PROGRESS ON MUON PARAMETRIC-RESONANCE IONIZATION COOLING CHANNEL DEVELOPMENT*

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Abstract

Parametric-resonance Ionization Cooling (PIC) is intended as the final 6D cooling stage of a high-luminosity muon collider. To implement PIC, a continuous-field twin-helix magnetic channel with a correlated behavior of the horizontal and vertical betatron motions and dispersion was developed. A 6D cooling with stochastic effects off is demonstrated in a GEANT4/ G4beamline model of a system where wedge-shaped Be absorbers are placed at the appropriate dispersion points in the twin-helix channel and are followed by short rf cavities. To proceed to cooling simulations with stochastics on, compensation of the beam aberrations from one absorber to another is required. Initial results on aberration compensation using a set of various-order continuous multipole fields are presented. As another avenue to mitigate the aberration effect, we optimize the cooling channel's period length. We observe a parasitic parametric resonance naturally occurring in the channel's horizontal plane due to the periodic beam energy modulation caused by the absorbers and rf. We discuss options for compensating this resonance and/or properly combining it with the induced half-integer parametric resonance needed for PIC.

INTRODUCTION

Combining muon ionization cooling with parametric resonant dynamics should allow much smaller final transverse muon beam sizes than conventional ionization cooling alone [1, 2]. In the PIC concept, a half-integer parametric resonance is induced in a muon cooling channel. The beam is then naturally focused with the period of the channel's free oscillations. The horizontal and vertical betatron periods must be correlated in such a channel so that, at certain locations, focusing occurs in both planes simultaneously. Absorber plates for ionization cooling are placed at the focal points and are followed by energy-restoring RF cavities. At the absorbers, ionization cooling limits the angular spread while the parametric resonance causes a strong reduction of the beam spot size. The final equilibrium transverse emittances in this scheme should be at least an order of magnitude smaller than those achievable with conventional ionization cooling [2]. An emittance exchange to maintain the longitudinal emittance is introduced by tapering the absorber and having a proper dispersion at its location.

SIMULATIONS IN A TWIN HELIX

A continuous-field twin-helix magnetic channel was proposed for the implementation of PIC [3-8]. The channel is a combination of two helical dipole harmonics pitched in opposite directions but otherwise identical. Additionally, a continuous straight quadrupole is superimposed on top of the helical fields [3]. Such a system is adjusted to meet the correlated optics requirements. Half-integer parametric resonances are induced in both planes using a pair of opposite-helicity but otherwise matching continuous helical quadrupole harmonics per plane [7]. Locations of the focal points are set by the phases of the helical quadrupole pairs.

Figure 1 shows 250 MeV/c muon tracks in a twin-helix channel with 1 m helix period. 2 cm thick Be absorbers with 0.3 thickness gradient are placed every two helix periods where the beam can be simultaneously focused in both the horizontal and vertical planes. The absorber location within a period is chosen at a point with 3 cm dispersion providing the necessary emittance exchange.

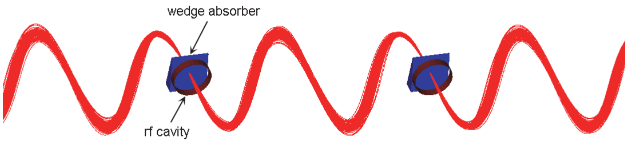

Figure 1: 250 MeV/c muon tracks in a 1 m period twin helix with wedge absorbers and rf cavities.

We used the system shown in Fig. 1 to demonstrate 6D cooling without parametric resonances and with stochastic effects switched off. The simulation was done using a GEANT4-based G4beamline code [9]. The obtained evolution of the three 2D emittances along the channel is shown in Fig. 2. We intentionally chose relatively small initial emittance values to stay close to the linear regime to be able to compare this result to the first-order simulations in [8]. The initial emittance oscillations are caused by a phase-space mismatch because the initial bunch was generated using independent Gaussian distributions for the 6D phase-space coordinates without taking into account proper correlations between them. The later oscillations are probably due to a finite number (only 1000) of particles in the bunch. Since the stochastic effects are off, the emittances cool virtually to zero. This confirms that our basic system is setup correctly.

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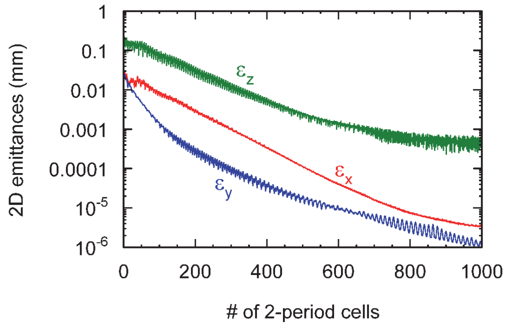

Figure 2: Horizontal ε_x, vertical ε_y, and longitudinal ε_z emittances along the cooling channel plotted vs. the number of 2-period cells (equal to the number of passed absorbers).

