower acceleration gradient range that may incur MP. When decreasing R1 and increasing R2, the corresponding acceleration gradient range moves leftward on the growth rate vs \(E_{acc}\) map, leaving away from its normal operating accelerating gradient range. As for the symmetrical roof and outward convex roof, their accelerating gradients that may incur MP start from about 1.6 MV/m and can reach up to 5 MV/m. MP may be a quite severe problem if designed like that.

For the plane roofs, MP may occur in three of them. There is no sign of MP for outward plane roof 2. As for the other three roofs, the corresponding accelerating gradient ranges that may incur MP are much lower and...
narrower than the round roofs. They are all below 2 MV/m and last less than 1 MV/m.

The reason of the differences between the round roofs and the plane roofs can only be explained qualitatively now. For the round roofs, the specific locations of MP electrons are close to the connection point of the two filleted corners. The electrons are almost symmetrically distributed at the two sides of that point. It is quite easier for the electrons to satisfy the resonance condition on a surface with relatively symmetrical smooth transition. However, for the plane roofs, the smooth transition is replaced by abrupt right angle. The relatively asymmetrical roof in geometry makes it more difficult to satisfy resonance condition. So the plane roofs can suppress MP more effectively than the round roofs for QWR.

CONCLUSION

In general, according to the simulation results, there is no sign of MP during the normal operating accelerating gradient range from 6 MV/m to 8 MV/m for this particular superconducting QWR. However, MP trap may exist in the accelerating gradient range from about 1.4 MV/m to 3.2 MV/m and the places where MP may occur are mainly located at the top part of the QWR.

The effect of different top geometries on MP has also been studied in depth. The MP cases of several QWRs with different round roofs and plane roofs are compared. Simulation results show that inward convex round roof is better than other round roofs, and plane roofs have an obvious advantage over round roofs on the suppression of MP. While considering the optimization of its EM design, our initial designed model is also acceptable. This study may provide a useful reference on the suppression of MP for the latter QWR design.

REFERENCES