To proceed to cooling simulations with stochastics on, compensation of beam aberrations is required. This was verified by running a first-order simulation with stochastics on [8]. One approach to aberration compensation [7] is the following. Since in PIC regime the beam has a small size and a large angular spread at the absorber, a set of particles with systematically-arranged initial angles is started from a focal point on the reference trajectory and tracked to the next focal point to determine the aberration-induced beam smear at that location. A general optimization procedure is used to minimize the beam smear by introducing various-order continuous multipole fields.

Figure 3 shows an example of such optimization. The particles' initial angles were distributed on a grid of azimuthal angles from 0 to 2π in π/4 steps and polar angles from 20 to 220 mrad in 40 mrad steps. Straight sextupole and octupole harmonics as well as helical quadrupole and decapole pairs were added to the system. The aberrations were minimized by varying the strengths of the straight field components and both strengths and phases of the helical harmonic pairs.

A more systematic approach to aberration compensation is to use COSY Infinity [10], a matrix-based code, which works by expanding a particle's trajectory around a reference orbit to an arbitrary order in the 6D phase-space coordinates. Therefore, it can be used to unfold and analyze individual aberrations [5].

Another avenue to mitigate the aberration effect is to optimize the cooling channel's period length. The helix period is an important parameter for a number of reasons. From the engineering point of view, it determines the channel's total length and sets the requirements on the magnet and rf cavity parameters. From the performance point of view, it determines the cooling rate per unit length and muon loss due to decay. It also determines the channel's optical properties, such as its focusing strength, periodic orbit amplitude, and values of the beta functions and dispersion.

The helix period was optimized by minimizing the particle loss after an initially zero-emittance muon beam passed through a large number of 1 mm absorbers and short rf cavities in a twin-helix channel. For long helix periods, the particles were getting lost due to large absolute aberration sizes. For short helix periods, the loss was apparently due to reduction of the dynamic aperture. A balance between these effects seemed to be established for a helix period of about 20 cm, which is also reasonable from a practical point of view.

We also determined that the main mechanism for the particle loss is apparently the longitudinal dynamics. Figure 4 shows the beam's longitudinal phase space after 900 absorbers. Shown in red are the particles that are lost after 100 more absorbers. There is a clear loss pattern in the longitudinal phase space, which is not observed in the other dimensions.

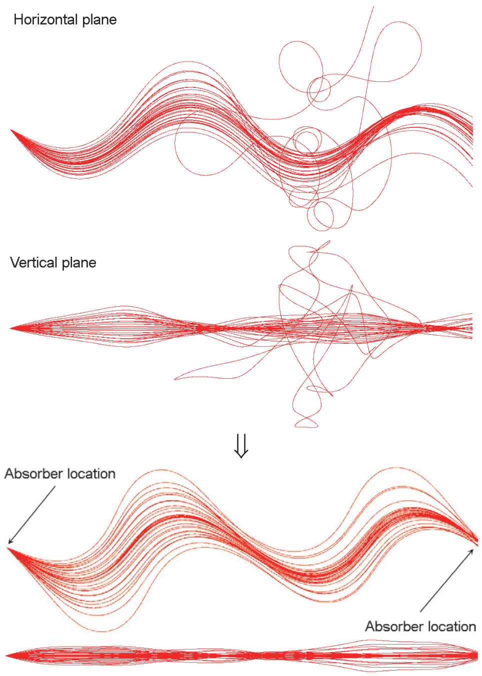

Figure 3: 250 MeV/c muon tracks from one focal point to the next before (top) and after (bottom) aberration compensation using field harmonics up to decapole.

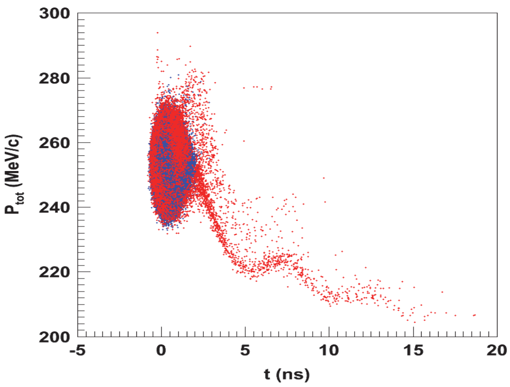

Figure 4: Longitudinal phase space after an initially zero-emittance bunch passes through 900 thin absorbers. Shown in red are the particles that are lost after 100 more absorbers.

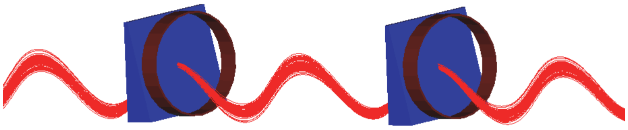

Figure 5: Muon tracks in a 0.2 m period twin helix.

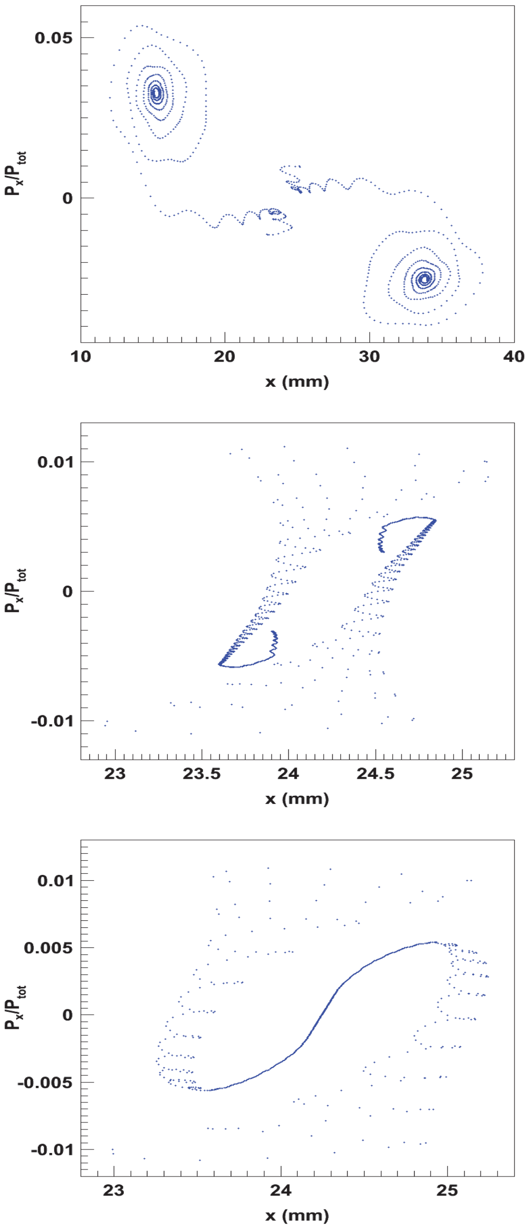

Figure 6: Horizontal phase space trajectory of a single muon at the absorber locations in a 0.2 m period twin helix without induced parametric resonances (top), with horizontal and vertical resonances induced with the nominal phases (middle), and with the phase of the induced horizontal resonance shifted by 200 mrad (bottom).

To address this issue, we introduced more longitudinal cooling. We again used high thickness-gradient 2 cm absorbers as in Fig. 1. Since the dispersion scales together with the helix period, we accounted for its reduction in the now 20 cm period twin helix by moving the absorbers to the maximum dispersion points corresponding to the points of maximum reference orbit offset as illustrated in Fig. 5.

Tracking in the shorter-period system showed a strong parasitic parametric resonance [6]. The resonance is excited in the horizontal plane due to the periodic beam energy modulation caused by the absorbers and rf, which happens at twice the frequency of the horizontal betatron oscillations. Its effect is more pronounced for shorter helix periods because relative impact of the absorbers and rf cavities is greater. This effect is now well understood. Symmetric positioning of the rf cavities should help correct the parasitic resonance. One can also account for the parasitic resonance by adjusting the phase of the induced horizontal resonance so that the net effect of the two resonances is what is needed for PIC.

Figure 6 (top) shows a muon's horizontal phase space trajectory at the absorber locations in a system without induced parametric resonances. The two stability islands in the phase space are an indication of a half-integer resonance. Figure 6 (middle) is the phase space trajectory of the same particle when horizontal and vertical parametric resonances are induced in the twin helix with their nominal phases. The parasitic and induced horizontal resonances interfere together resulting in a change of the stability island locations; however, their orientation is not suitable for PIC. Adjusting the phase of the induced horizontal resonance by 200 mrad gives the picture shown in Fig. 6 (bottom). The particle cools to a single point in the phase space. These results are preliminary but they demonstrate the general approach to treating the parasitic resonance when setting up PIC.

REFERENCES

[1] Ya.S. Derbenev and R.P. Johnson, AIP Conf. Proc. 821 (AIP, Melville, NY, 2006), p. 420.
[2] Ya.S. Derbenev et al., arXiv:1205.3476v1 [physics.acc-ph].
[3] V.S. Morozov et al., AIP Conf. Proc. 1299 (AIP, Melville, NY, 2010), p. 664.
[4] J.A. Maloney et al., in Proc. PAC'11, MOP050.
[5] J.A. Maloney et al., in Proc. PAC'11, WEP074.
[6] V.S. Morozov et al., in Proc. PAC'11, MOP036.
[7] V.S. Morozov et al., in Proc. IPAC'11, WEPZ009.
[8] J.A. Maloney et al., "Studies of the twin-helix parametric-resonance ionization cooling channel with COSY Infinity", this conference, TUPPD011.
[9] G4beamline, http://g4beamline.muonsinc.com
[10] K. Makino and M. Berz, NIM A558, 346 (2005).